Satellite Attitude Dynamics and Control
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Lecture - 49
Reaction Wheel / Gyrostat

Welcome to the lecture number 49. So, in the past we have been discussing about the Gyroscope. So, that was a simplified case where the weight of the outer and the inner frame it was neglected and therefore only the rotor mass was considered and rotor inertia was considered. Now we will, today we will start with the Reaction Wheel and what we call as a Gyrostat. Basically the gyrostat we can broadly gyrostat is a term where we have some part of the system it is a moving with respect to the other part.

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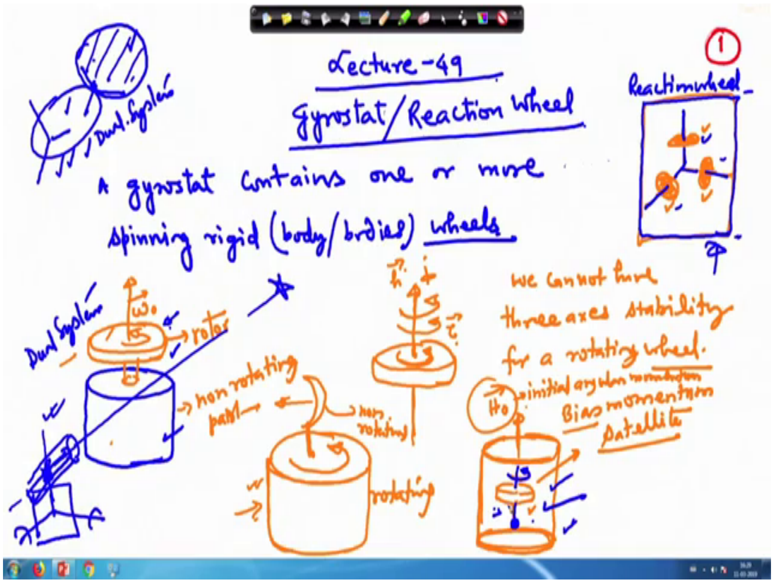

So, for I will write the formal definition, a gyrostat one or more rotating rigid body inside it ok. So, let us consider this is a cylindrical satellite ok, to this we have another rotar is attached. So, say this is the rotor and this is a rotating part and this is non rotating part. So, about this axis, this is the upper part is rotating. So, thereby you know that already we have discussed that, if we have any rotating wheel as in the previous example; in the previous lecture we have observed that if we have a rotating wheel, which is rotor spinning about would not axis. So, then it tries to maintain its direction. And if it is perturbed ok, so, if you perturb there is an angular vector associated with this, say this is the h angular vector. So, if you try to perturb it, so it tries it resist it, resist this motion, basically you need an external torque to change this orientation however. If you apply a torque along this axis itself means you are applying a torque tau along the axis means, along the axis of rotation ok. So, in that case it will not be able to resist this torque ok. So, basically this will speed up.

So, its stability along this axis is stability due to torque along this line that as shown here ok. So, the stability in rotation it is not maintained. So, this implies that we cannot have three axis stability for a rotating wheel because in the direction of the axis about which it is rotating along that axis it cannot tolerate disturbance it will simply speed up along that direction. Now, so, this is the same case. So, if you have a rotor either this part may be rotating or either this part may be rotating. Sometimes it may happen that you have a satellite where there may be antenna for communication purpose or it may be transponder. So, this transponder is pointing towards the earth or pointing towards some it may be some optical instrument which is pointing towards some particular direction, while this part is rotating and this is non rotating. And this is rotating part. So, larger the size of the satellite, larger this part here this part is a small we can see that this rotating part is large here. So, this will resists the external torque more, but not along the axis of rotation ok. It can resist along the other 3 axis, but not along the axis of rotation.

So, the another configuration for this we can have like I have a satellite, and inside this itself there is a wheel. And this wheel is rotating. So, this kind of configuration which is inside the system. This is often this is called the Bias momentum satellite. In it is quiet often that a bias momentum means a permanent H 0 is given along certain direction and moreover by rotating this wheel you will be able to change the orientation of the main body. If you try to speed up this wheel H 0 you are giving right in the beginning. So, this is the initial angular momentum of the wheel this is the initial angular momentum of the wheel. So, thereafter if you speed up this particular wheel then; obviously, you can control the rotation along this axis for, along this axis for this satellite ok.

So, in this particular part we call this as the bias momentum satellite or it can happened at inside the satellite along the 3 axis, you have 3 wheels along this axis, then along this axis and along this axis. So, you have three along the 3, you have 3 wheels and by controlling the speed of this wheels then you are able to so these are the wheels, and by rotating the wheels or speeding up applying torque to this wheel using certain motor then we can orient the satellite along particular direction we can change its angular velocity and so on. So, this kind of configuration it is called the Reaction wheels, reaction wheel so, this are acting as the reaction wheels.

And already we have discussed about the gyroscope. So, instead of that, this axis remaining being fixed; this axis can be tilted by a applying a motor here in this place and thereby also the torque can be generated and that configuration we call as the control moment gyroscope. So, we will take up that issue later on, first we are today going to discuss about the mechanics of this gyrostat and especially we will start with that, we have this configuration and in this particular configuration then, we will derive the equation of motion for this and then we will general generalize this through this configuration. And with this configuration this is also applicable. Thus same equation can be applied here, but if you have a situation something like this that this is a satellite and this is rotating along the 3 axis and together with this say here some telescope is mounted. There is a telescope here which you are using for measuring some distant angle for the stars or whatever it may be. So, in that case you may be rotating this telescope along the maybe the about this axis or about an axis perpendicular to this paper which is going inside.

So, the 3 axis control can be done for this telescope. So, in that case what happens this becomes 2 system one system is here and another system we can assume is to be attached to this point. Here we have the axis for this and this system is moving with respect to this and this system is also moving. So, this kind of system, it is a, Dual System. So, this is also a dual system. But this case is simplified because this axis is going inside this and it just appears like this, inside being in instead of being inside the satellite it is just a line outside. But this 2 cases are the same. However, this case is different because, it can rotate, it can have this part can have 3 degrees of freedom with respect to this part. So, this kind of system we call as the dual spin system where this part is rotating separately, this part is rotating separately. Here also this part it can rotate in certain direction, but inside this is also rotating along with this and together with this it has its own rotation. It has rotation on its own axis also. So, let us take this problem.

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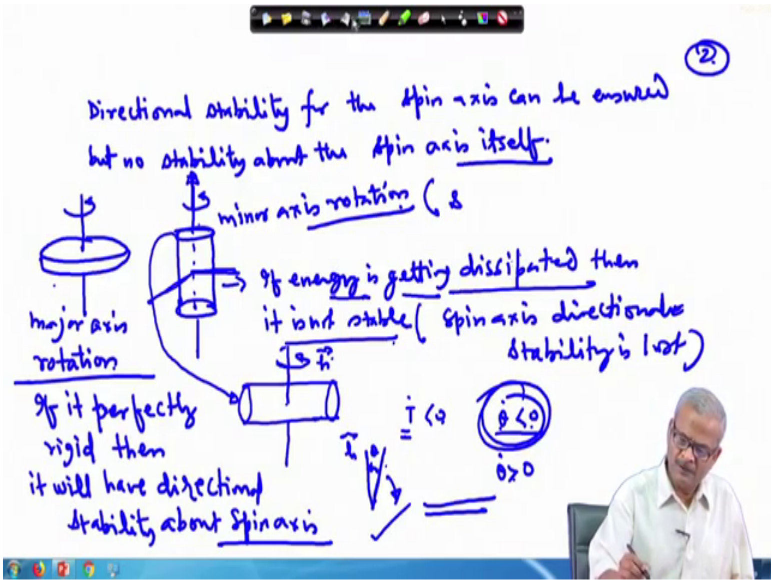

So, what we conclude from the previous discussion the directional stability for the spin axis can be ensured, but no stability about the spin axis itself. Moreover we have already observed that, if you are given a system, say this kind of system and you have another the cylindrical system. This kind of system if it is rotating about this axis and this is rotating about this axis and if there is certain dissipation of energy inside, so if energy is getting dissipated. This issue we have already discussed and derived also, then it is not stable. Say a directional stability cannot be maintained, Though you may be thinking that this about this axis at least a the spin axis stabilization can be done means directional stability of the spin axis can be maintained, but this is not true as we have discussed earlier also because this is the minor axis rotation minor axis rotation.

So, therefore, the spin axis stability is not possible if the energy this is getting dissipated. Then it is not stable spin is axis directional stability. Directional stability is lost. However, this case this is rotating about its major axis, major axis rotation and therefore, if, in normal cases that we have if it is perfectly rigid, perfectly rigid which is an idealization then, it will have directional stability about the spin axis. So, this kind of configuration which is rotating about this axis and if there is any energy dissipation, so, ultimately what will happen that, it will come a configuration, it is, a because the angular momentum vector in the absence of external torque it cannot change so still it will keep rotating along this direction. The angular momentum cannot change in the absence of torque. So, it will start rotating about this axis and this axis will come to this position and the one of this minor axis will come to this position.

So, the rotation axis will get interchanged. And this we have discussed also when we have done the derivation etcetera for theT dot case when it will be negative. And theta dot when it is less than 0 so, if it is theta dot where we have written as the if this is the h vector so, from here we are measured that theta, means the deviation of the axis from this is a (Refer Time: 16:50) angle. So, if this is negative means the theta will be decreasing it will be stable, theta dot if it is greater than 0 means the theta dot will grow and the system will unstable, will become an unstable. So, this case we have already discussed in quiet details. So, this we are just recapitulating before getting into the derivation of what we are looking for.

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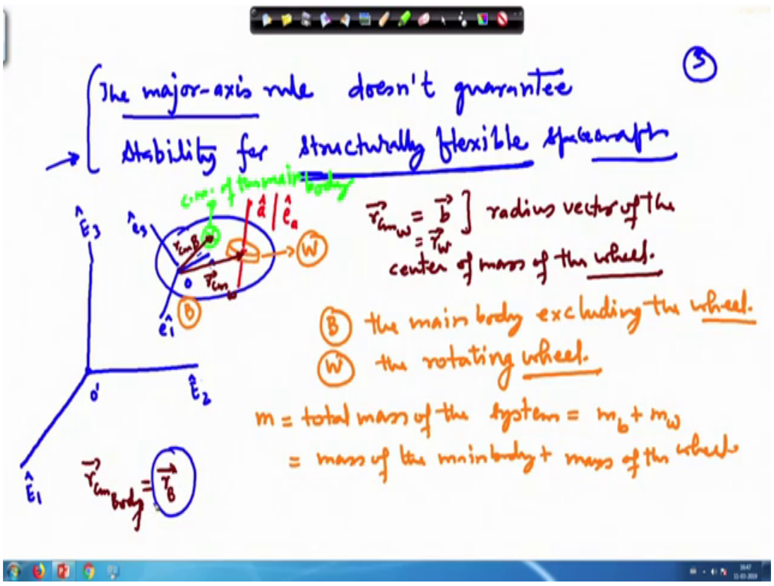

So, what we conclude here the major-axis rule means, if it is rotating about the major axis does not guarantee stability for structurally; say we have assumed that energy dissipation is there, but we have not assumed that the system was structurally flexible. So, this case we have not done. So, as you grow professionally you will see that you will have more and more complex system and if you know the basics the very important part is here that if you know the basics, how to approach the problem then you will be able to deal with the problem.

So, let us consider the system as follows. So, this is your initial frame and then you have a body here, this is the point o about which we fix the body axis e 1 cap, e 2 cap these are the unit vectors or the basis vectors . And then we have rotating disc here, and along this direction let us say that a cap or either we can use the notation e a cap this is the unit vector along this direction along the axis of the rotating wheel. And the centre of mass of the wheel is here. So, the vector to this, this is your e 2 cap here on this side. So, this vector we can write as r centre of mass of the wheel ok. So, r cm wheel we can write this as let us say that we write this as the b vector. This is the radius vector of the centre of mass of the wheel.

So, what we are interested in? That say if I can just approach in a shortcut way that assuming that this wheel is located on one of the body axis and I simplify the system and do it very quickly. But I just want to go through a general approach which will be very useful if you have to really work out certain problem. So, the centre of mass of we will tag on the system as say this we tag as B, means the B is stand for the main body excluding the wheeland this we tag as W. So, w stands for the rotating wheel. Accordingly, we can define the m which is the total mass of the system, system equal to m b plus m w. So, mass of the body this equal to mass of the main body plus mass of the wheel. So and let us assume that somewhere here the this is the centre of mass of the main body ok. So, the command centre of mass therefore, we can write again we will have to show it by a vector. So, I need to rub it out and ok, let us say this is the centre of mass of the r c m this is the r c m wheel we have written by b. So, this is r c m body, this is r c m capital body.

So, r c m, the body this can, we can give some other notation to this maybe we can write this as the something like r c. We write this as the r capital B, just to indicate this is for the main body and this we have returned for wheel. So, instead of b we can also write this has r w. So, these are the notation configuration we can choose on our own. So, let us proceed and wherever the way we require. So, we will define the corresponding variable.

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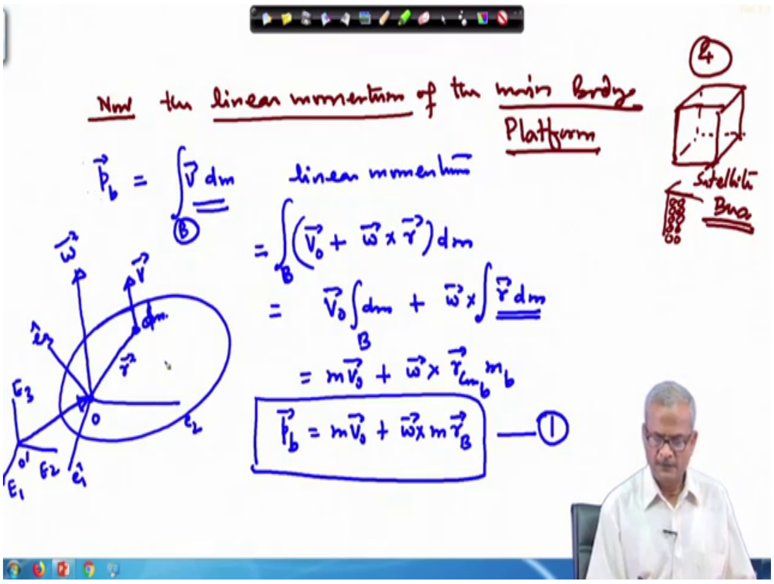

Linear momentum of the main body, this main body also this is called the platform.. Sometimes the also the casing of the satellite say this is a satellite, so, this is the satellite casing. So, this is basically a honeycomb structure means the casings are made of panel which inside just like the honey bee is having it is high. So, the same way this structure is there. So, you will see if this is hollow, and on the 2 sides of thin plates are pasted to make it light and also give it a strength.

So, this is also called something the Satellite Bus. So, for calculating the linear momentum of the main body which we will write as p b; we write this as p b. So, p b will be equal to we have to linear momentum is mass times velocity. So, the velocity of any point, so, this is your main body and inside that there is a wheel so that wheel you just remove it ok. So, and this is your point o and already we have derived this also this is e 1, e 2 and e 3 these are the 3 body axis ok. This point we have taken as o if this is any mass delta m or d m ok.

V is the velocity of this mass. So, V times d m this is the r the distance of this mass from the point o. So, this becomes your linear momentum and this integration is to be done over the main body. And as you know that we are referring this with respect to the inertial frame which is E 1, E 2 and E 3. So, therefore, we can be written as V 0 plus this we have discussed. So, I am not taking up with again and say this is the omega is the angular velocity of this main body. So, omega cross r d m this becomes its linear momentum. So, V 0 d m this is the integration over the body plus omega cross r d m. So, m V 0 and plus omega cross r d m this is nothing but r centre of mass of the wheel times mass of the sorry centre of mass of the main body plus mass of the main body. And this notation we have kept it for this as r b. So, in the short we can write this as m V 0 plus omega times m this is the centre of mass of the main body The same way we can also derive the linear momentum of the your rotating wheel.

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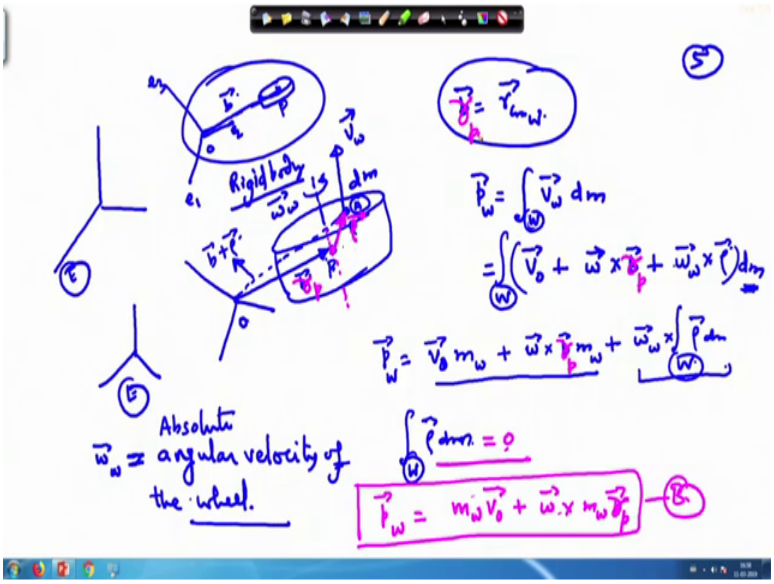

So, again we have this is E frame and this is the body frame e 1, e 2 and e 3 this is point o and say the wheel is located here, centre of mass is here whose radius vector we are writing as b . So, now I will expand this say this is the radius vector of this wheel I will make it little larger to look it good. So, this is centre of mass of the wheel which we have b equal to r c m w. And here is your frame, if we take any mass in this body which is d m and let us say this is the velocity V and we can type this has V w. So, according to the, our earlier derivation so this is your point o this is frame E here. So, p will be equal to V w d m and V w will be equal to V 0 plus velocity of the point b with respect to point o.

So, that it is a because we are assuming this to be a rigid body; so, the only part that can arise here is omega 0 or omega cross where, omega is the angular velocity of the main body. Omega cross then, this distance which is b and plus if we have the angular velocity of the say this wheel is also rotating about this axis and let us say omega w. So, omega w we write as the angular velocity of the and this is the absolute angular velocity of the wheel. So that means, with respect to this point this mass is also rotating ok so, that we need to take into account so, we have to write for that also. So, that will be equal to omega wheel cross rho where rho is this vector that means from the point this is point o and this is the let us say this is point we can write this as the point maybe point P ok. So, this is point P here and this is any point here some arbitrary mass which we can refer as point let us say A. Whose radius vector from this place to this place is rho. So, from this place to this place the radius vector this becomes, this vector we have defined as b, this victor we have defined as b. So, and this is rho. So, this vector becomes b plus rho. And then if we integrate, so, you should always do a systematic analysis and this habit will make you solve any kind of problem ok.

If we try to memorize certain things then it becomes difficult to reproduce it. But we are if we are systematic at working then it is always possible that whenever the problem is given we can do the problem. Therefore, the p w this we can write as V 0; obviously, V 0 a constant. This is not dependent on the not constant I mean this does not depend on the integration. Integration is over mass. So, this is mass independent. Therefore, we can write this as V 0 times m and this is over the wheel. So, this is over the wheel not over the body. So, here we need to correct it this is over the wheel. So, we will write as m w and then plus omega cross b m. So, this these two parts they will stay. And what about this part? This part we have rho d m an integration over the wheel..

So, you can see that this is the point P is the centre of mass and rho is being measured from this point ok. So, rho is being measured from this point, this is your rho vector. It is a being measured from point P which is located at the centre line or this wheel ok. Therefore, being the centre of mass this quantity is going to be 0 and we get p w this equal to m w, m w V 0 plus m w or omega cross.

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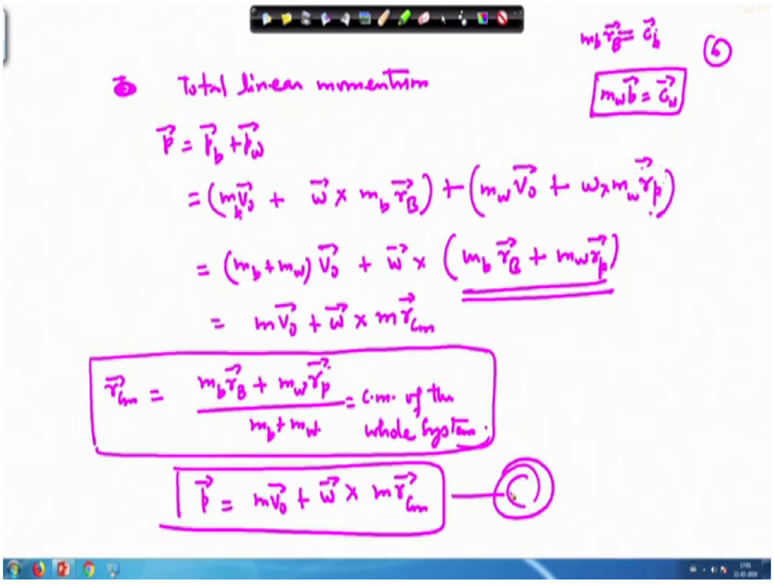

So, the total linear momentum the p can be written as p main body plus the p wheel. So, the p main body is this quantity m V 0, m V 0 plus omega cross m times r B, m b times r B. Here we have not written we should write here m b this is for the body; so this is m b. m b we have written here so, this should also be there. So, this and plus from the previous step m w times V 0 m w times V 0 plus omega cross m w b, omega w times omega cross m w b. We can combine them together, this m w times b, this we can also write as c. Or we can put a tag c w here similarly m b times r b; where r b is the location of the centre of mass of the main body this can be written as c b ok.These notations should not become confusing at any stage. So, maybe this, this we can change it to r p. So, if we change it to r p. So, this will become r p omega cross r p and then we can write this as r p. Here also we can change it to r p. This is r p and this is the combined centre of mass multiplied by total mass. So, this becomes m this part m V 0 plus omega this is the angular velocity cross total mass times the combined centre of mass.which obviously, we will write as r c m this is the r c m total . So, your r c m this equal to m b plus r B plus m w times r p. Instead of p we could also have written here the w to indicate this is the wheel centre of mass. So, divided by m b plus m w. This is the location of the this is the centre of mass of the whole system combined with a wheel. So,our conclusion is p equal to m V 0 plus omega cross m r c m.

So, we will continue in the next lecture and in the next lecture we are going to discuss about the, we have just now worked out the linear momentum and discussed some preliminary regarding this. So, in the next lecture we will be discussing about the angular momentum of this system and thereafter if there is some external torque acting on the system then how its angular momentum will change? So, the basically we are going to derive the equation of motion for both force and the moment, related to the force and the moment. So, rotational dynamics as well as translational dynamics.

Thank you very much.