Satellite Attitude Dynamics and Control
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Lecture – 43
Control Moment Gyroscope

Welcome to the 43rd lecture. What we have been discussing about earlier it is the spin stabilization, now we enter into the reaction wheels and control moment gyros. So, reaction wheels and control moment gyros, they are used to control the attitude of the satellite.

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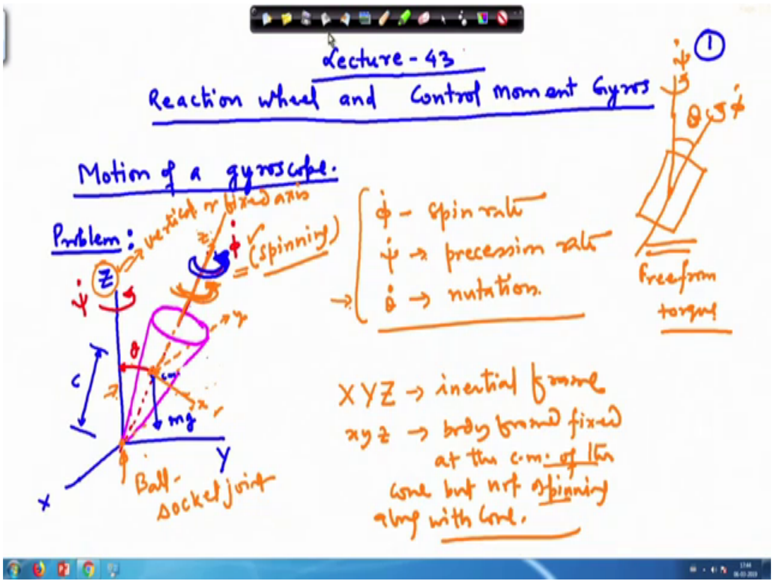

But, before we enter into that topic, so we will have some video for that, and we have a problem here also that the problem is like we have a top say, you know the play top ok. So, this play top is hinged over this point. So, this point is a ball-socket joint. So, this gives a 3 degree of freedom ok.

Now, this cone is rotating on its axis. Basically, this is the spin here, the spinning ok, this phi dot is the spin rate. And psi dot this is your the precision rate, and theta dot will call as the nutation rate. Already we have discussed, this thing that rigid body dynamics ok. So, we will do this problem. So, this will expose you to how the rigid body dynamics problem can be solved ok.

Now, here in this case the as you know once you play with the top, so it is a spinning at some spin rate, which is given by this phi dot ok, and it makes certain angle with the vertical. So, this vertical I have shown it here in this place ok. So, this is your vertical fixed axis ok. And the gravitational torque, it is acting here ok.

So, x and y we have shown x, y, and this is a small z, this is small z. So, this is capital Z, there is a difference. So, X, Y, Z capital X, Y, and Z, these constitutes inertial frame. While the x, y, z small z, it constitutes the body frame ok. So, here this is fixed to the body. So, x, y, z, this is fixed to the fixed at the body frame. Fixed at the centre of mass of the cone, but not spinning along with the cone ok. So, why we are not fixing, you will see later on, what is the advantage of not fixing it if we can also fix the body axis such that it is a fixed here, and there thereafter, it is a rotating along with the cone as shown in this direction. This is the anti-clockwise as seen from the top ok, so this is the rotation here.

Now, we have to solve certain problem related to this, which I am going to a state later on ok. So, here the torque acting is about the y-axis. This y-axis here it seems to be this is coming out here, we can may be we will change it little bit the y-axis; y-axis we can show like this, so this is your y-axis. So, the gravity x-axis is inclined like this, what it has been shown here.

So, x-axis is making theta angle, because this particular line the axial line of the cone, it has got inclined with the this vertical line by theta degree here. So, this x-axis will also get inclined with the horizontal by theta angle. So, this will be theta angle, which I am not going to show here, because this will unnecessary complicate the figure ok. We will take up this issue later on also.

So, here basically due to the gravity, there is a moment about the y-axis ok, and under this moment, so this is not a torque free case. Earlier if you remember in the rigid body dynamics, we have taken the case of a cylinder which is precissing, and there is the nutation angle involved, and it is spinning phi dot here, this is theta, and then there is the psi dot is present ok.

But, that case we have taken, where it was free from torque, torque free rigid body dynamics we have done free from. But, here in this case, it is not free from torque that is a torque acting at the centre of there is a force acting at the centre of gravity. And therefore, a torque will act along the y-direction, if we put here, x and y are shown here ok. If I push it here to the bottom point, so you will see that along the y-direction the torque is acting ok.

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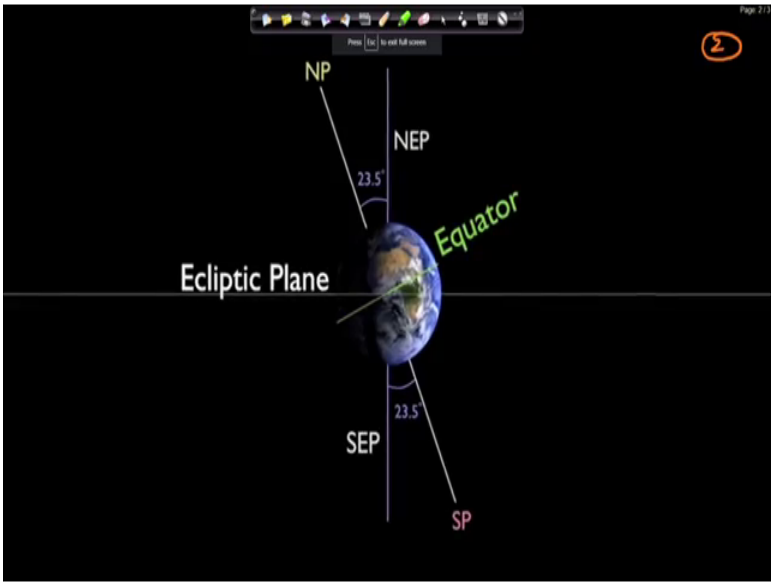

So, let us look into this video is from the YouTube. If you see here, this particular line ok, this is the equatorial plane.

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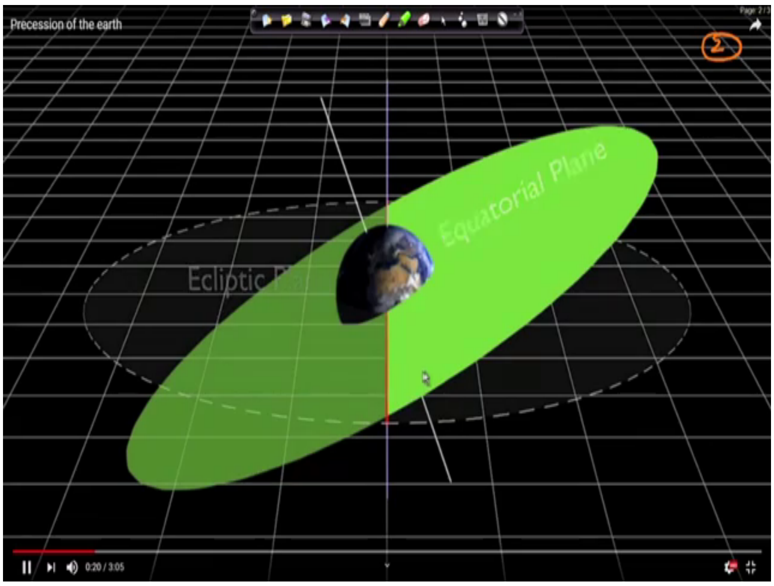

The green plane which is shown that is a equatorial plane, and the dotted line here ok. So, here this is the dotted line, it is a call the ecliptic plane ok. Already, we have discussed about this just in the beginning, and the green line which is shown here, this is the equatorial plane. You are seeing from the equatorial plane from the side, and there this is a view of the ecliptic plane from the top.

So, see if now you can see that the ecliptic plane and the equatorial plane, they are cutting along this line, which is shown by the red line here ok. This you can see the cursor, it is a moving. So, this is along the red line, this called the line of nodes ok. So, here in this case what is happening that as this is the earth ok. And earth equatorial plane, it cuts the ecliptic plane, and it cuts along this nodal line ok, and earth is precissing.

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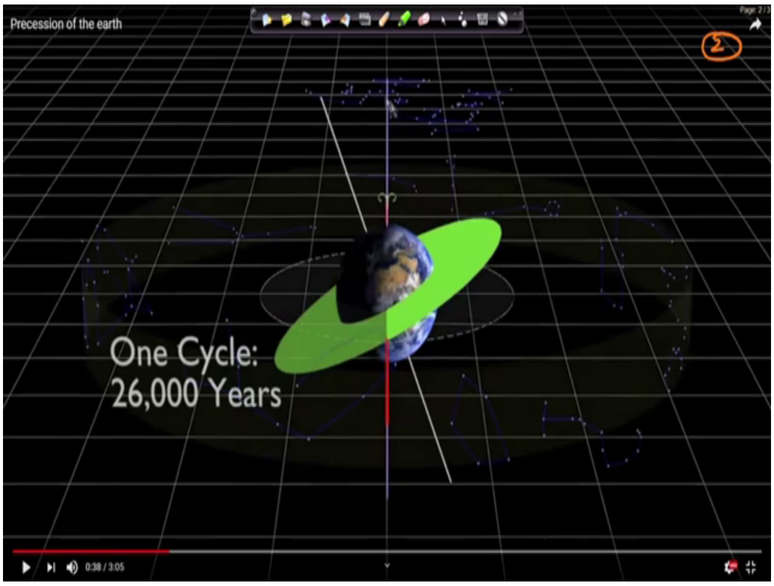

So, if you see the let us go further and see, you can see that this axis, which is shown here, this axis it is a rotating about this vertical line ok. So, this motion is the motion that takes place like this, as you see by cursor. So, this line this motion, it constitutes the precision motion ok. So, this is see the small this dotted line is getting generated here, so this is the precision ok.

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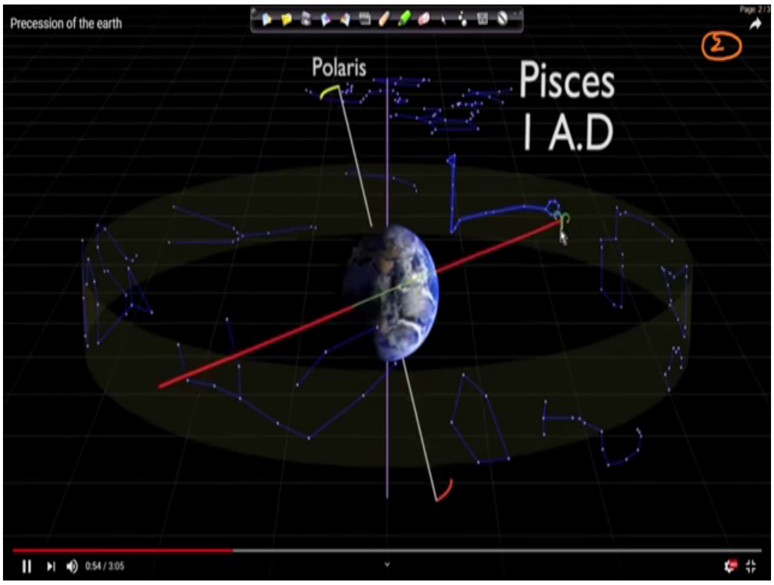

And your line of nodes which is shown here by the red line, it is a rotating.

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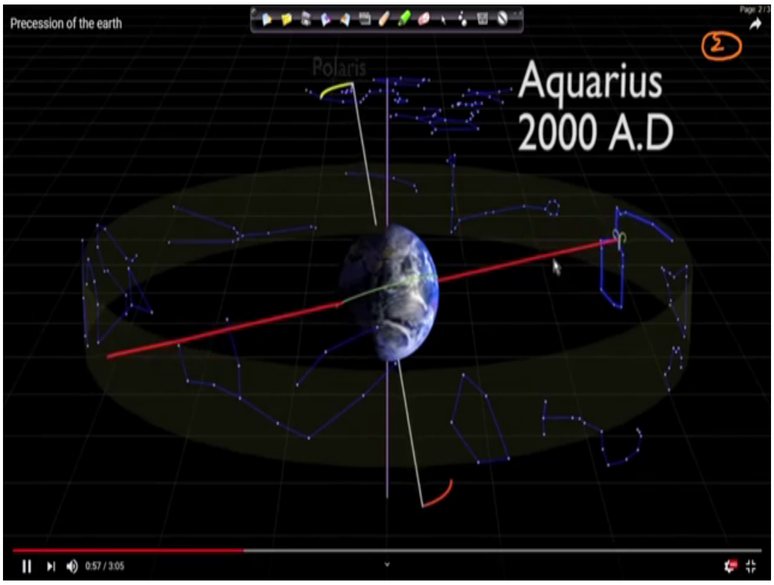

So, this rotation, it takes place nearly around 25800 means 26000 years roughly you can say, it goes through 360 degree ok.

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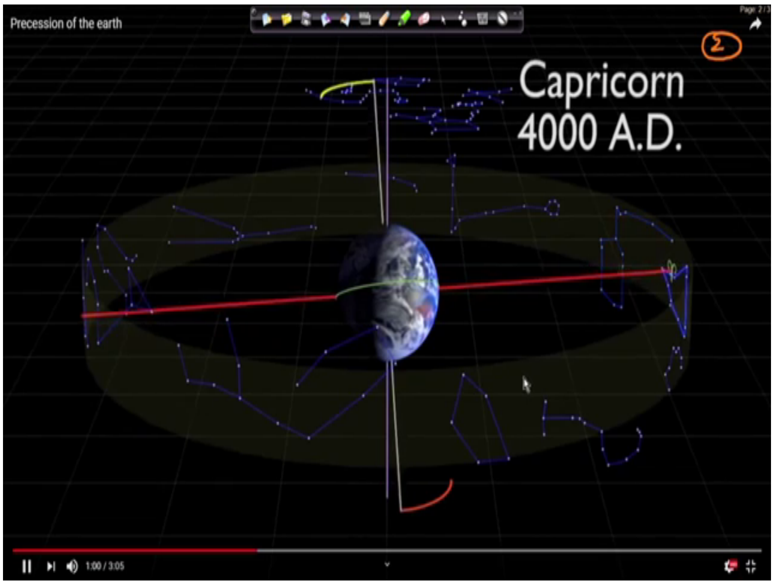

So, that means in 26000 years this line which is moving here, it will go ones round like this.

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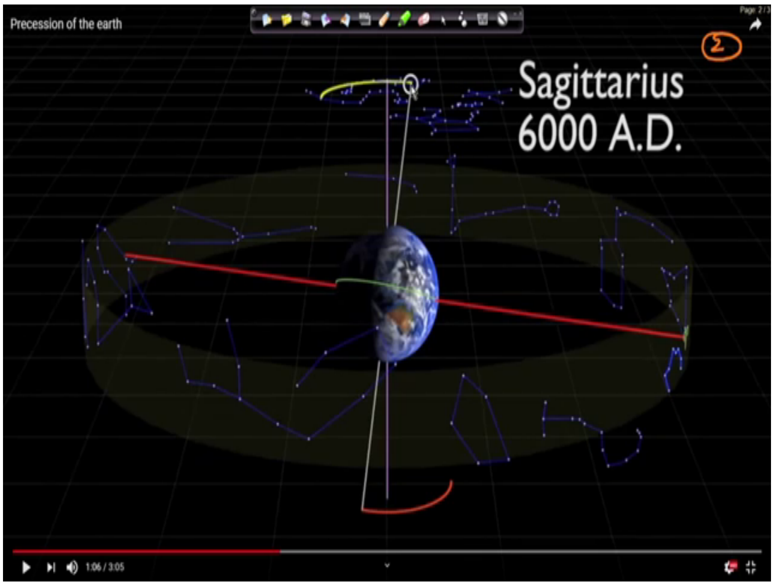

So, right now this line the where I have put the cursor ok, so about this where the circle has appear. So, this line right now, it points toward the Polaris ok, what you call as the pole star ok. After 5000 years, there will be no pole star for the earth ok. Right now the axis of the earth about which it is rotating, so that is pointing toward the Polaris.

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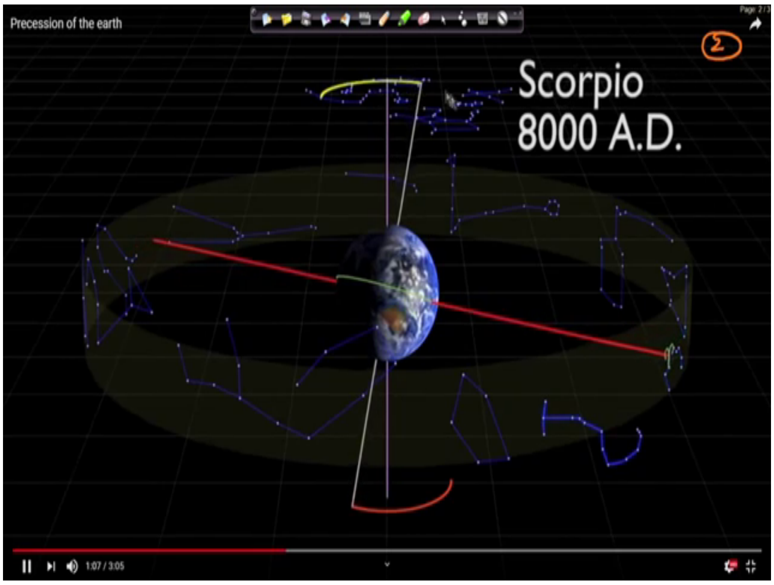

So, and thereby it is a describing a circle (Refer Time: 9:23), so this is the precision motion.

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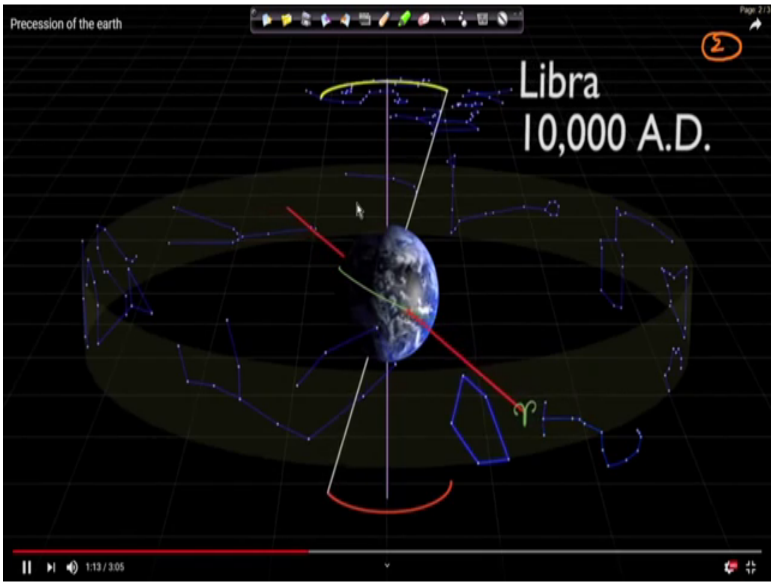

In addition, there is a small nutation motion, which is not visible here that takes place about the green line itself.

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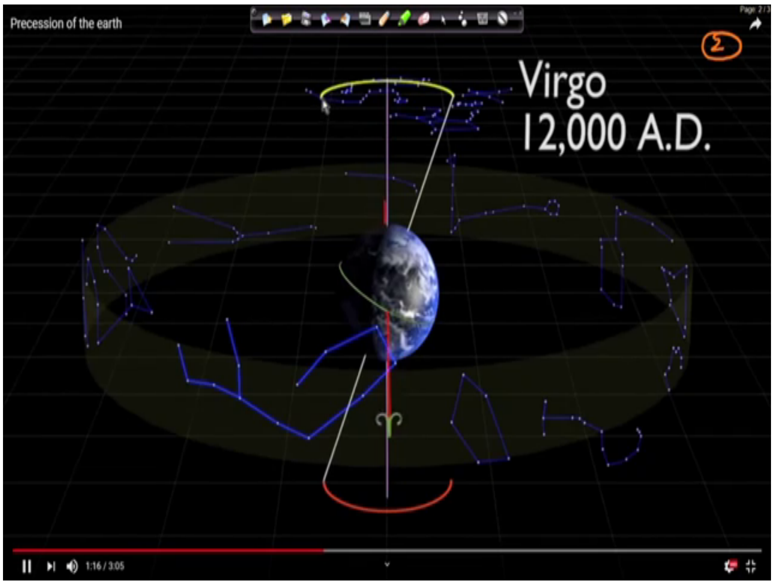

You can see that, I am showing it by cursor, it does like this. See the cursor, it is a moving like this. So, it will do this kind of motion ok, so this called the nutation motion.

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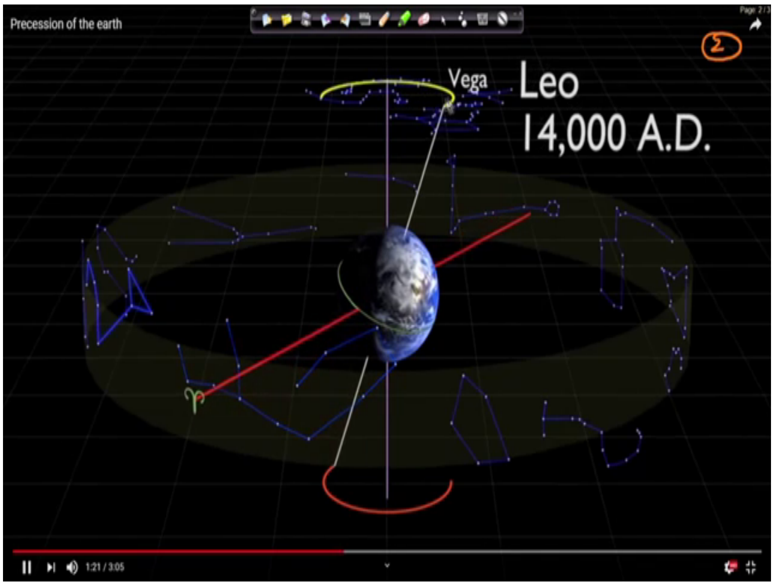

So, the axis the angle between the earth axis and this line, this line and this earth axis, it is a 23 and half degree roughly, and because of the nutation it is a average value. So, without because of the nutation, it keeps changing ok, it keeps oscillating about 23.5 ok. And this is you the north pole of the earth.

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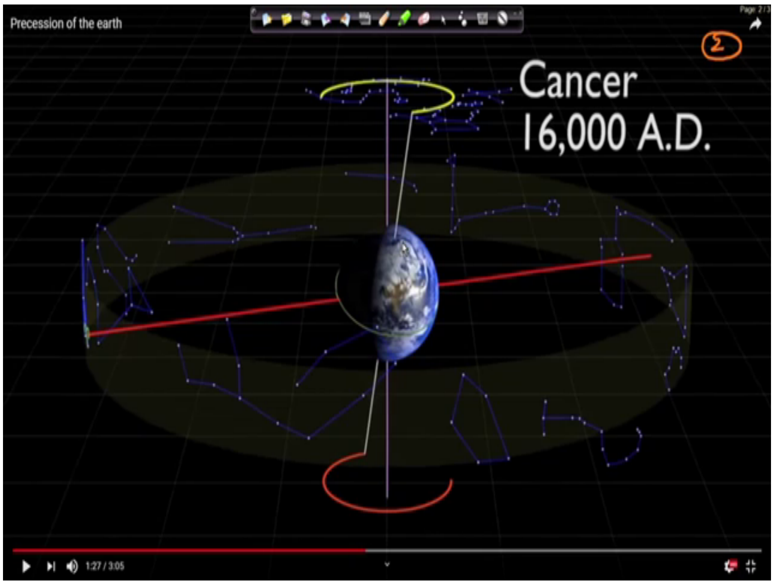

So, now if it is a showing here that at what time, where this polar axis will be pointing ok. So, and this cancer and other things, so what has been shown here. So, it is a showing this line of nodes as it is moving, so sometimes it will be in the constellation of aries, sometimes in the constellation of cancer and so on.

So, there are overall how many constellations are there, as you know from your astronomy part or the even in the astrology part, this is quite well known. This is the total 12 constellation, we call it Libra, cancer, Scorpio and so on, Pisces. So, your nodal line will be along the different constellation during different period of time ok.

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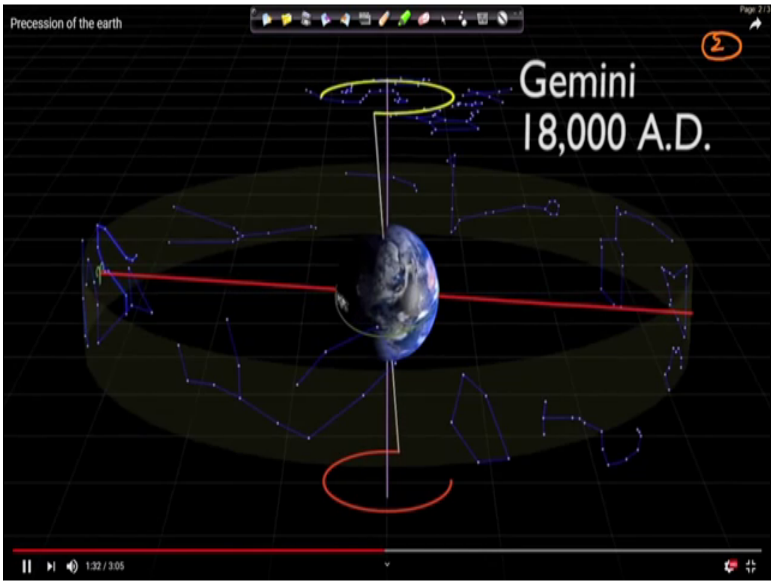

So, what are my objective here is to show you that even this earth ok, so this precisses and this nutates also ok.

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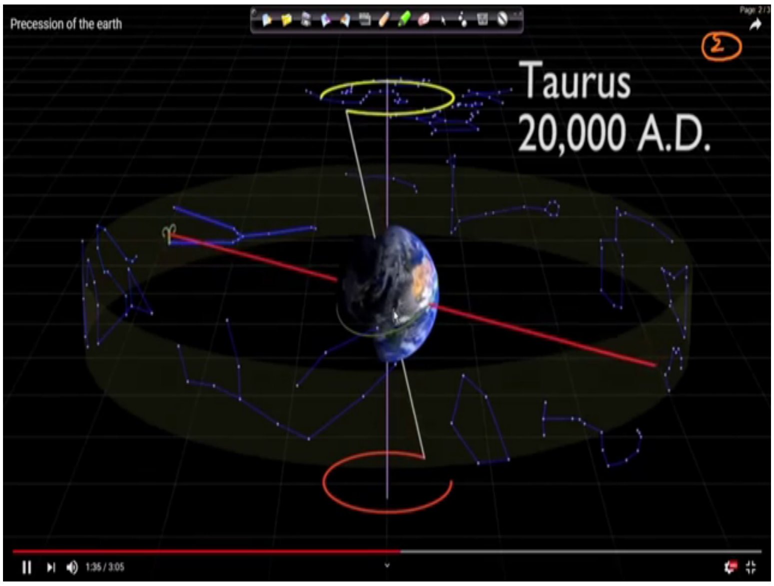

And as a result, it is a polar axis.

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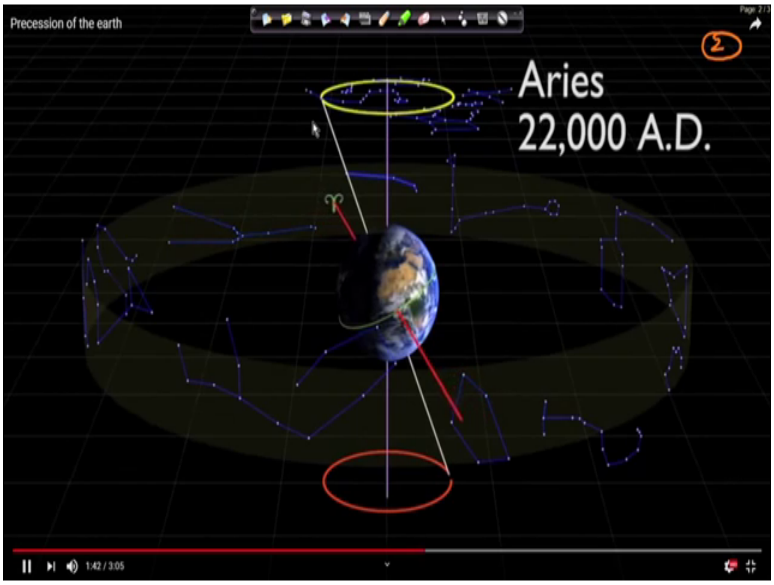

Right now it is a toward the Polaris, as we know our pole star is the Polaris, and it will keep changing over the period of time ok. So, why does this happen? This happens because of the torque acting about the centre of gravity of or centre of mass of the earth about the centre of mass of the earth, there is a torque acting. And that torque is responsible for is responsible for; that torque is responsible for your this precision motion ok.

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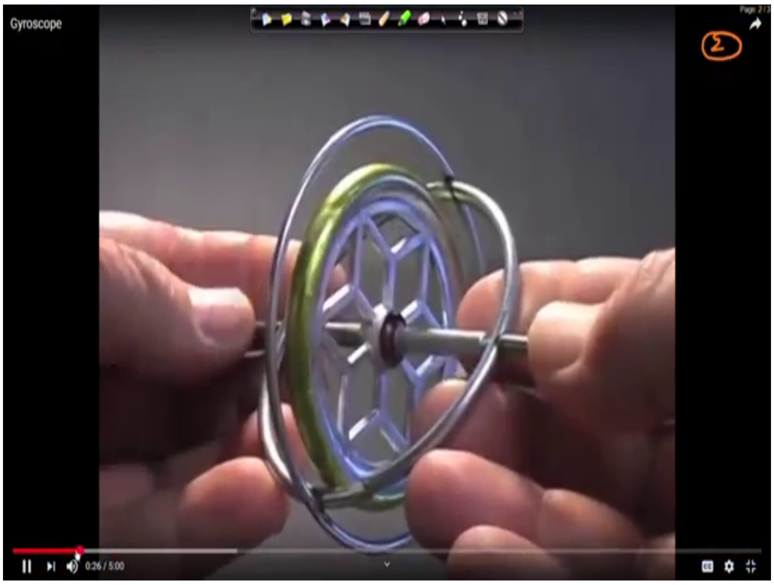

Let us see another video.

Wheel and axle spin freely secured in a metal frame, this simple device.

That is a simply a wheel is rotating about this axis.

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Where the cursor is there ok, about this axis the wheel is rotating.

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So, if you spin the wheel at a very high speed, so this is equivalent to a disc rotating about this axis ok. Now, if we do this thing on the earth, so we cannot make it free from the gravity ok, the torque will be acting on the system, and because of this torque the system will precess and this will nutate. So, this I am going to show you through the mathematics. So, this will be problem that will be doing it completely ok. So, let us look into this video. So, how the system it works?

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Here the spin stabilization concept also comes into picture.

Try this standard on a ended falls over.

Ok, see this normally if the wheel is not rotating.

Standard vibration (Refer Time: 13:04).

If you hit it like this, so it will simply fall.

Apparently (Refer Time: 13:07) by gravity.

It is a falling.

That spin we obtain hold actually.

But if you rotate it, rotate the wheel,

A (Refer Time: 13:16) by the simple.

Then it is spinning about certain axis ok. And there is the angular momentum associated with this. And wheel tries to maintain that particular direction until unless disturbed from that.

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So, here in this case the gravitational torque will be acting, and that will tend to disturb it. And therefore, as a result of that it precesses and nutates ok. So, you will see shortly in this video that you can look such through such videos ok, you can see that it is a maintaining this direction or here it was not able to stand, but now it is standing up. And this and once the velocity of the wheel, the angular velocity of the this wheel, it becomes a small, so thereafter it will come down.

Comparably this force was support the gyroscope like this.

Now, you see the earlier it was not a standing there, now it is a standing and it is rotating ok. It is a precessing ok, and there is certain nutation angle involved with this.

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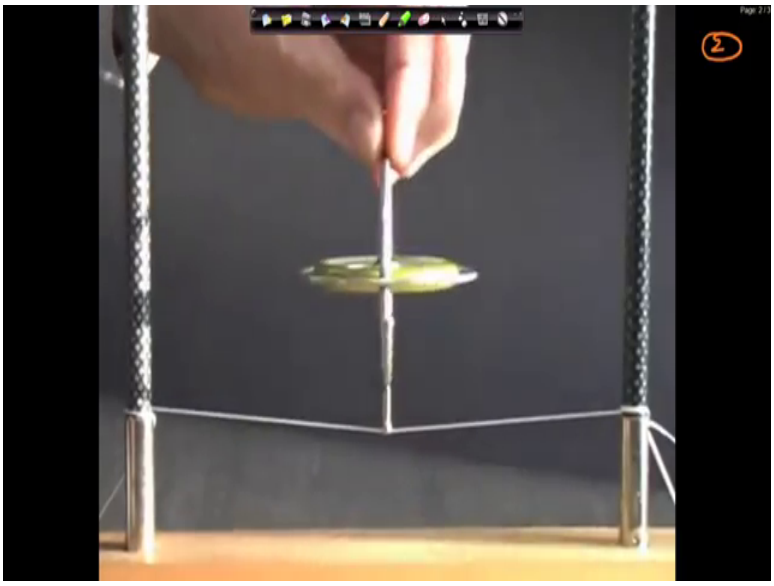

Spinning (Refer Time: 14:17).

So, this happens because it is having certain amount of angular momentum ok. And this we have used for the spin stabilization of the satellite ok. So, by a spinning any object, you can make it point along certain direction ok. And if you perturb from that direction ok, then it will tend to rotate, you can see again ok. So, this is undergoing precision motion and there is the associated.

This interestingly (Refer Time: 14:52).

Nutation angle also ok. So, we are stop here with this video ok. So, this cone simulates that gyroscope what we have seen ok. And it is rotating about this point ok, this is the ball socket joint, and it is spinning about this axis ok. So, derive the equation of motion for this analyse, how much will be the angle and other things. So, as I am going state it on the next page.

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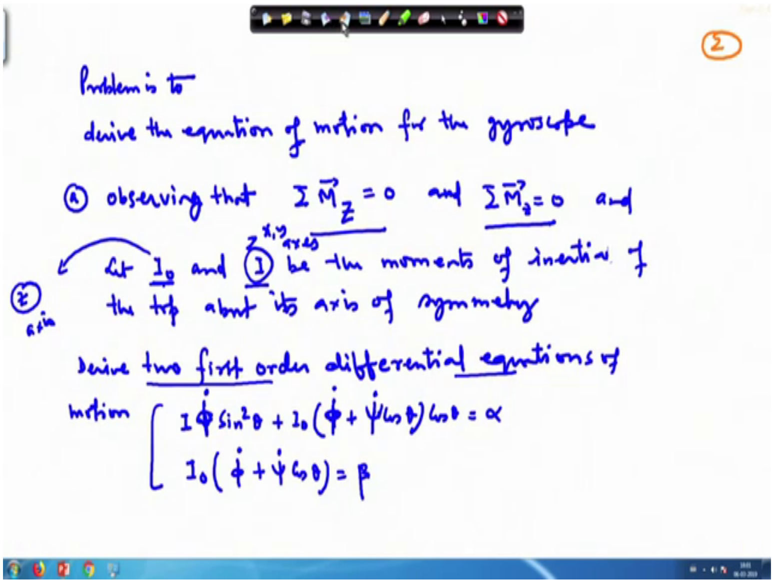

So, problem is to derive the equation of motion for the gyroscope. So, we state here the following problems, this is capital Z, and this is a small z ok. There are two axes we have already shown, this is capital Z here ok, and the small z is shown here in this place.

So, observing that E M Z and about the vertical axis, and about the central axis of the body about this axis there is no torque, because whatever the gravity is acting. So, torque due to this it will arise along the y axis ok. And similarly, along this axis there will be not be any torque, though it is a precessing along this, you can see the gravity is has acting downward. So, in no way it is the corresponding torque can be along this direction. So, therefore these two conditions are satisfied.

Let with the moments of inertia of the torque about its axis of symmetry. So, this is associated with this small z axis ok. And these are associated with x, y axis axis. So, these two are if this is along z, so this is along the respective transverse axis, which is x and y.

So, derive there is so some change of symbol, we will do here, because the symbols we have used phi dot for the spin. So, the same symbol, I will continue with, I will not use a new symbol. This is psi dot, and this will write as phi dot and this also as the phi dot. So, these are the two first order differential equation that you need to work out.

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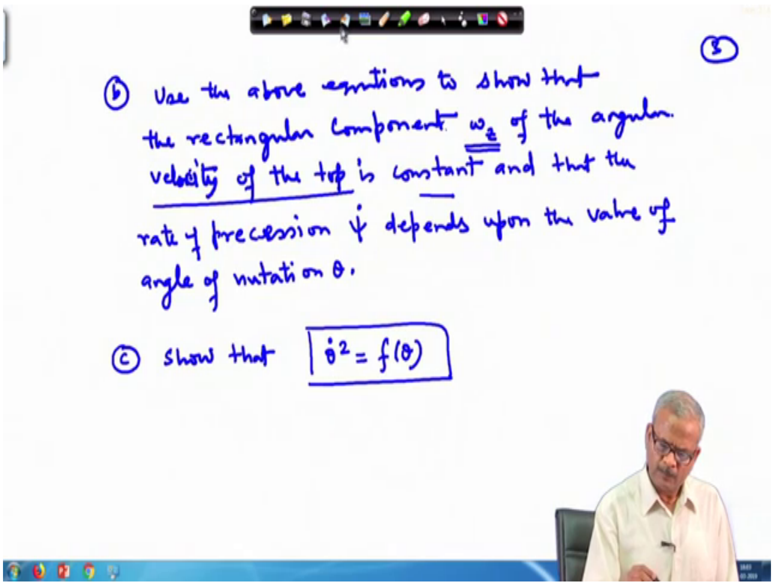

Use the above equations to show that rectangular components omega a small z, its written ok, angular velocity of the top is constant, so omega z of the angular velocity of the top is constant, and that the rate of precession. Also it can be shown that thus show that this rate of nutation, this can be written as a function of theta. So, these are the objectives that we need to fulfil.

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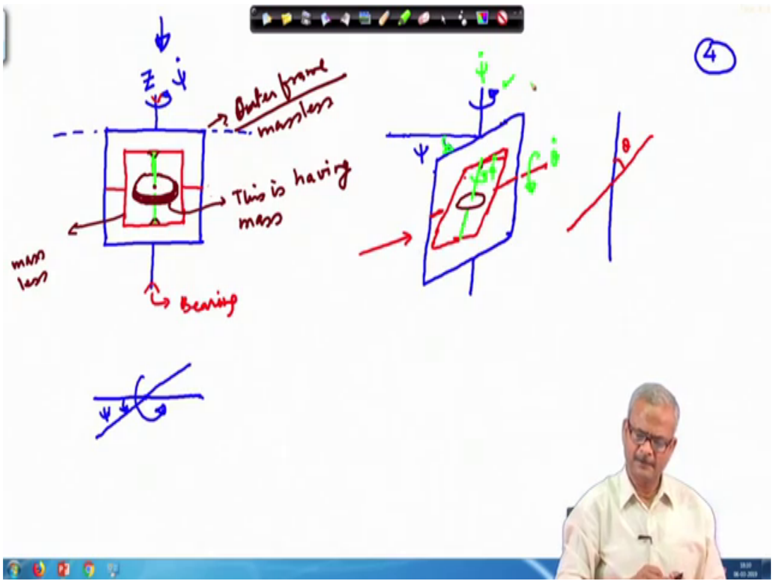

So, already we have discuss about the rigid body dynamics of a torque free motion ok. Here let us consider that there is a hoop like this or a square hoop which can rotate on this axis. And this we write as rotation rate as psi dot ok. And this direction, I will write here capital Z, then let us take another frame which is here attached to this. So, this is on the gimbal ok. So, it can rotate here, there is a bearing here, similarly if the bearing is here. So, I will not write here bearing, because I want to tag it with the corresponding directions.

And thereafter, there is another bearing here to which to a wheel is attached, so this is bearing here. So, what we have discussed earlier about the torque, it simulates this gyroscope, it simulates the same motion. How this frame this is the outer frame, we assume this to be mass-less ok. So, the associated moment of inertia will not represent, similarly this hoop also we assume it to be mass-less. And therefore, associated moment of inertia will not be present. This is having mass, and its rotating about this axis.

Now, consider this in a different shape. So, we have say this outer body, the outer frame, it has rotated to some other orientation which I am showing here. So, this has rotated here ok. And the rotation angle the corresponding rotation angle, we can measure ok. So, let us say that if I measure the rotation angle from, right now this is straight ok, so z we have shown here, and x and y we can show ok.

So, let us say this direction is x and y is somewhere or either we can do it in a little better way. So, first of all let us just look into the figure that it is a rotating. So, let us say that the frame has rotated by angle psi, we are not worried about from which axis, so this is the axis here. So, initially this frame was coinciding with this line, now it has rotated by psi angle here ok.

Then thereafter, we assume that this red frame ok, this is shown in the red. So, this red frame also this has rotated ok. So, this is the bearing here about which it rotates, and as a result it comes out ok. So, this rotation we can show it like this ok, it is a coming out. So, now it has got inclined ok. So, if I see it from the side, if I look from this side, so it appears like this. You have this frame, if you look this frame from the side, it will appear as a straight line ok. And the red frame, it will appear inclined like this; it will appear inclined in this way. And this angle be so it by theta that it has rotated by theta ok.

If you look at this frame, if I see it from the top ok, here as it is shown; so, from the top, so this frame from the top, it will also look like a straight line ok. And if I rotate it by psi angle, so this is going to look like this ok, it has rotated by psi angle here ok, so this rotation is like this. If seeing it from the top ok, so this is rotating anti-clockwise.

Similarly, this has rotated like this, and inside this then your wheel is there, the wheel that is rotating. So, you have a wheel here, this is mounted on this axis, and this wheel is rotating. So, this rotation is here given by phi dot ok. This rotation you are showing it by psi dot and the rotation which has taken place here. This was showing by theta dot, because this angle is changed from this place to this place, and therefore, this is psi dot ok.

As you can see like this is rotating like this, so this is giving rise to the psi dot ok. Similarly, the theta dot it is appearing like this. So, along the thumb direction theta dot will be appearing ok. So, this is the anti-clockwise motion shown here, so this is anti-clockwise motion. So, all the motion I have shown this is the anti-clockwise motion. Now, we are ready to discuss about the gyroscope what we have formulated, so we will continue in the next lecture.

Thank you very much.