**Satellite Attitude Dynamics and Control**

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**Lecture - 55**
**Control Moment Gyro**

Welcome to the lecture number 55. We have already finished the gyro state, the satellite with a rotating wheel or one part of the satellite may be rotating. So, that we have called at the gyro state. Now the satellite with the control moment gyros; so, this topic we are going to discuss, but before that we will discuss some of the mathematics from the previous lecture means about the gyro state and that is very important for deriving the relationship for of the satellite dynamics, under the motion of control moment gyros.

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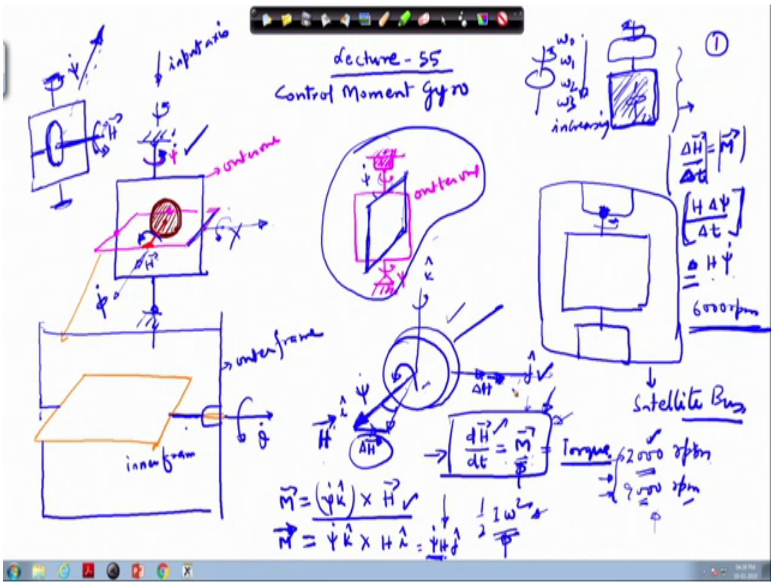


So, usually the control moment gyros pa earlier also I have talked about this. So, control moment gyros it looks like we have a say frame here, and this we call as the gimbal axis for this frame, then we have another frame -and there is a wheel what we call as the rotor and this is the rotor -and from the centre of this rotor, this conduction goes back here and comes here in this place.

So, this is another gimbal. So, this place we have; so, at this point this is gimbal and here the gimbal is at this point and toward the back here and where I am showing this by a pink dot. So, this gimbal is basically, if you look into here in this gimbal. So, it may look something like this, say this is the outer one. So, this is the outer one.

So, there may be a motor attached to this instead of here looking like this, it may look like this and this axis may look some like (Refer Time: 03:18). So, this motor may be responsible for rotating it or this motor maybe not here, but this motor may be fixed to the this support only and this is attached to like this. So, as this motor is rotating. So, this will rotate about this axis and this rotates by psi here. So, this motion we are showing by psi and then there is also a motor about this point ok. So, if you look for the inner one which I have shown here by the pink colour and I will show the same thing here in this place.

So, this is usually fixed like this, say if I instead of joining it here if I do it like this and take the outer frame like this and to this outer frame there is a motor attached to this, then this is this axel and this axel it is going like this. So, imagine this kind of situation ok. So, this is your the outer one outer frame, and this is the inner frame and once this motor rotates. So, this will also rotate and it will suppose it rotates about this axis. So, this will rotate.

And this rotation we show it by theta and similarly your wheel is rotating. So, this wheel also the rotation of the wheel can be shown like this. So, this is rotating here y on this we can show it by phi ok. So, theta dot will be here in this direction phi dot will be here in this direction and a psi dot will be here in this direction and phi dot will be here. The question arises the what is the benefit of using such a control moment gyros on a satellite.

So, thus say we have a satellite here and if from the ceiling of the satellite also this we call as the satellite bus. So, if I join this outer frame here and there is a motor here in this place itself. So, you can assume that this is rotating about this. So, what we can see that has a most simple case if I have this rotor. This is a rotor and if this is rotating about this axis and let us say that I show this rotor angular momentum by H.

So, instead of speeding up this rotor what we do that, I rotate it about this axis I rotate outer frame about this axis. Outer frame cannot rotate about any other axis because only the freedom is given about this axis. So, if it rotates means this H vector. So, using the motor in this place if using this motor if I rotate this frame from this position to this frame position, it goes like this and this rotation here is by psi. So, this vector will change its direction this is the angular momentum vector. So, it will change its direction from this place to this place. So, this change of momentum, it lies in the plane of the wheel ok. So, this is delta H. So, this delta H will lie here in this plane.

So, as a consequence of the rotation about this axis, we are able to generate a change in momentum which is given by delta H and this delta H is the torque generated which acts along this axis. Because you know that d H by d t we have learnt that this we are writing as M, the rate of change of angular momentum is given as a torque this equal to torque. But this is not the only way of producing the torque here if I apply this method that I change this H instead of rotating this angular vector from this position to this position, I start increasing the magnitude of this H means if I start building up suppose.

So, I can produce a torque,if I change this; so, a torque will be produced according to this scheme if you apply M. So, this will change d H by d t isn'tit? So, this will change, but here in this case what happens and as a result you also get the reaction, but the problem with this method, we which is also applied for the reaction wheels which pa we have called as a gyro state also. In the gyro state especially there is only one rotating wheel either inside or it may be the outer part maybe rotating here this part; there may be a wheel, which is outside the satellite and this wheel is rotating and this is the stabilize platform, this is the non rotating part.

So this mechanism we are going to discuss the control issues once we are through with the theory. So, this is the last portion of the theory and another one is remaining this magnetic attitude control along with Lorentz force these are the only two things remaining and once e we are finishing it. So, we will take few hours for covering the control issues and the related mechanism. So, what is the harm here in doing this? The problem is that you cannot apply last torque using this method and moreover the energy is spent in this method using the applying the torque and speeding up the wheel energy is lost here in speeding up the wheel.

Your energy is getting lost because the initially say the wheel is rotating at 2000 r p m or 6000 rpm something like this and then you are speeding up to the 9000 r p m. So, you are accelerating it. And as a consequence of this you get the reaction on the satellite and satellite gets reoriented along that particular axis; however, in doing so, what we are doing that this wheel is speeding up from this speed to this speed.

So, you know that the kinetic energy is given by 1 by 2 I omega square so that means, your energy whatever the energy you have supplied to a speed up the wheel that is getting lost in this wheel rotation, because it has increased from 2000 to 9000 rpms. So, this method of control this not only produces a small torque, but also this consumes energy. So, to overcome this problem, it is suggested that instead of doing this, you do the other part that is use this H equal to M equal to if I rotate this vector at the rate say here its rotating at the rate psi dot and psi dot is the vector in this direction in the k direction.

So, psi dot k cap this becomes omega and cross H. So, here your delta H is the change in the torque if I divided by delta t. So, that gives you the torque generated is not it the rate of change of angular momentum is the torque and what is delta H? Delta H is the quantity that we can write a delta H times this length, this length times this angle. So, delta H times delta psi. So, this is delta H by magnitude and if I divided by delta t. So, what we get magnitude wise you can see from this place that we get as this delta H is equal to H 0 times delta psi or H times delta psi.

So, what we see that this becomes H times delta psi is psi dot ok. So, this is what we are I am writing here in this place. Now look from this place psi dot k and cross H, H is suppose this is ;i if I assume this to be the k direction. So, this is i direction and let us say this is the j direction.

So, what we can see the H is H times i cap and that becomes psi dot H times k times psi is nothing, but j cap. So, this is a torque along the j direction. So, this is magnitude wise this is what you are getting if we look magnitude wise. So, magnitude wise this is what we ge; H times psi dot. So, H H times psi dot is also here in this place. So, this implies here in this case I am not imparting any is speed, I am not speeding up the this rotor only by applying some small torque, I am rotating it from this position to this position and as a result.

So, we forget about this because we are discussing about this figure. So, this is not concerned with this, this is only we are talking about the rotor. So, here we can take up this figure. So, once we suppose we are not rotating here in this place, this is not being rotated what we are doing that we are just rotating about this. So, the angular momentum vector which is along this direction. So, this will rotate from this place to this place. This is the torque that gets generated here M and which is acting along the j direction.

So, and for rotating it by psi dot we apply a small torque about this axis ok. So, the applied the input axis is this we call in that case is the input axis. So, as a result your output is along this direction. So, torque will be available along this direction. So, this way of producing a torque using a rotor wheel without is speeding it up; so, this we say as a control moment gyros in the reaction wheel we speed up the wheel itself.

Here initially this is omega 0 later on it will be omega 1 after wards its omega 2, omega 3. So, a speed is increasing up here in this direction, it is increasing, while here the speed I am not increasing. So, using this property of the control moment gyros large torque can be produced. Suppose if h is very large in magnitude a wheel which is rotating at some 6000 rpm ok. If the wheel is rotating at this speed and this is also a massive wheel ok. So, you can see that H will be quite high and then by applying certain psi dot about here in this place, we will be able to if we apply this psi dot we will be able to generate a large torque along the j axis.

So, about the j axis the torque is produced, and that torque will be transferred to the satellite. Similarly if we apply a torque now say it depends on how many gimbals you have. So, let us consider the case that we have only one gimbal. This is the only one gimbal this is rotating about this axis and then you are rotating about this axis by psi dot ok. So, if this wheel is a spinning about this axis, this is the axis about which it is spinning. So, H here is in this direction. So, you consider this H will go inside and the torque produced will be along this direction because this frame is fixed in the satellite.

So, in that case a torque will be directly produced along this axis and that torque is being tolerated by the satellite. So, the satellite orientation will start changing.

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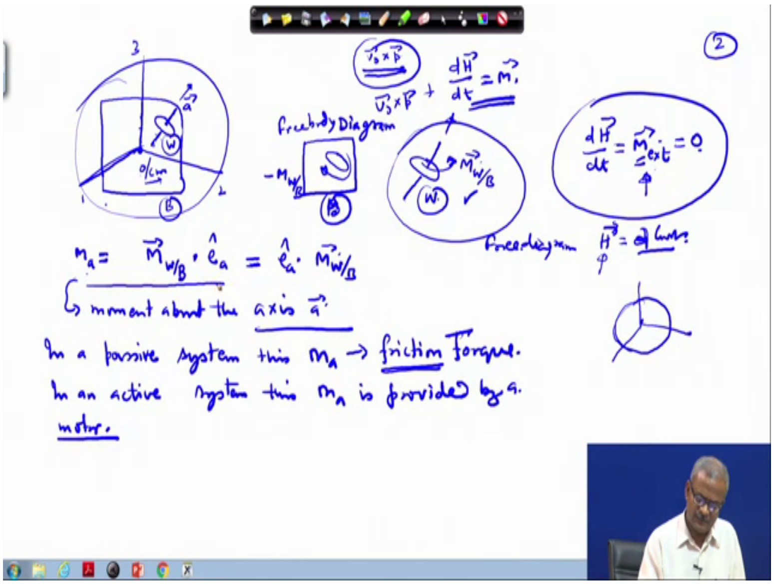


So, this type of technique, it is a used for large satellite like the space station and others where a small wheels will not work and this is also energy efficient as we I have told you that here you are not speeding up the wheel. So, energy is not getting lost in speeding of the wheel only a small energy we have to spend about this axis and as a result you are getting the last torque about the other axis. And therefore, this system is energy efficient while the reaction wheel system is not energy efficient with because it is consuming energy in speeding up itself.

So we will discuss up the mechanism of this control moment gyros further. So, this is just an introduction and what today I want to do that we have been discussing about the mm gyro state. So, in that context we were discussing few concepts and I told you that either I will set this as an assignment or I will give you as an supplementary material; but I want to discuss it today itself so, that that concept can be utilised for the control moment and gyros also.

So, in a satellite, this is 1 2 and the 3 axis and somewhere a rotor wheel was there and if rotor wheel is aligned with some of the axis means the rotor instead of rotor will be here in this place if the rotor wheel is located on this axis itself, then the things gets simplified and o is also the this is the centre of mass this point. So, that in the moment equation the V 0 cross p term we were getting that will be vanishing and your equation of motion then looks like d H by d t this equal to M. Otherwise we have the extra term V 0 cross p if o does not coincide with the centre of mass.

So, in this context, I wanted to discuss few things. Let us write; so, what we have earlier done that we have a rotor and this is the a axis in this direction and its rotating about this axis. So, for this, the torque about the a axis we write as M a and suppose we make the free body diagram of the this satellite. So, free body diagram here there is a hollow, this is a hollow and your rotor comes here in this place and how we have modelled? We have applied a torque on this M w on the wheel due to the main body.

So, this is the main body and this is the wheel. So, here this is the main body B and this is your wheel. So, if we consider independently this part. So, we can write M w B, M a the torque along this axis that we can write as e a cap and it is the same as e a dot a times M w B and the same torque also if this is acting here in this direction. So, this wheel here an opposite torque will be applied here in this place. So, M w B will act on the satellite, but these are the internal torques and the forces. So, they occur in there and they cancel each other. So, therefore, such a system if no external force is acting on this system or external torque is acting on this system.

So, once we write the equation of motion. So, this is d H by d t equal to M. If M this is the external one, this is 0. So, only we get here H has a constant. So, for such kind of system the total angular momentum will remain constant if there is no external torque, but in the case of the satellite this does not happen because there will be a torque from the aerodynamic forces, torque due to the solar radiation there can be; however, those things can be model as a periodic and constant terms.

So, those are different issues, then if it is free from the torque then we have the gravity gradient torque ok. So, the M external if in the; it is in the lower orbit. So, you cannot avoid it gravity gradient torque will be there the only way to avoid it is that, the satellite is a spherical in the same in that case you will not have any gravity gradient torque.

So, we are discussing about this part. So, this is M w B this is the moment about the axis a. In a passive system this M a will be the friction force or the friction torque for a passive system means there is no motor inside this or motor is switched off. So, whatever the friction is acting between the bearing and this axel; that is the torque available. So, that is the friction torque. In an active system this M a is provided by a motor.

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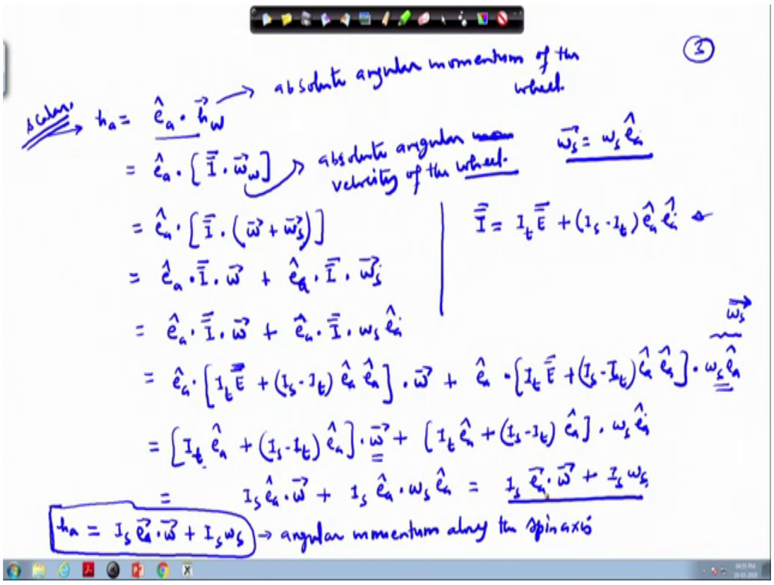

So, h a we can write as also e a cap times angular momentum of the wheel; remember this is the absolute angular momentum, absolute angular momentum of the wheel. Because we are discussing about this figure this is the free body diagram, this is the free body diagram.  This is also a free body diagram.

So, we can write it in this way and as we know that this h w can be written as  this is the absolute angular momentum; absolute angular momentum of the wheel. Absolute angular velocity of the wheel and I is obviously, the inertia dyadic. So, this omega speed of the wheel we have written as  this we can expand and omega is been as we know this quantity is nothing, but omega s times  e a cap as per our earlier discussion.

So, this gets reduced to; Moreover if you remember that in your assignment also this appeared this I double r this is the inertia dyadic; this we have written as I t times  E double bar this unit dyadic  plus I s minus I t times  e a cap e a cap. This is the inertia; moment of inertia of the wheel or the more inertia dyadic of the wheel.

Now, if we insert here in this place, see why I am doing this exercise because the satellite control any way many people from the electrical engineering they may be working, but understanding the physics of the system it is a totally different. You are given as equation and you know about the controls, you can apply to the system and you can produce the result; this is happening that is happening, but without much knowledge about this, behaviour of the system. This behaviour of the system until and unless we go through the physics and the proper mathematics in a systematic way, we never learn the system behaviour.

So, there is a difference between doing the controls of the satellite somebody is doing the controls of the air craft, but no knowledge of the aircraft system or either how the aircraft behaves, it is a just given that these are the (Refer Time: 29:58) and say the elevator, the these are the three primary control surfaces on the aircraft. So, you can use them to control the aircraft. Somebody can give the control engineer one equation; ok do this and this is the result I am looking for; your air craft should behave like this.

So, the control person he is not knowing about much about the system dynamics about the aircraft behaviour how it is designed, what are the factors which will affect it. So, only through the person who is designing the aircraft who is aware of the aircraft dynamics and also he is aware of controls he can do such things that the aircraft design my aircraft will behave like this, if this is done.

So, it is very important that we learn the dynamics of the systems and then we do to the control. Here this in this course we are not going to do the controls any more extensively because we have only two weeks late left including this we are running for the last 11th week and then there will be one more week. So, what I will try to cover some of the control issues, the control block diagram, how to do that and there after the problems are to be solved using the MATLAB; we cannot do using the there this hand calculation.

So, there is no meaning to discuss the proper controls and all those things, and seeing the number of letters limited. So, we will confine our self to the certain ideas which we get from the block diagram and how the system is going to behave.

So, in that context I am working this so that you understand what we are really doing. So, here this we can write as can see from this place, this is the unit I t. So, we get here I t times e a cap plus I s minus I t times e a cap dot e a cap this becomes one, this is e a cap and then dot omega plus the; Same thing we have to do here in this place but on this side here we are missing this part omega s times e a cap.

While here it is omega here in this place this is quantity here is nothing, but omega s vector. So, doing the same exercise here we get here, It times e a cap plus is minus It times e a cap dot e a cap becomes one times omega s times e a cap.

So, as we see it this is I t and I t here. So, that term cancels out and we get I s times e a cap dot omega and plus similarly here in this place we get here I s times e a cap dot omega s times ea cap. So, this gives you I s times omega s. So, therefore, this is here we have to do the modification, this is we are taking the dot product and therefore, this is the scalar here. So, therefore, our h a this becomes I s e a dot omega plus I s times omega s, this is angular momentum along the spin axis.

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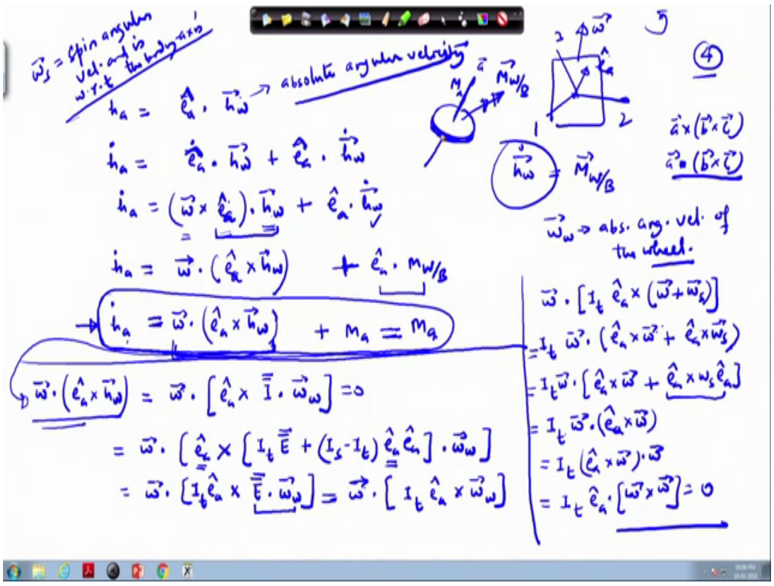


Now this h a we are writing as e a dot h w. So, h a dot this quantity we can write as; differentiating this, and this quantity is nothing, but omega cross e a cap, this is cap here, let us write this as cap we are using cap here.

I want to tell you something very important that is what I am doing this exercises. You will come to know, what the, and remember what we are doing here. So, e a is a vector, e a cap is a vector inside the satellite, and this satellite this is the 1 2 and 3 axis and this satellite is rotating at the angular velocity of omega. Therefore, this e a vector will. So, e a dot e a cap dot; it is a given by omega plus e a, this is the standard thing we have already done.

Now, h a dot; so, there we can write as omega cross e cap and this is I double bar dot omega w. This is also a vector and what this quantity will be? H w dot; hw dot this is nothing, but the torque acting on the wheel which is coming from the body M w B. So, this quantity we can replace this as M w B in the next step we do little bit modification.

So, we could have done at this stage itself rather than expanding let me remove this and first do that part. So, because this is a triple dot product; we call as vector triple product. So, vector triple product is different for a cross b cross c. This we call as the vector triple product, and dot triple product we write as a dot b cross c. So, for using this particular notation e a cap cross h w which change. This is the property of the dot product.

So, we put it here in this format. So, this quantity then this is if you look back, we have defined this as the M w B dot e a cap equal to M a. So, we replace this as M a. This is a scalar and this is h a dot. And here we get omega dot e a cap cross h w. Now this quantity we have to work on and see how much this quantity turns out to be.

So, we have omega dot e a cap cross I double bar dot h w is nothing but I double bar dot w w means the absolute angular velocity of the wheel; Absolute angular velocity of the wheel. We need to expand this part, now, we need to work out both these parts separately. So, this is omega dot e a cross I t somewhere let us write here in this place this is I t; once we expand it . So, this is e double bar dot omega w and plus e a cap cross e a crap this part will drop out, this is the cross product here.

So, this will be 0; so, that part drops out. So, we dropped at here itself. So, this gets reduced into this format. Now this is the unit adding. So, this term get reduced as omega dot times I t times e a cap cross omega w and we need to work out this part. So, we are just looking at this particular part which is written here in this place, omega w is nothing but omega plus omega s.

And therefore, omega dot I t we take it outside, this becomes e a cap cross omega and plus e a cap cross omega s. And e a cap cross omega s we have written as times e a cap because this is the angular velocity vector along the a axis. This is with respect to the body axis. This omega s is the spin angular velocity. So, this is spin angular velocity and is with respect to the body axis.

So, therefore, this part will be 0 and what we did here the I t times omega dot e a cap cross omega, and you know that we can write this as e a cap cross omega dot omega this can be exchanged and thereafter we do a final change here, omega cross omega . So, this quantity is 0; so that means, this quantity this turns out to be 0. So, this implies this quantity s dot becomes equal to M a.

So, M a is the torque applied along the a axis. So, this is the a axis and total torque suppose this is applied here, it lies here in this direction, this is the torque M w B which also, I have shown it like this, but in the vector notation you can show it like this . So, its component along this direction this is M a.

So, that M a is responsible for changing the angular momentum of the; this rotor about this a axis. So, this results will be important while we discuss further. So, we will continue in the next lecture.