Satellite Attitude Dynamics and Control
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Lecture - 56
Control Moment Gyro (Contd.)

Welcome to the 56th lecture. So, this is about the gyrostat what we are going to discuss, but this gives us an important result which will we can use for everywhere in the satellite.

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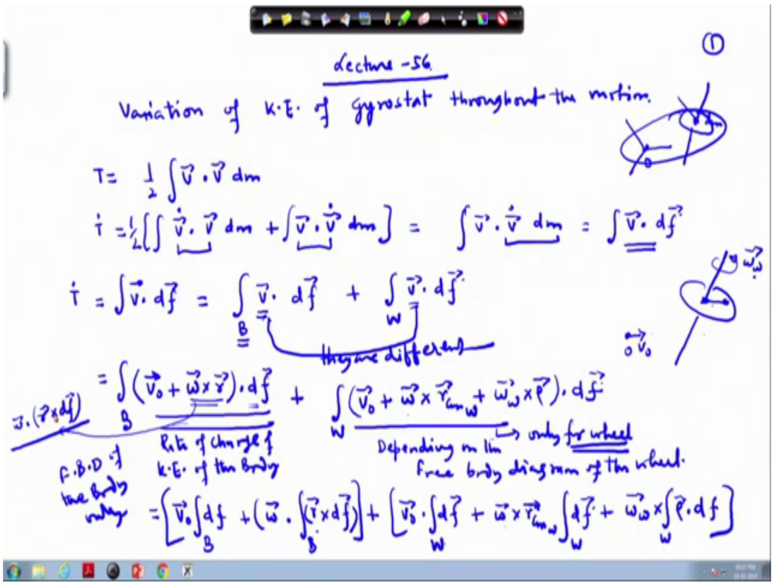

So, let us look into the kinetic energy expression this is given by v dot v dm. This earlier also we have done while we were discussing about the attitude dynamics of the satellite. So, T dot we can write as v dot v dm and this can be written as v dot v dm because the dot product is a commutative and therefore, we can write it like this 1 by 2 will vanish.

So, this quantity can be written as v dot v dot dm is nothing, but on an elementary particle dm, the force acting from Newton's law. So, T dot this becomes equal to v dot d f and this we have to break into two portions; one for the main body and another for the wheel. So, for the main body the velocity this is for the main body B it is written here. Accordingly this v and this v they will be different. So, they are different, they are different. So, this v is equal to v 0 if you recall from the gyro stat v 0 plus omega cross any point we are taking.

So, in the main body this is r and then dot d f which appears here, this is over the main body and plus in the same way we are writing here v 0 plus omega cross r c m wheel and then, we have written as with respect to the body axis there is omega s we are modelling the wheel separately. So, we have to look into the behaviour of the system in a different way. So, what we have done that this is the point o and this is the point where your wheel is located. So, here there the wheel is located. So, this is kinetic energy of the body rate of change of the kinetic energy of the body, rate of change of kinetic energy of the body and this is separately for wheel. This is only for wheel.

So, if we look for only for the wheel, so if we take the free body diagram of the wheel, so this wheel velocity of the point o is here known this is v o and this point velocity then will be obtained by multiplying by omega r c m. Now, what is the angular velocity of the wheel? So, here the angular velocity of the wheel as we have done for the gyrostat this is omega w means this is the absolute angular velocity of the wheel not omega s. So, here if we want to write it, so then we have to write it like this one in terms of omega.

Otherwise, first we write like omega and then expand it, write in terms of omega w and cross rho. This is how we have written and this dot d f. So, this is for the free depending on the free body diagram of the wheel, this is depending on the free body diagram of the wheel. Similarly, this is on the free body diagram of the body.

Now, we can work on this. So, v 0 is a quantity which is not dependent on integrand and this d f we can write it like this. This is over the body and similarly omega cross this is not dependent on integrand. So, this can come outside and this r dot d f, this is over the body. The same way we can do here v 0 dot d f, however this is on the wheel. This is over the wheel. The other term we will separate it out; omega cross r c m wheel. Again this quantity is not dependent on integrand. So, we can take it outside and write here this as d f and plus omega wheel cross rho dot d f.

So, these are the terms we are getting. Now one by one we can put it all the terms.

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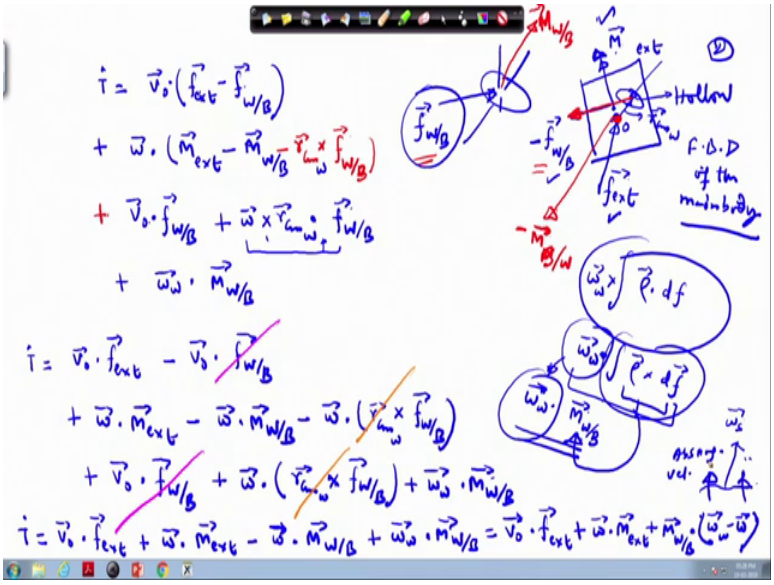

So, the first one; so, on the left hand side we have T dot and the first term is v 0 d f . So, v 0 and d f what is the force acting on the main body say here there is a hollow, this is a hollow which was used for this keeping the wheel rotor, ok. So, this is not there. So, this is the free body diagram, free body diagram of the main body. So, the force acting will be somewhere the f external where this is acting at the centre of mass or at the point o. If we take at the point o, this is the point o we have been referring.

And then there is a force also acting because of the wheel or the rotor. On the rotor we have assumed that there is a force acting on this f w B on the wheel due to the body. So, just opposite of this, it will act here in this direction f, f w B , but with a minus sign. So, how many forces are acting; f external here and this is the force acting here. So, as far as the force is concerned, so we can write as f external minus f w B plus the next term w cross r dot d f.

So, w cross r dot d f; w cross r dot d f, this term is together; remember this term is together. This is not dot product with d f first. It is omega cross r first on this you are operating by d f. So, we can exchange also this part and we will do the changes here itself to make it convenient. I do not need to carry it here. So, if we can change it, so it will look like this. So, this particular term we are writing as omega dot r cross d f. So, we have changed the order of the dot and cross product as per the property of the dot triple product, so that becomes omega dot and then, take this term r cross d f. So, r cross d f is what; r cross d f is the torque acting on the body.

So, what are the forces acting on the body that we have to write. So, in addition what we have assumed that there is an external torque acting on the body, this is M body, this is external or this is simply M external we have written. So, we can write this as this B we can draw because the external torque will act only on the body. So, we can write here M external, this is the external torque and what else the torque can arise? So, this; the f external we are showing it passing through this point o. Therefore, this does not have any moment here, but this will have. So, this will depend this force will have because here this is the r c m will is the radius vector. So, this part we can write as moment due to the two things and here this part is also there.

So, M what we are showing here. So, this M is the moment acting on the actually we should show it first on the wheel. So, if I show it on the wheel M, so here it should appear with a minus sign M w. This is we have shown as the external moment.. So, that we have shown as the external moment. So, leave it and let us show by some other direction I will choose say there is a torque acting in this direction here and just opposite to this there is a torque acting on the wheel here in this direction. So, M w B torque on the wheel due to the body. So, here this will be torque on the body. Due to the wheel this will come with a minus sign here. So, this total torque we have to take into account for this particular part. This is the torque term.

So, we need to take all that into account. So, then that becomes here M minus M w B the torque acting because of the wheel on due to the body and plus this because this is the offset. This force on the body due to the wheel this part this is offset from this point o and this distance is r c m, so that we have to take into account so that we can write here as r c m cross times f w B. The minus sign automatically comes from here. So, we are putting here then take the other terms.

So, next we go to this term v 0 d f. This is for the wheel. So, v 0 and d f what are the external forces what are the forces acting on the wheel? This is only this part. So, that makes it f w B here dot is there, this dot is there dot will be appearing. So, this is v 0 and then we have another term which is this part omega cross r c m w omega cross r c m wheel and then, this is dot here, this is dot and this is coming with d f.

So, accordingly we can write that term also. So, r c m and so, this is dot with the force acting on the body which is f w B because this integration is only over the body only over the wheel due to the body. So, this is f w B. So, f w B we are writing here in this place and in the next step we will change this order a you will write here dot and this will replace by cross and then, the last term we have last term we have omega cross rho dot d f omega w plus omega w cross rho d f. So, that term rho d f rho dot d f this is a what our last term is.

So, this also we need to express and what that term is. So, this we can modify little bit here itself and write this as rho cross d f and what this quantity will be. This quantity will be nothing, but M w B the moment acting on the this part is M w B and here w w dot. So, this is what we get here. So, the last part we can replace by omega w plus M w B. So, this part, this integration part we are replacing by this M w B and this is the separate, this omega dot part here this is coming here in this place.

And now let us check what are the terms; this term I need to rearrange. So,I will write it simply in the next step T dot equal to v dot f external minus v 0 dot f w B plus w dot M external minus M w B minus w dot r c m wheel. This is r c m wheel cross f w B plus v 0 and this part we are changing by writing it like omega cross r c m cross f w B and then the last term we have.

Now, see what are the terms which are cancelling, which are the terms? This, this get cancel out ok, then this term and this term they cancel out. Just other terms are not cancellable. So, now we can write the equation. So, we can write T dot this equal to v 0 times f external plus omega dot times f external and then, minus omega dot times M w B and plus omega w dot M w slash B. These two can be combined together. So, we can if we combine it, so we can write it as v 0 dot f external plus M external and then, plus M w B and dot in bracket we can write w w minus.

So, this is the absolute angular velocity of the wheel, this is the absolute angular velocity and this is just the angular velocity of the main body. So, if we subtract it, so this quantity together this gives me omega s.

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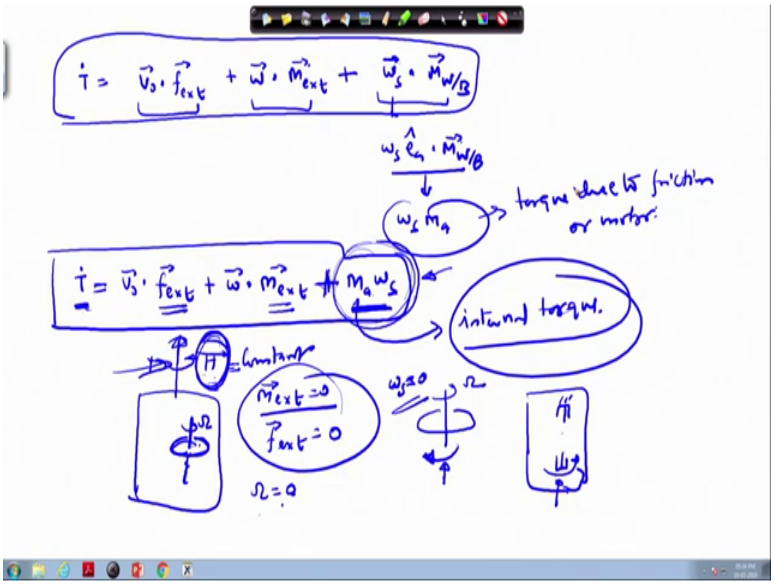

So, therefore, this equation can be written as T dot, this equal to v 0 dot f external and already we have written this term. This as you see; this term can be written as w s times e a cap dot M w B. So, what this term is? This term is omega s and this is nothing, but M a . So, omega s times M a.

So, this is torque due to friction or motor or maybe both. So, depending on the system, therefore T dot can be written as v 0 dot f external plus omega dot M external and plus M a times omega s. Here this M a is internal torque. So, we can see that in the kinetic energy term it is changing because of the internal torque. The angular momentum cannot change if you have a system here which is rotating at the say the H 0 is angular velocity and inside there is a rotating wheel. This wheel is rotating.

So, initially this wheel is rotating at the say capital omega and I is the angular momentum along this direction. So, this will have the associated this I is the its a moment of inertia. Let us remove this. So, this wheel is rotating about this axis and angular momentum of the satellite, this is H which is constant because it is free from the external torque M external.

This we are setting to 0 and f external also we are setting to 0. So, there is no external torque or the force acting on the system. So, in that situation h is wheel not going to change, but here what we will see that because of the friction if the wheel is rotating, it will slowly slowly come and it will come to a stop. So, this capital omega will ultimately become 0 as T becomes large.

So, obviously why it is a have, this is happening because of the if this is rotating about this axis. So, your wheel is rotating about this axis as capital omega. So, that means a friction torque is applied along this direction. So, and this implies that if this is the hollow or the say the mounting point inside the if this is the mounting point inside the satellite in which your desires are mounted, so here your wheel is mounted on this. So, obviously if the friction torque is acting here in this direction, so in this part friction torque will act like this. So, we can see that as a result of this friction torque.

These are the internal portion. So, H is not going to change because these are the internal forces and the torques; they will not affect your equation of motion, but they do a flip the rate at which the kinetic energy change. So, here the kinetic energy it is getting dissipated. The total kinetic energy of the system it is getting dissipated because of the friction and therefore, this will come to a stop. Ultimately, this is going to stop, but your angular momentum vector this is not going to change at any cost ok.

So, in your rate of change of kinetic energy if f external is 0, M external is 0. So, only thing that rate of kinetic energy, then it depends on this. So, if you are applying the deriving your energy from the motor, so M a times omega s that will come from the battery. If there is no motor, no battery applied there, so this will be just a dissipative term which will make the energy die out whatever the initial energy of the system was there.

So, M a times omega s because a omega s will be finally this will be set to 0. Once it is coming decelerating and becoming slowly to 0. So, this part will go down, but your angular velocity vector can never get affected. This is remaining in the same direction until unless there is external torque acting on the system.

So, this is a learning which we are going to utilize later on for the control moment gyros and this is very important. This part what we have written here, this part is very important. So, this way we have worked out the basics which were required for understanding the control moment gyros. Now, we can discuss about the control moment gyros, ok. So, we go to the next page.

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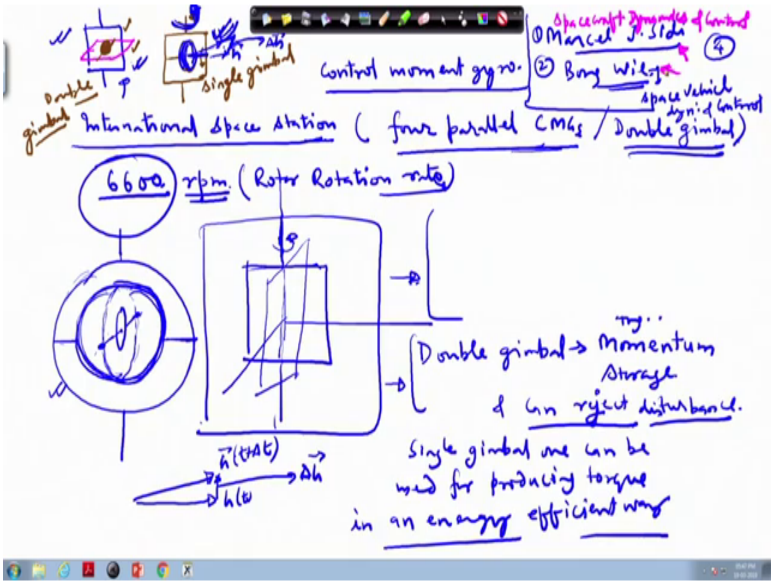

So, now we take the control moment gyros a flow full placed discussion of the control moment gyros and to be used in the control. So, I can discuss the dynamics here, but discussing the control part of this will be out of a scope of this course. So, we will discuss the control for using the control moment gyros only it is using, it is simplified format while I will do the rigorous derivation of the equation of motion for the satellite using control moment gyros. Say for the international space station, it is equipped with control moment gyros, and those are the four parallel parallaley mounted or the parallel c m gs. What is meant by parallel and other things, we will come due course of time and this is also double gimbal. Double gimbal already I have explained you that there is an outer gimbal and then, there is an inner gimbal and thereafter we have the wheel which is rotating about this axis.

So, this gimbal and this gimbal this together they form the double gimbal. If we have the single gimbal, that means I will have only this portion and a wheel is mounted on this and this wheel is then rotating and here this is free to rotate about this axis. So, this is single gimbal while this is double gimbal. So, the this c m g on the international space station it may rotate at the 6600 rpm ok; so, rotor rotation rate.

So, if; this is double gimbaled one, Therefore this is of this configuration and it is not necessary that this is in the form of the frame. It can look in some other portion say I can have a system where this is in the form of a spherical symmetry. This is a cell on which there is another inside which there is another cell and on this cell then from, obviously this is a cell means this is a spherical cell what I mean here. So, inside this cell there is a wheel. So, this is exactly the same thing, but here this is a spherical cell which I cannot show here.

So, inside a spherical cell there is another spherical cell which is gimbaled like this and then, inside this cell then you have say this is 1 diameter. So, this is another perpendicular to this, there is another diameter. So, from this end to this end; this is going. So, in this kind of frame this system will be easy to analyse because of the spherical symmetry, but this kind of system as it rotates, so the moment of inertia of the satellite will change. Right now as we can see that the moment of inertia about of the satellite if it is like this.

So, if it rotates about this, so this is moment of inertia about this axis will not change ok, but what about the order 2 axis, about this 2 axis? So, obviously once it becomes it changes its direction like this and finally, it becomes a line like this, we are seeing from the side. So, it will appear like a line. So, in that case the moment of inertia of the satellite has changed. So, this is going to affect. So, if you take into account all these thing, the system becomes very complex. So, we will try to keep the things as simple as possible and the single gimbal and the double gimbal they have their own purpose. Double gimbal one can be used for momentum storage, while the single gimbal and it can do the disturbance rejection.

So, double gimbal, double gimbal it can do momentum storage angular momentum; angular momentum and can reject disturbances and obviously, it can also be used for reorienting the satellite, but this will not be as energy efficient as the single gimbal one; this one. This is more of an energy efficient as compared to this, but the problem here is that it can activate because if a you are not a speeding up the rotor here, this rotor is remaining this rotor speed is not changing.

Therefore, you are just changing the angular momentum direction. This is the angular momentum direction you are changing by rotating about this axis. So, it can produce the torque once it rotates. So, for if you are rotating in this direction, so this will be the change in the angular momentum vector from this place to this place. So, this is your delta h. Iam showing like this is h and then it is going like this. So, this is another h this speed is not changing only the direction has changed. So, this is a change which is shown as delta h.

So, the single gimbal one can be used for producing torque in an energy efficient way ok. So, we have already finished the basic mathematics required for going into this topic and obviously the mathematics is also associated with this.

So, in the next lecture we will wind up. It is mathematics and also we will discuss about the various modalities. First we will discuss about the how they are acting, how they are able to activate in which direction they are producing torque. So, after we finish this, then there after we will take up the mathematics and work it. There after we will take a simple model for the control moment gyros and look into the satellite control problem, but not solving the control problem.

We will do the block diagram and get a qualitative understanding of the system, not a quantitative one because we need the Matlab and you have to be on the other end on the Matlab to do this. So, we will not do that. We will go through the basic requirement, so that you understand the basic control and then, you can refer the necessary book which is by Marcell J. Sidi. This is on the Spacecraft Dynamic. One book already the name I have given you and the another book related book by is by Bong Wie. This is also on the spacecraft dynamics, the book name; let me see that the exact name I have right now or not. It does not seem to be with me right now. So, next time ok.

This is space vehicle dynamics and dontrol. This is by; space vehicle, space vehicle dynamics and control; dynamics and control. So, you can look into this book and Marcell J. Sidi that book name is read only one l may be there a spacecraft dynamics and control. This book name is Spacecraft Dynamics and Control. So, for the control issues you can look in to this book and some system dynamics has also been discussed.

But it is not that elaborate. The understanding of the; now, once we have come to the control moment gyroscope, we will see that the very fine issues associated with the rotation of the body and how to write the equation of motion for the system andthat is very important. This is what we are going to discuss that will be the most important part in learning the system dynamics of the rigid body.

So, thank you very much for listening and next time we will continue.