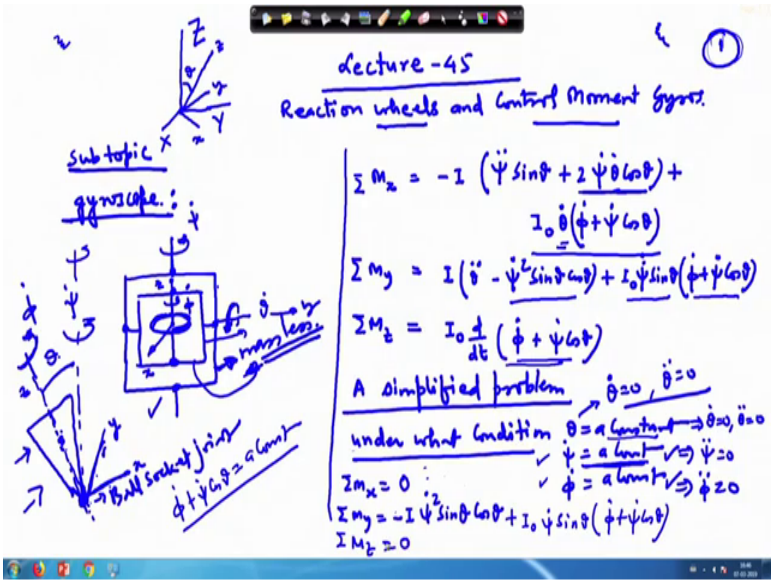
Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture – 45
Gyroscopic / Top Motion

Welcome to the 45 lecture. So, we have been discussing about the Gyroscope. So, actually our ultimate aim is to get into the reaction wheels and control movement gyros. So, control movement gyros are used for controlling the attitude of the spacecraft or the satellite while the gyroscope itself it is a device which can be used for measuring the angular displacement and then the angle rates.

(Refer Slide Time: 00:53)



So, in that context, we have been trying to before going into the entering into the reaction wheel and the control movement gyros, we are trying to enter into this gyroscope, how does it work. And already we have looked into the rigid body dynamics of a cylinder which was a symmetrical case, it can be unsymmetrical also, but here the gyroscope it consists of two frames as we have discussed earlier that it consists of two frames, one the outer frame and the another will be the inner frame which we have.

So, this is on the gimbal here. And then you have another frame which is inside this ok, and then we have another, then there is a wheel attached to this, and this is also a gimbal.

So, these are the gimbal points. So, why we were discussing that we wanted to look into the behaviour of the system which is nothing but a top playing top.

So, it is devoted here about with a ball socket joint and then it is displaced from the vertical by theta angle and a spinning about this by angle phi. And then we draw it, we drew it on the right hand side the last time, it does not matter whether it is on the right hand or the left hand side. So, to understand the behaviour of this system we were looking into this ok, because this system and this system they do not differ much.

Here if we assume this to be massless, this two frames we are assuming to be massless ok. And therefore, this moment of inertia it is a getting ignored and we are just concentrating on this field ok. So, this wheel will be have rotation because of the rotation about this axis because of rotation about this axis ok. So, it is a anticlockwise rotation here, so theta dot here we have written, psi dot here, and then it is rotating on this axis itself a spinning; so which we have written as phi dot.

So, over all the behaviour of this system it will appear to be the same as the behaviour of this system. So, behaviour of this system means without the frame mass, these two frames are this and these two frames, they are massless ok. So, understanding this it will help me in solving this particular problem. And here we instead of taking the reference frame at the centre of mass, I can choose the reference from here in this place ok. And we can write here as x y and this is the z-axis ok, and we know that this is spinning here spinning on this axis has its shown by this arrow.

But as in the gyroscope last time we have discussed that this x-axis which is here and the y-axis which is going here in this direction and the small z-axis which is here. So, this is not a spinning along with the discs, similarly here in this case this times and y, they are not spinning along with this one, but it is nutating and this axis is nutating, and this axis is also processing and this is because of the symmetry ok.

So, in that context we derived certain equations and those equations once we use it, so it will give us the result we are looking for. So, we derive the total movement acting about the times axis. So, this was minus I times psi double dot sin theta, so this was one of the equation. The second equation we have written ok.

Now, let us consider a very simplified situation, before we go into the actual problem. So, in the simplified problem what we are looking for that the under what condition, what condition theta dot will be 0 means theta is a constant, psi dot this is a constant and phi dot this is also a constant. So, we have to find out that torque condition basically from this, so let us explore this.

So, if we look here in the first equation. So, if your theta is constant, so this implies theta dot, this will be 0 and also theta double dot this will also be 0, because it is a constant all the time. So, you can look that because theta dot is present here and theta dot is also present here, so this term will drop out, this is dot here. So, this term will drop out and ok, this term drops out and this term also drops out. Living out only the first term that means, E M x will be equal to minus I times psi double dot sin theta. So, this term is going out because of theta dot equal to 0, this term also gets deleted.

If you will look here in this part, so E M z this quantity I 0 d by dt phi dot psi dot. So, here as per our original problem we wanted to solve this problem. So, along this axis no movement is acting, this is the vertical axis we have written this as the capital z-axis, so this is capital X, this is capital Y and with respect to this your X Y and Z from if your this cone is on the right hand side, so we can show it like this is, this is the small z-axis and the x and the y-axis is here, a small x and the small y-axis, this angle is theta ok. So, with the vertical this x-axis also that makes angle theta.

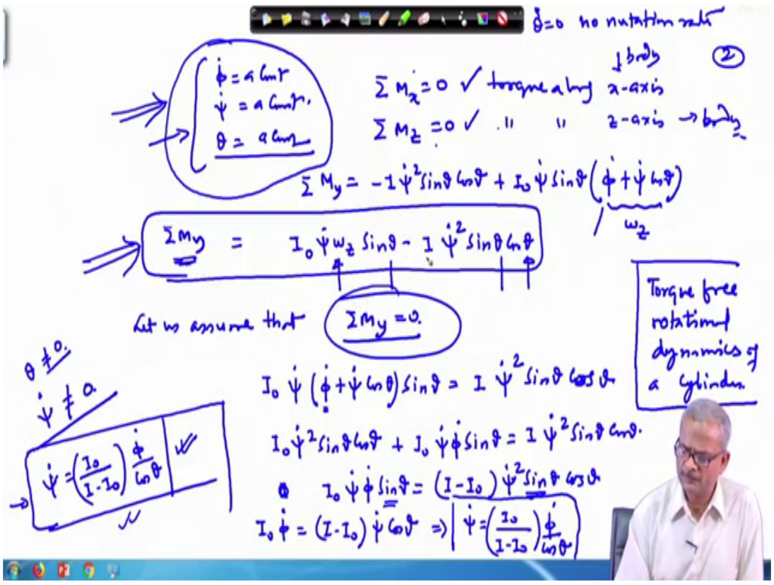
So, we get one equation here, you will look here in this. So, from there we will have E M y. Now, again one more thing here, psi double dot this is also 0 psi dot is constant, because this condition we have not taken psi dot is constant, we are assuming here for simplified problem.

So, psi dot is constant and phi dot is also a constant, so that means, this term should also vanish, so that means, this simply implies. So, from here we can write on this side this implies psi double dot this equal to 0, this implies theta dot equal to 0 and theta double dot this is also 0, and this implies phi double dot this will be equal to 0.

So, therefore in the first equation all the terms they are drop out, so this becomes simply 0. There after we pick up the second equation, here theta double dot this will be 0, but this term is not 0. And here if we see, all these terms are present ok. So, this becomes I times minus sign we will have to put here, minus I times psi dot square sin theta cos theta, sin theta cos theta and then from this place I 0 psi dot ok.

Now, here in this part if we look phi dot is a constant ok, psi dot is a constant has shown here, psi dot is a constant, phi dot is a constant and theta is also a constant as per our assumption. So, this part so this implies that phi dot plus psi dot cos theta this is also a constant. And therefore, derivative time derivative of this will be 0, so this implies E M z this is also 0.

(Refer Slide Time: 11:53)



So, the condition we were looking for that phi dot equal to a constant and psi dot this is equal to a constant and theta this is equal to a constant, list to the condition that E M x this should be 0 and E M z this should also be equal to 0. And what is non-zero is E M y and that E M y is written here in this place, this is the net torque on the along the y-axis minus I times psi dot square.

So, we will take it on the next phase minus I times psi dot square sin theta into cos theta, sin theta into cos theta plus psi 0 psi dot sin theta plus I 0 psi dot sin theta and times phi dot plus psi dot cos theta in this particular part, we can rearrange it. So, from here then we get the desired solution ok.

So, our solution will be E M y means the what is required that torque along x-axis should be 0, remember this is the body axis which is nutating and precessing x-axis. So, here in this case because theta dot equal to 0 theta is a constant, so no nutation rate no nutation rate, but nutation angle is present. And this is torque along the z-axis, all these are the body axis ok.

So, if we rearranged it, so we can write here as, this quantity is nothing but your omega z ok. So, we can write it in a simple way I 0 times psi dot omega z time sin theta minus I times psi dot square sin theta into cos theta. So, this is the amount of torque required you can see that theta is a constant here ok, only psi dot and phi dot which has gone into this omega z. So, phi dot these two quantities are present ok.

Now, we can do little bit more simplification to this. Let us assume that E M y is also 0 ok. One more condition we impose, already three conditions we have imposed and one more condition we are imposing E M y is 0. So, what does this mean already you have no torque along the x-axis, no torque along the z-axis, no torque along the y-axis.

So, then this gets simplified to a situation we have been discussing for the rigid body dynamics, which is torque free rotational motion of the rigid body ok; this problem we have already discussed. So, let us look into this problem psi dot square sin theta into cos theta. We will separate out the phi dot term and psi dot term and so write the equation correspondingly.

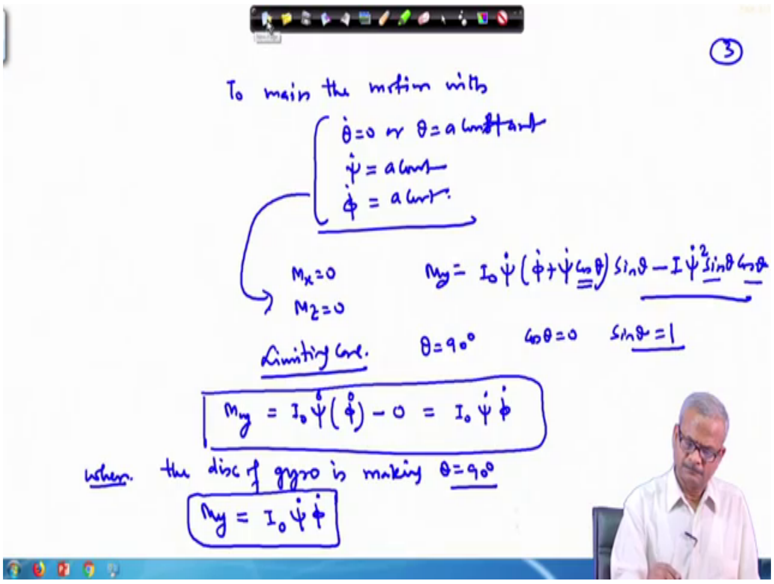
So, this is I 0 psi dot a square sin theta cos theta psi dot square sin theta cos theta plus I 0 psi dot phi dot sin theta, on the right hand side sin theta times cos theta. And what we are interested in let us first take psi dot on this terms we are going to rearrange. So, we will write this as I 0 psi dot phi dot sin theta this equal to I minus I 0 psi dot square sin theta into cos theta.

Because psi dot is non-zero, psi dot this quantity is non-zero ok. And theta also the theta which is present here, so this is on the both hand side ok. So, theta is non-zero. So, in that case we can put it like this times I 0 times phi dot, this equal to I minus I 0 psi dot cos theta. So, this implies psi dot equal to I 0 divided by I minus I 0 times phi dot divided by cos theta means, the equation we have got here this is psi dot equal to I 0 divided by I minus I 0.

And this is exactly the expression we got for the three body. So, what we have got for the torque free rotational dynamics of a cylinder or either disc ok. So, through this route also what we are getting that the same result for the rotational motion of a cylinder or a disc which is free from torque, you will get psi dot equal to this contains the expression which is written here, which we are getting through some indirect route. So, the same problem it can be solved in multiple ways.

Now, we are interested in that if we sphere it that E M y is not equal to 0. So, in that case only and this conditions they apply, so we will have to apply this much of torque along the y-axis to maintain the motion ok.

(Refer Slide Time: 19:51)



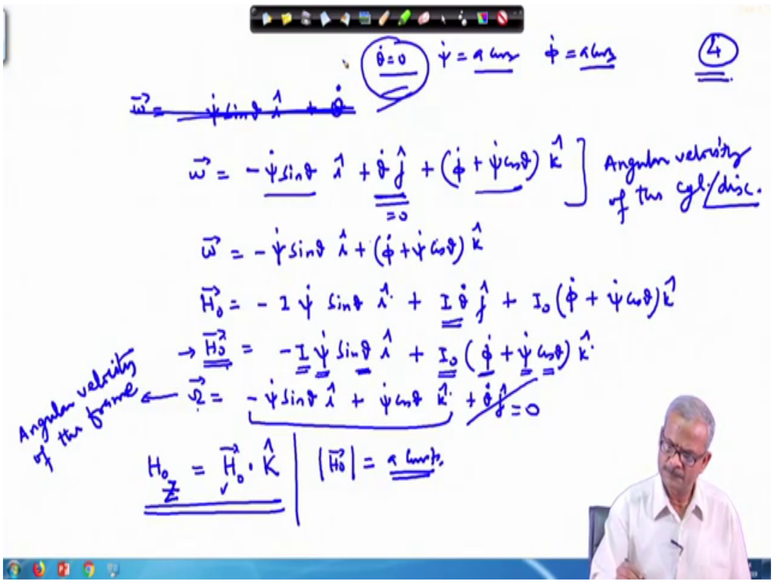
So, to maintain the motion with theta dot equal to 0 or theta equal to a constant and psi dot this is a constant and phi dot also this is a constant. So, under this assumption what we are getting M x equal to 0, M z equal to 0 and M y is given by this expression, which is written here in this place. So, I 0 times psi dot omega z psi dot omega z is phi dot plus psi dot cos theta, this part and then sin multiplied by sin theta minus I dots I times psi dot square sin theta cos theta psi dot square sin theta cos theta.

Now, let us look into a limiting case. If theta equal to 90 degree ok, so in that case what result we get ok. So, for theta equal to 90 degree, cos theta will be equal to 0 and sin theta is equal to 1 ok. So, we get M y equal to I 0 times psi dot times phi dot this part will be 0, sin theta becomes equal to 1.

So, on the other part we have I times psi dot sin theta is 1, but cos theta becomes 0, so this part other part is also 0. So, what we get I 0 psi dot times phi dot. So, when the disc of the gyro is making theta equal to 90 degree, so at that time the torque required to maintain the motion along the y-axis will be given by I 0 psi dot phi dot.

Now, we are ready to take the original problem, we wanted to discuss about this problem ok. So, now we are in a position that we can discussed about this problem. So, this way you need to analyze all the problems.

(Refer Slide Time: 23:47)



So, for our present case ok, there are few more things, before we go into that we will discuss about. So, your omega is minus psi dot sin theta times theta dot, see if omega the equation it is a long equation we have written. So, under the assumption that theta dot this equal to 0, and psi dot this is a constant, and phi dot this is also a constant.

So, I am exploring the same issue from the other prospective and it is important that we take care of all these things and basically we are trying to learn here ok. So, we should look into all the angles ok, so we should look through all the angles. So, here we have the omega, we have written earlier derived one particular equation, which is omega equal to minus psi dot sin theta i cap plus theta times j cap.

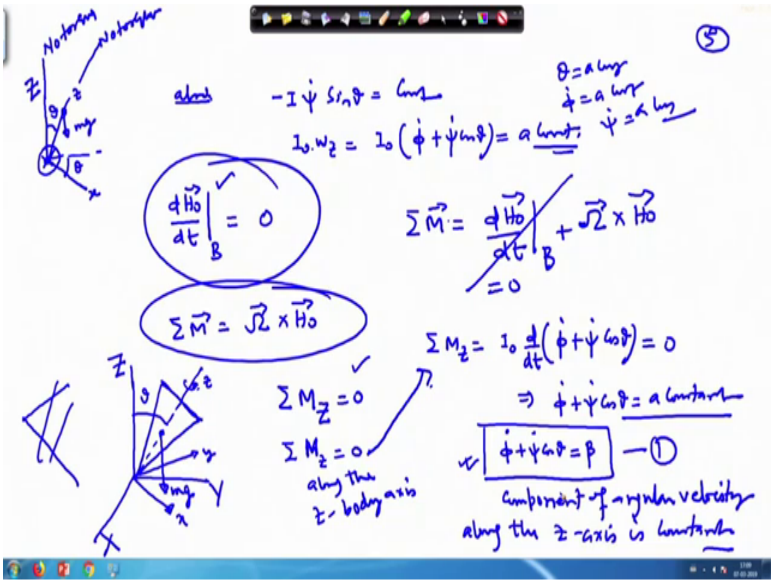
So, when your theta dot equal to 0 ok, here this is theta dot. So, this term will drop out, so this term becomes 0 and omega gets reduced to minus psi dot sin theta i cap plus theta dot j cap this part is gone plus phi dot plus psi dot cos theta times k cap this is omega.

And similarly, the H 0 vector then what we have written earlier, so that gets reduced to H 0 we have written as minus I times psi dot sin theta times i cap plus I 0 times basically multiplying this by I, this by I, and this. So, here because theta dot is 0, so this particular j term will drop out here also. So, what we get here I time psi dot sin theta times i cap plus I 0 phi dot plus psi dot cos theta times k cap.

And there are few other things that we also need to look into. And capital omega this term is minus psi dot sin theta i cap plus psi dot cos theta k cap. Capital omega we have written by the equation, we know sin dot there was one more term which was theta dot times j cap. So, this term this drops out because of this condition and we get only this term. So, this is your angular velocity of the frame ok, not the angular velocity of the disc; this is the angular velocity of the frame and this is the angular velocity of the disc or the cylinder, cylinder slash disc ok.

If we look for the momentum component along the capital z-axis, so what we need to do, just we need to take the dot product of this with respect to capital K and that will give you the momentum component along the z-axis ok. So, few more things we can write here. If you look here in this equation the angular momentum equation; I is a constant, psi dot is a constant, theta is a constant, I 0 is a constant, phi dot this is a constant, all these are constant. Therefore, this implies that H 0 magnitude, this is also going to be a constant.

(Refer Slide Time: 29:47)



What else we can do here further? This also implies we have I dot sin theta, this is a constant and already we have written I 0 times omega z which is equal to phi dot plus psi dot cos theta, this is also a constant ok. Therefore, dH 0 by dt with respect to the body axis ok. So, your which vector is constant and therefore, this quantity this become 0. So, what is left in the equation, E M this we have written as dH 0 by dt with respect to the body axis plus capital omega cross H 0.

So, here this part this drops out under the condition we have mentioned, where theta is a constant, phi dot is a constant and psi dot is also a constant ok. So, you see that the situation get simplified to capital omega cross H 0 ok. Moreover in this problem, where the cone is treated like this, this axis is making theta angle with this one and here your x-axis and y-axis are directed this is capital X, capital Y, capital Z; this is a small z here. So, we can see it from this place that E capital M z this quantity is 0, because your M g this is acting just downward ok, there is a torque about this axis ok.

So, if we assume that the situation is something like this, this is the z-axis and x-axis is going down like this ok. So, with respect to this capital Z-axis the small z-axis is tilted and with the horizontal, then your x-axis is making angle theta ok. So, under that situation you can see somewhere on this somewhere M g force is acting and because of that you get a torque here in this place. So, this torque will be directed along the y-axis, which is going inside the plane of the paper ok.

So, this quantity is 0, similarly we do not have torque along this axis. So, here no torque along this axis also no torque ok. So, to torque is acting along the y-axis, E M a small z, this quantity also is 0. And you can solve for this, so if we try to work it out, So, E M z this equation we have written as I 0 d by dt phi dot plus psi dot cos theta. So, if this quantity is 0 from this place this particular one, this is along the z body axis. So, this implies that phi dot plus psi dot cos theta this will be a constant.

And let us assume that phi plus psi dot cos theta, this quantity we write as we can give some name to this let us say this we write as beta. So, this is the one relationship we are having, this simply says that along the z-axis. Your component of angular velocity along the z-axis, this remains constant this is simply it implies. So, component of component of angular velocity along the z-axis is constant. So, it is not changing.

So, we will continue looking into this particular topic, still lot more has to be done. So, we will continue in the next lecture.

Thank you very much.