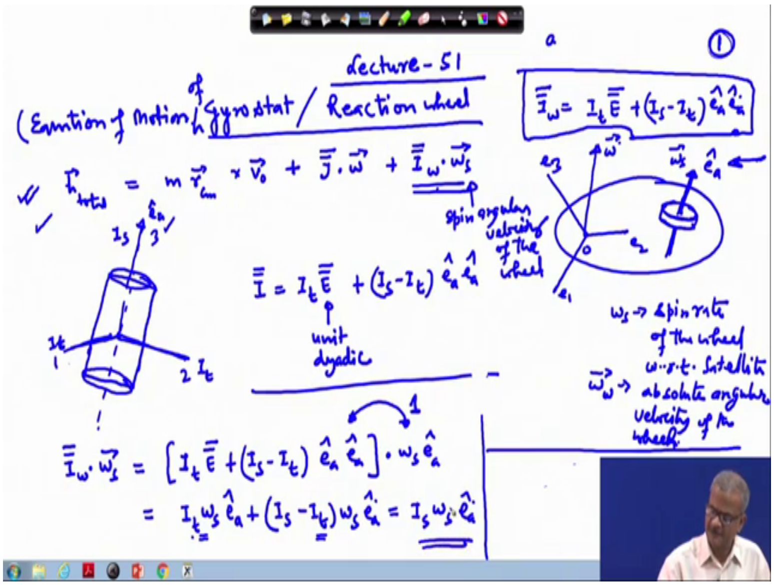
Satellite Attitude Dynamics and Control

Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture – 51
Gyrostat

Welcome to the lecture number 51. So, we have been working with the Gyrostat or the reaction wheel actuated satellite. So, in that context we were deriving the equation of motion. So, till now we have worked out the angular momentum of such kind of system.

(Refer Slide Time: 00:34)



So, today we will be working for the equation of motion of the system; equation of motion of Gyrostat or reaction wheel. So, this was the equation that we have derived for the satellite which consists of reaction wheel or the rotating wheel ok. So, here a direction is shown that a direction is nothing about the axis of the this rotating wheel and omega s this is the spin rate of the wheel with respect to the satellite. And omega w we have written as the absolute angular velocity of the wheel or the rotor and a is a unit vector here so a or we can write this as e a cap.

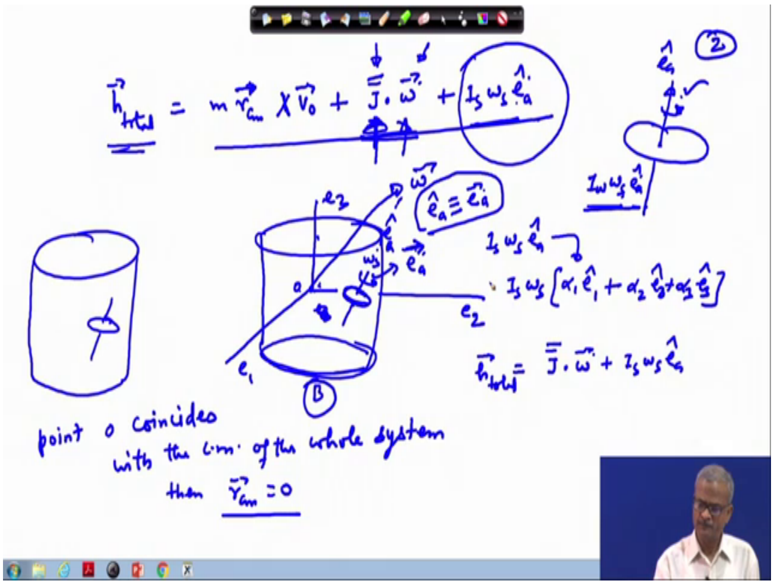
So, in the last assignment we have posted one solution to a problem which gives you this result. That if we have a system like a cylinder or a disc which has initial symmetry means if this is say these are the 2 axis is 3 axis here, 1 2 and the 3 axis for this wheel and this has got symmetry about this axis ok. So, this is the initial symmetry means this 2 inertia will be the same. So, in that case we can write this as I t inertia along this direction, I t inertia along this direction and I s is the inertia along this direction.

So, then the inertia of this wheel can be written as I t times E double bar where this is the unit dyadic plus is minus I t times ea cap times ea cap, where ea cap is the vector along this direction here as it is shown in this place ok. So, this I have solved in that assignment solution to that assignment. So, I am not working on this here. Now given this system we need to find out the equation of motion for this system.

So, before we do this, just we want to simplify this particular term. So, this term we can write as I t times E double bar. So, omega s and omega cap they are along the same direction ok. This is the spin angular velocity, this is the spin angular velocity of the wheel. So, therefore, this omega s instead of writing it like this we can write in terms of this and the unit vector along the e a direction and then just simplify this. So, if we simplify this gets reduced to it times omega s and here and we can see this dot product with the unit dyadic.

This will held the same vector on this I s minus I t times again here omega s and e a dot e a that will be equal to 1 that is the unit vector. So, this product dot product will be 1 and therefore, we get this as omega s times e a cap. So, you can say that it term is present here and also present here. So, that term cancels out and we get here highest times omega s times e a cap. So, your this equation then it gets reduced to I double bar equal to I t times we are trying to reduce this equation not this one. So, we do not have a space here from the next page we will write here, write this equation.

(Refer Slide Time: 06:22)



So, h total is m r cm cross b where m is the total mass r cm is the centre of mass of the whole system, cross V 0 plus J double bar this is the combined moment of inertia J 1 this particular part and the other part then this can be reduced during this I s times omega s I s times omega s times e a cap. So, this part is nothing, but the wheels angular momentum, this wheel in angular momentum along its particular axis e a cap. So, I s times omega s is the corresponding angular momentum about this axis and if we multiply by this e a cap. So, this gets into a vector format and this is the main body, this is the main body angular momentum together. So, what we are getting that, if we have any satellite and inside this there is a wheel.

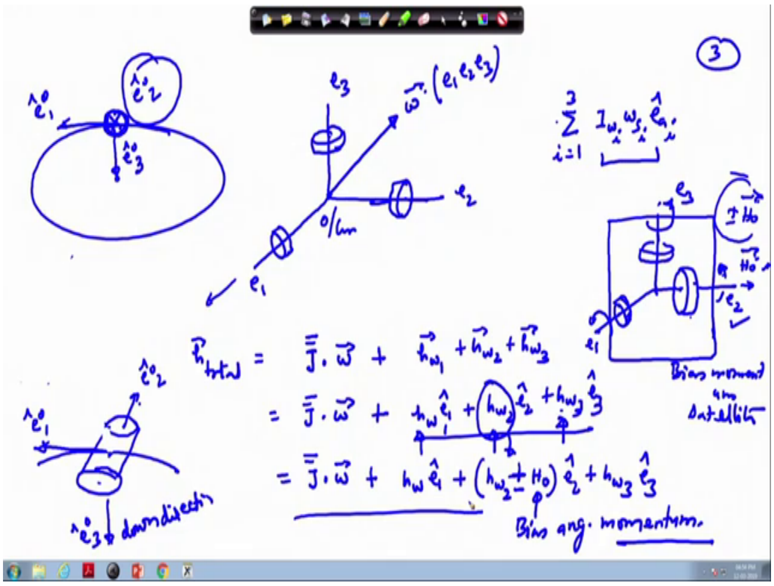
So the complete moment of inertia and unit vector along this direction is e a and let us say that e 1 e 2 and e 3 these are the 3 mutually perpendicular direction, this is e 2 and o is located here ok. So, in that case this is not necessarily the centre of mass as per this one ok. So, what we need to do that assuming that this wheel is not rotating. So, this constitutes one single system this whole thing along with this wheel and wheel is not rotating. So, for what will be the moment of inertia of this complete system? It gets into this place then what is the angular velocity of this main body which we have earlier written as B? So, take dot product with that, there after if this wheel is rotating so at the angular momentum for that. So, this wheel is rotating with respect to this body has the angular speed omega s. So, I s times omega s times e a cap that becomes extra angular momentum.

Obviously we will have to convert this whole thing in terms of the body access. So, let us say this is alpha 1 or alpha 1 e a e 1 cap, this can be this unit vector can be written in terms of e 1 e 2 e 3 ok. If its r because it has a vector is having some arbitrary direction this is the a vector here in this direction this is the a vector direction. So, its arbitrary and therefore, this ea cap or ea cap as I write it like this or e a it indicates the same thing its the same vector they are identical.

So, this way your equation angular momentum equation it gets simplified. Now here V 0 if the point o coincides with the centre of mass of the whole system then rcm this will be equal to 0, and therefore, our h total in that case this gets reduced to m the first part gets dropped out, we have only the second part.

And now we can see that if we have multiple wheels say we have a situation we can go to the next page.

(Refer Slide Time: 11:07)



We have a situation where this is e 1 this is e 2 e 3 this point is o. So, one wheel is rotating along this axis another wheel is rotating along this axis, another wheel is rotating along this axis ok. So, in that case the last term that we are writing I w times omega s times e a cap, and b if we have multiple wheels ok. So, we need to add for all of them and say this is the it th wheel and that is and it is a corresponding a spin rate is omega s or omega i and corresponding e a direction is e a i ok. So, if we sum it over 1 to 3 if you have 3 wheels.

So, I could have a started solving this problem right at this stage assuming this point to be the centre of mass, and I could have done the whole thing and that would have been much easier for me to show the results for you. But a systematic approach is what we have discussed till now and this is much more generalized result what we have written ok. So, if we take into account this. So, h total this becomes m times r c m and if o is the centre of mass itself. So, let us drop this term if we drop this term.

So, we get here J double dot omega, where omega is the angular velocity of this body axis e 1 e 2 e 3 this is related to e 1 e 2 e 3 and then you have the corresponding term here in this place. So, for each of them let us write this has h wheel 1 plus h wheel 2 plus h wheel 3; obviously, we can write this as if they are aligned along the this body axis ok. So, this become gets very much simplified and we can write this as e 1 cap h w 2 e 2 cap plus h w 3 e 3 cap. So, these are for the 3 rules.

Sometimes it may happened that one of the wheels like if you are going for doing the spin stabilization of the satellite. So, in the orbit the satellite is going. So, this is your e o 1 direction and e o cap 2 which is going inside the page. So, I am not showing it here. So, this is your e o cap 2 and this is toward the centre of the earth ok. So, this is your e o cap 3 and your satellite is spinning ok.

So, we can set the satellite to a spin. So, if along the one way of already for the spin stabilization what we have seen that, we can spin the satellite about the y 2 axis at this axis and therefore, we can get the corresponding directional stability for this axis ok.

However if we have the satellite here in this place instead of this here we have the satellite which looks like, say this is going in the orbit. So, it will appear something like this. So, this is your e o 1 direction and along this direction then e cap o 2 is coming out and vertical and toward the centre of the earth this toward the centre of the earth we have; then towards the centre of the earth it will be shown down, it will be shown down this is the down direction. So, e o 3 cap.

So, sometimes there may be a wheel and say these are the 2 faces of the wheel which we are seeing here and this is one direction, this is 2 direction and this is 3 direction ok. So, in this case the way it has been shown 3 direction turns out to be a different one 1 and 2, 1 and 3 they turn out to be equal. So, one and this is the one and the perpendicular 1 this is 3. So, we can show it like this one and then say what I am trying to show you here that inside the satellite in a particular direction I can set the wheel rotating, say along this direction this is the e 2 directions of I set the wheel rotating right in the beginning ok.

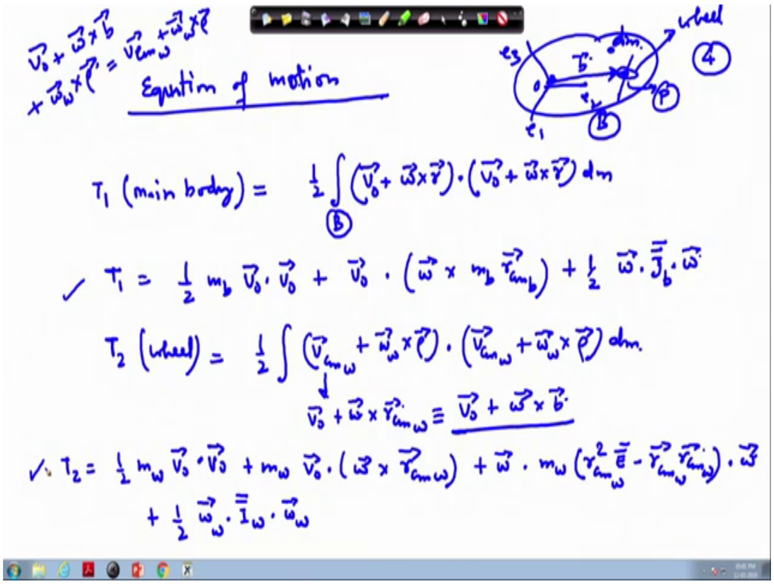
So, this is the anticlockwise rotation and we can write this as the H 0 and depending on if this is anticlockwise this will be H 0 if this is clockwise then this will be minus H 0 the vector will be in the opposite direction. So, this can be plus minus H 0 depending on the rotation ok. And if this happens that other wheels are not rotating, but one wheel you have rotated for giving some bias momentum to the satellite ok. So, in that case we call this as the bias momentum satellite; bias momentum satellite.

So, here in this small figure I am not able to make it, but this is the whole purpose that depending on this the previous rotation direction, we can have this H 0 either plus or either minus ok. There after you a speed up the wheel along the e 2 direction you are a speeding up along the e 1 direction similarly you are speeding it up along the e 3 direction. So, accordingly you will also get this terms ok. Because then the angular velocity will be along this direction for the corresponding wheels and accordingly you get all this terms here ok.

So, we will return back to this again and let me complete this part ok. So, h w times e 1 cap. So, this part instead of writing it like this, this is often written as h w 2 and if you have the bias momentum say accordingly H 0 e 2 cap plus h w 3 times a 3 cap. So, you can see that I have put here plus minus to indicate, it depends on in which direction of the e 2 axis e 2 axis you are h 0 this vector is the bias momentum is directed.

So, this has been converted to this from where this is the bias angular momentum. So, we will return back to this equation again before right now we go for doing the deriving the equation of motion.

(Refer Slide Time: 20:27)



So, we now derive the equation of motion. Kinetic energy also we can write here. So, let me summarise just the kinetic energy term you can try to derive yourself ok. So, here kinetic energy for the main body this is for main body, I am just writing the result here this will be given by 1 by 2 integration over the whole this main body V 0 plus omega cross r, r is any point in the body.

So, e 1 e 2 e 3 already we have referred to this figure. So, any mass here d m and this is a body B we are not considering at this time the wheel here, wheel we have to add separately for that. So, this dot always right your velocity term like this while you are calculating kinetic energy in terms of dot product.

Otherwise you are bond to do mistake and if you expand it and combine the terms. So, this will look like m b times centre of mass of the main body plus 1 by 2 times moment of inertia of the main body, I will try to scan some of the pages and provide you a supplementary material.

So, you try to work out this portion yourself and you will find these things later on as a supplementary material similarly t T 2 this is for the wheel.

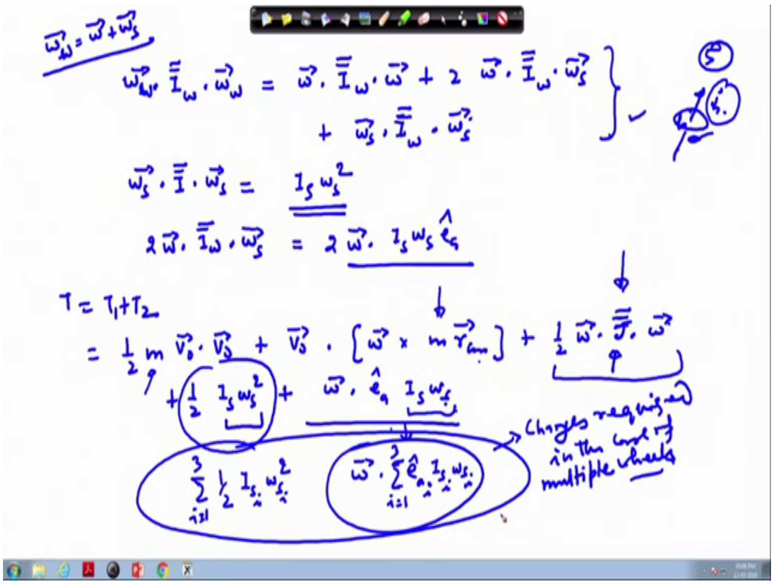
So, that will be V 0 plus the location of the wheel is here. So, from this point to this point this vector is b which is indicating the location of the centre of mass of the wheel ok. So, this is your point we have earlier taken as, I do not remember let us write this as this is the point p and this is your wheel. So, either I can break it and write it here like the V 0 plus the velocity of this point with, this is the velocity of the point o velocity of this point with respect to this. So, that will be omega cross b and plus now any point on the wheel itself will have certain velocity with respect to the centre of mass.

So, far that if omega is the angular velocity of the wheel in the initial frame omega w which we are writing the absolute angular velocity as per our earlier notation rho is any point. So, we need to write it like this and the same thing can be also written as v centre of mass of the wheel, velocity of the centre of mass of the wheel plus omega w plus rho ok. So, both way we can write. So, let us make it compact here first and then we can omega centre, velocity of centre of mass of the wheel and plus omega w plus rho dot and d m.

Now, this quantity has already written above this quantity is nothing, but V 0 plus omega cross centre of mass of the wheel which we are writing as V means this is nothing, but your omega cross b; b is the vector form here to the centre of mass of the wheel ok. Then we need to expand and work with this we expand and work with this, this gets reduced to. So, T 2 gets reduced to 1 by 2 m wheel times V 0 dot V 0 centre of mass of the wheel plus omega dot if we write in terms of V. So, we do not have to carry the centre of mass of the wheel.

So, that will be little easier where all the terms have their usual notations as per our earlier discussion. Now we can add this T 1 and T 2 combine this terms.

(Refer Slide Time: 27:22)



So, its a little time taking, but you can always do by combining all of them ok. And especially your term like omega w dot I double bar w dot omega w, this can be broken up in terms of the angular velocity of the main body and spin rate of the wheel where we have use the information that omega w, this equal to omega plus omega s ok. Inserted here and expanded it and then combine the terms to get this equation finally, here the term omega s.

So, I am going to post this particular material ok. So, this can be written as I s times omega s square ok, but try on your own also ok. So, these are the way this can be written. So, total T will be T 1 plus T 2 and if we had and combine all these terms. So, we get this equation one times and V 0 V 0 dot, this is the total marks and rcm, this is the total rcm for the whole system, this is the total moment of inertia ok. If you have multiple wheels so these are not going to change because in this capital J what we have written here this is the moment of inertia of the whole system. So, here all wheels will be accounted in this one, here m this will include mass of all the wheels. So, nothing is going to change here, nothing is going to change here. So, this equations they remain as it is this part, where it will change? Here instead of a single wheel then we will have multiple wheel.

So, this term then we are going to replace this by summation 1 by 2 this is the spin rate of the wheel all the wheels can be rotated at different speed with respect to the satellite and that is required in order to control the system. So, basically in controlling what is important that the rate of change of angular momentum of the wheel about its own axis. So, this part is used for controlling this the satellite altitude, very soon we will get into the control topics also although we have less number of lectures, but as far as possible I will try to cover, similarly here is omega is appearing. So, this part also we can write as omega dot then e cap a i this is along one particular axis.

So, you will be putting along 3 orthogonal directions. So, for that it is standing here similarly this is e s i omega s i. So, you have to sum over i equal to 1 to 3; i equal to 1 to 3 for the 3 wheels ok. So, these are the changes required for multiple wheels, required in the case of multiple wheels ok. So, we have stop here and in the next class we are going to discuss about the equation of motion of the motion of the system, which will be involving the torque acting on the system. Though I wanted to finish the torque part here, but covering the basic materials it has taken time. So, let us start the next topic in the next lecture ok.

Thank you very much.