Satellite Attitude Dynamics and Control
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Lecture - 42
Spin Stabilization (Contd.)

Welcome to the lecture number 42. We have been discussing about the Spin Stabilization of the Satellite.

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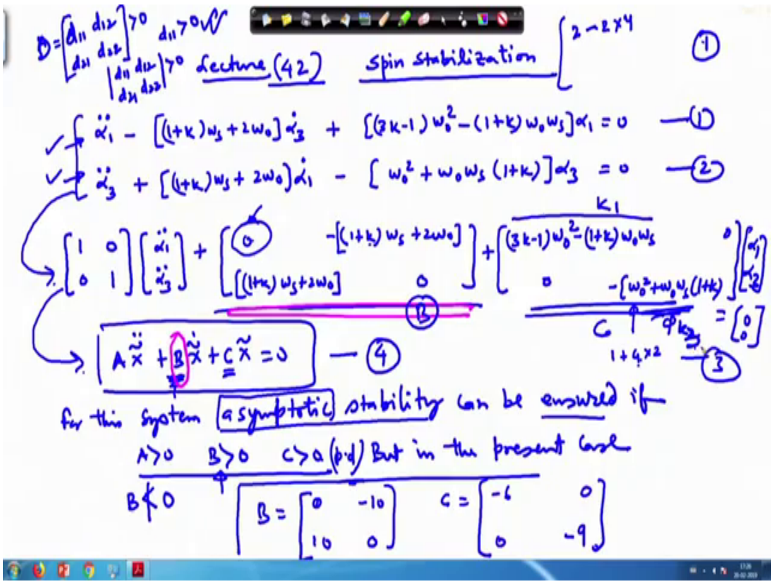

So, in that contest we derived these equations and these 2 equations can be combined into one. And this one can has been written in this format. So, what we were discussing that for this kind of system asymptotic stability can be ensured if A is greater than 0 B is greater than 0 and C is greater than 0.

But in the present case B is this is not B is not positive definite ok. So, these are greater than 0 implies this is positive definite. So, B is not less than 0, but this is not positive definite. Why? Because if we look here in this particular term. So, the initial entry this is 0 the first entry here. See if know from the previous criterion for any matrix say A equal to or let us name something else v C let us name it as D.

So, d equal to d11 d1 2 d 2 1 and d 2 2 for this matrix to be positive definite it is required that d1 1 should be greater than 0 and also the determinant d11 d1 2 d 2 1 and d 2 2 this should be greater than 0. So, at least this condition it is not satisfied here in this case, because here this entry is 0 ok. And therefore we cannot ensure asymptotic stability of the system; however, this system still can be stabilized because as I told you earlier this is a this term is appearing this B here.

This is not our damping term rather it is a gyroscopic coupling term and because of this the stability can be showed, but not the asymptotic stability. For the asymptotic stability the strict requirement will be that B should be greater than 0 which is not the case here. So, we continue discussing with this. So, let us pick up this equation the first equation.

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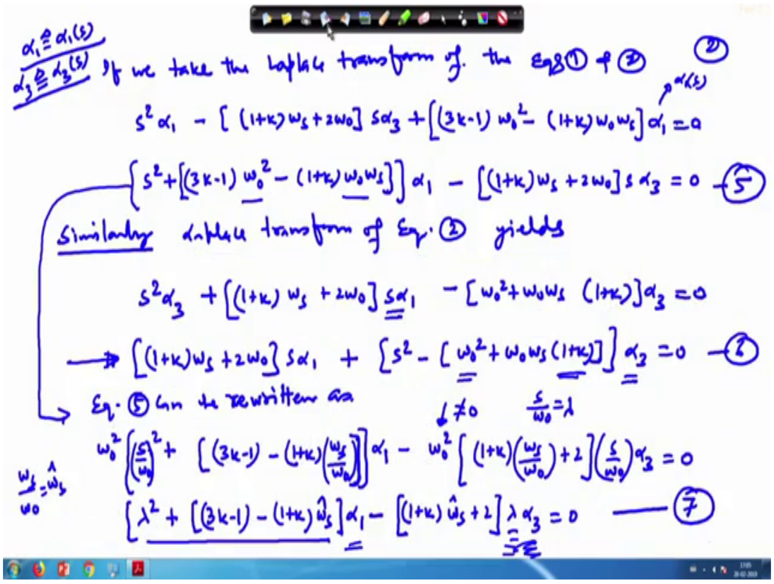

So, the first question we can write as if we take the Laplace transform of the equation 1 and 2 equations 1 and 2. So, these 2 equations we are going to take the Laplace transform on them. So, this held s square alpha 1 s. So, for shortcut I will remove this alpha 1 s and I will simply write this as alpha 1 assuming that alpha 1 is nothing but alpha 1 s here ok.

So, for it we need not put this extra because it is a function of s. So, I am not going to indicate this for the simplicity. And there after rest of the terms we can write for this equation. So, the order terms in this equations are minus 1 plus k omega s plus 2 omega 0 and then alpha 3 dot. So, this will be s times alpha 3. Again alpha 3 here is nothing but here alpha 3 s.

And here this also the Laplace transform this will be nothing but alpha 1 s, so, this equal to 0. So, these 2 terms can be combined together. So, this be s square plus 3 k minus omega 0 s square minus 1 plus k omega 0 omega s. And then we are going to take alpha 1 outside this minus 1 plus k. So, alpha 1 and alpha 3 terms we have separated out.

Similarly, Laplace transform of equation 2 is s square and then that the first term is of alpha 3 double dot. So, this becomes alpha 3 1 plus k. And then again we combine the terms. So, the first term will be 1 plus k omega s plus 2 omega 0. This term alpha 1 plus s square minus alpha 3 this equal this is the equation number 6.

So, what we can do that in this equation 5 can be re written as. We take omega 0 s square common out of this s by omega 0 s square omega s by omega 0 here times alpha 1 and minus omega 0 square, if we take it as common. So, this will be 1 by k omega s by omega 0 plus 2. And here we will write as s by omega 0 times alpha 3. This equal to 0 and we see that omega 0 this quantity is not equal to 0. So, therefore, we can remove it and let us write s by omega 0 is lambda.

So, this becomes lambda s square plus 3 k minus 1 minus 1 plus k. There is a bracket here, 1 plus k omega s by omega 0 we write as omega s square this is a non dimensional quantity, is this spin rate which we have non dimensionalized. So, this we write as omega s cap, times alpha 1 minus the same way here we will have 1 plus k times omega s square plus 2 times lambda times alpha 3 this equal to 0. This is equation number 7; following the same line of treatment we can reduce this equation number 6.

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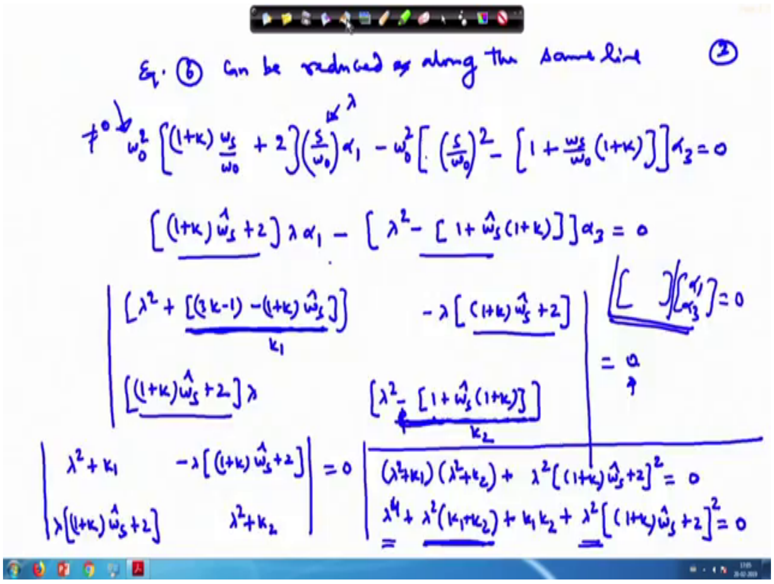

So, we will get here equation for the equation 6 1 plus k omega s plus 2 omega 0 1 plus k omega s omega s plus 2 omega 0. So, this becomes 2 and the other term this term here we write as s by omega 0 times alpha 1 s by omega 0 times alpha 1. And omega 0 we will write here as omega s square. So, one omega 0 will we can cancel 1 1 once inside the bracket 1 here in this place. The other term this is s square. So, here also omega 0 square we can take it outside. Then this is plus here it is minus omega 0 s square.

So, this is minus sign here. So, this becomes 1 this term it becomes 1 and we get here omega s by omega 0 this is omega s by omega 0 times 1 plus k. And then finally, here alpha 3, so, alpha 3 is taken out of the bracket this is equal to. So, what we see that because this quantity this is not equal to 0. So, we can eliminate and we can write this equation as 1 plus k omega s cap plus 2 and this quantity we are writing as lambda.

So, this is lambda times alpha 1 minus lambda s square minus 1 plus omega s cap 1 plus k this equal to 0. So, we have these 2 equations in alpha 1 and alpha 2. So, we can get the Eigen values from these 2 equations or we say for the non trivial solutions we cannot determinant form by this it. So, this is equal to 0 and that gives a first the required Eigen value equation. So, we combine both of them together, so, combining them together. So, let us go this is the first term here, lambda s square 3 k minus 1 lambda s square plus 3 k minus 1 k 3 minus 1. And then minus 1 plus k omega s cap; omega s cap, so, this is the first term the second term arises here in this place which is with minus sign.

So, here we will get here minus lambda times this lambda ok, 1 plus k omega s cap plus 2 ok. And this we pick up from this place. So, this is 1 plus k omega s cap plus 2 times lambda. And here this part becomes lambda s square minus 1 plus omega s cap times 1 plus k ok. This should be equal to 0. So, this 2 equations soap have been combined together. So, for the non trivial solution like what we have done that you take out alpha 1 and alpha 3. You can write it in the matrix format by alpha 1 and alpha 3 this equal to 0 those alpha 1 and alpha 3 is appearing here and also it is appearing here in this place ok.

So for the non trivial solution for this kind of system the determinant of this matrix must be equal to 0. So, this is what we are doing. And from there we get this Eigen value equation here or set of Eigen value polynomial. So, if this part we have lambda square 3 k minus 1 plus 1 plus k omega s cap 2. This is fine this equation you can check that here it is the symmetry here it is with minus sign, while here the sign is not there and this is lambda square 1 plus omega s plus k it is fine.

So, let us assume that this quantity is k1 and we assume that this quantity is k 2. So, this will help us little bit to reduce the problem. So, we can write this as lambda plus k 1 minus lambda times 1 plus k omega s cap plus 2, this equal to 0. Now if we expand this open up this determinant and write this equation. So, we get f delta f 4 degree polynomial in lambda. So, this k 2 what we will do we will put here a plus sign.

So, that we include this sign here also. There is a minus sign here. So, that k 2 will include this sign. So, that will make it little bit more covariant. And then here then this becomes plus lambda s square times 1 plus k omega s cap plus 2 whole square. And this quantity will be equal to 0. So, we have here lambda to the power 4 plus lambda square times k 1 plus k 2 and k1 k 2 plus lambda s square times 1 plus k omega s cap plus 2 this whole square this equal to 0.

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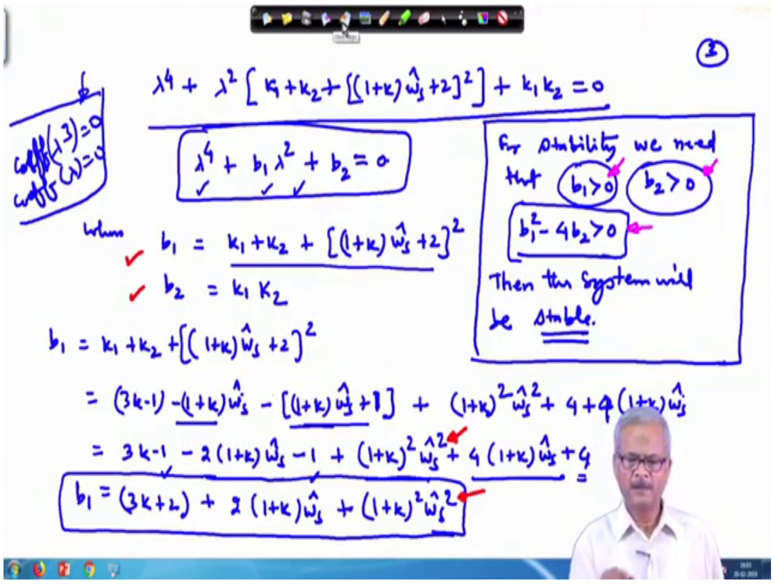

Lambda to the power 4 this part and then lambda is square term or p s here lambda is square term or p s here, so, we had both of them. So, the lambda square k 1 plus k 2 plus 1 plus k omega s cap plus 2 1 plus k omega s cap plus 2 this whole square plus k1 times k 2 this equal to 0. So, if we look here in this equation. So, this appears like lambda to the power 4 plus b1 times lambda square plus b 2 time plus b 2 0 this equal to 0. Where b1 equal to k 1 plus k 2 plus k omega s cap plus 2 is whole square and b 2 is k1 time k 2, this can be expanded and this can be reduced.

So, we have b1 equal to k 1 plus k 2 plus k whole square. So, if we insert the values for k1 and k 2, this we have written as k1 and this we have written as k1 and this we have written as k 2 with minus sign. So, this becomes 3 k minus 1 and then minus 1 plus k omega s cap minus 1 plus k omega s cap. So, this is the k1 term and k 2 term we have 1 plus omega s cap plus 1. So, there is a minus sign ahead of this. So, here so, this will appear with a minus sign the minus sign is here.

So, 1 plus omega s the bracket 1 plus k times omega s cap plus 1 ok. 1 plus omega s cap 1 plus k. So, this is what we have written here. And of course, the other terms we can expand it. So, if we expand this will be 1 plus k whole square omega s cap square plus 4 plus 2 times 1 plus k omega s cap. So, 2 into 2 this makes it 4 1 plus k omega s cap.

And then we can rearrange the terms. So, this is 3 k minus 1. Here this term and this term both are having the minus sign. So, 3 k minus 1 minus 2 times 1 plus k times omega s cap ok. And then this term comes with minus sign here plus 4 times 1 plus k omega s cap plus 4. So, this term we can look into this and this term is a separate one. So, this becomes and here we have minus 1 minus 1.

So, this becomes 3 k plus 2. If we take into account this so, 3 k plus 2. And from this place we will have 2 and this is plus this is coming with a this is coming with minus 2 sign this is plus sign. So, this becomes 2 times 1 plus k times omega s cap. And lastly we have this term plus 1 plus k whole square omega s cap.

So, this is your b1 term. So, if you remember that in the gravity gradient topic, we got the same kind of equation. We have ductily got the same kind of equations. So, for stability we must have we have discussed it in detail. So, I will again repeating the same thing for stability, we need that b1 should be greater than 0 and b 2 should be greater than 0. In addition, we should have b1 square minus 4 b 2 this should be greater than 0.

So, this is what for the gravity gradient also we have written. And I have explained it in details why the b1 and b 2 all the things need to be 0 ok. As you see that here in this case the coefficient of lambda cube, lambda cube term is not there it is a 0, it is a coefficient. Coefficient of lambda cube term this is 0 and coefficient of lambda term this is also equal to 0.

So, for this kind of system, if we want that all the roots they are lying on the negative real axis. So, there should not be any sign change here in this polynomial. And all the coefficients must be present, but here in this case this is those coefficients are 0. This way we have discussed and detailed it. So, I am taking it for granted that you have gone through the earlier lecture. So now, if b1 square minus b 2 4 b 2 this is greater than 0 and this quantity if these are satisfied, then the system will be the system will be stable, but not asymptotically stable. Again and again I am repeating because in this case. We do not have the B matrix the capital B matrix we have written this B matrix; this B matrix is not positive definite ok. As we can see here this is not a positive definite matrix.

So, we cannot ensure asymptotic stability here in this case, but at least we can ensure that the system remains stable means the once you disturb the system from the equilibrium state the disturbances they remain bounded. And for that these conditions must be satisfied at b1 is greater than 0, b 2 is greater than 0 b square minus 4 b 2 this is also greater than 0.

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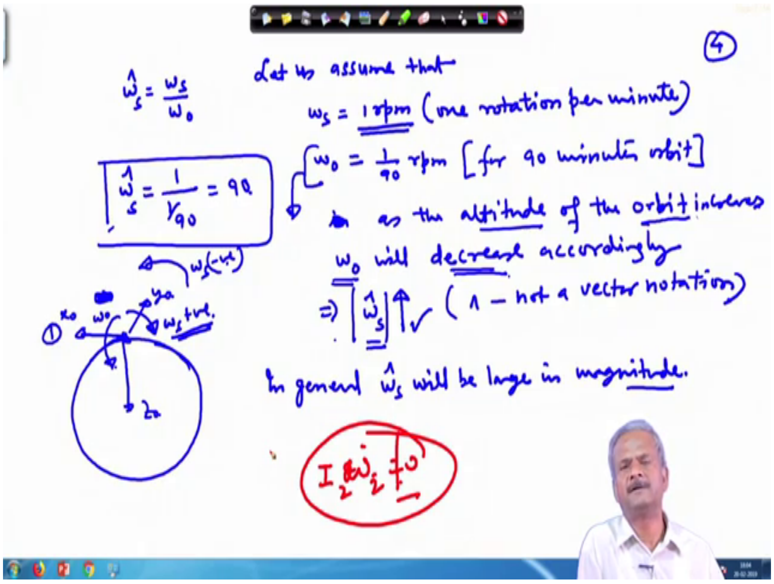

Now, we will look into this omega cap term which we have defined as s omega s by omega 0. Let us assume omega s equal to 1 rpm 1 rotation per minute ok. And omega 0 if it is a 90 minutes orbit, so, that implies 1 by 90 rpm for a 90 minutes FL orbit ok. So, omega 0 will be 1 by 90 rpm. So, omega cap in that case becomes 1 by 90 so, this is 90 ok.

So, in general or say as the altitude of the orbit increases; orbit increases omega 0 will decrease accordingly. And this implies that omega cap this omega s cap we have written. So, omega s cap this will grow in magnitude. As you remember that omega s the spin rate this can be positive or this can be negative because the satellite is moving here in this orbit here in this direction and your 1 axis is here or f 0.

We have taken here in this direction while the y 0 is along this direction and z 0 toward the centre of the. So, this is your orbital at system. So, here in this case your if omega s here in this case what we have taken that our this is the direction of omega 0. While this is omega 0 here while your omega s we have taken in this direction. So, this is a positive omega s. Omega s can be negative if the rotation is this way. If the spin rate is this way so, this is negative omega s. So, omega s can be both positive or negative, but if the as the altitude increases. So, the magnitude by s if we take it magnitude by this is not.

I am not showing a vector, this omega cap for just we have indicated for the indicating this as a non dimensional number. So, cap it does not indicate a vector here in this place.

This hat this is not a vector notation. Therefore, here in this place as we see that have as this omega the altitude increases. So, this quantity will keep decreasing. As the altitude of the orbit increases, this is the separate word here increase orbit increases.

So, omega 0 will keep decreasing accordingly in magnitude ok. And therefore and this implies omega s will go up it will become larger and larger in magnitude. So now we can discuss about the stability part again. So, in general omega s cap will be large in magnitude ok.

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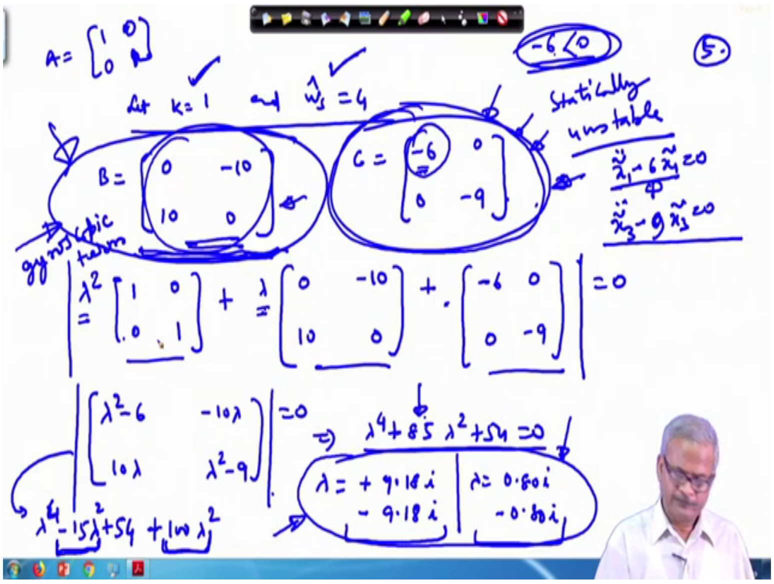

Now let us take one example. Let us say that let k equal to 1 and omega s cap this is equal to 4 ok. So, accordingly we can calculate this matrix the B matrix. If we go back for k equal to 1 and omega s equal to 4, this is your B matrix and this is your C matrix. So, k equal to 1 and omega equal to 4. So, I will write here in this place ok. So, the B matrix becomes if k equal to 1. So, this remains 0 and here k equal to 1 this is 1 plus 1 2 1 omega s cap we are taking.

So, if I will assume that it is already divided and therefore, this is minus 8 and plus 2 minus 10 this becomes minus 10 and this is of the opposite signs. So, this becomes 10 and this is 0. And accordingly the C matrix this is k equal to 1. So, 3 minus 1 this is 2 and this is omega square we have already eliminated. So, I will have to do it here, otherwise this part we are eliminating. So, 3 omega s cap we are taking equal to 4. So, 3 k minus 1 k equal to 1. So, this is 2 and then minus 1 plus k this is 2 into omega s cap. So, we get here 4. So, this is minus 6.

So, this is minus 6 and this element it is a 0. And this is 0. And here in this place we will have ah. So, here this will appear as one this term will become 1 and omega s cap. So, this is plus 4 times k equal to 1. So, into 2 4 into 2 8 plus 1 this is 9 and this with a minus sign. So, we put a minus sign here. So, once we are taking this example, this is 0 10. So, we go back to the. So, b equal to 0 minus 10 and 10 0 and c turns out to be minus 6 0 0 and minus 9. So, for this we can get the Eigen value of the system. So, your the first matrix A is nothing but one 0 1 1 and 0 0. So, directly also we can write the Eigen values like this.

So, determinant of we have to take the determinant of this quantity. This is 1 0 0 1 and plus lambda times 0 minus 10 10 0 and plus this term minus 6 0 0 minus 9. So, this way also we can get the Eigen value equation. So, from here just what we are doing that this is your a matrix, this is B matrix this is C matrix. And the corresponding double differential term has been replaced by lambda square single differential term by the second differential term by lambda square first differential term by lambda and because here there is no differential term. So, nothing is appearing here.

Now if we expand it and write it, so, this becomes lambda square. So, add all of them ok. So, we have to take the determinant of this becomes lambda square minus 6. And from this place we get minus 10 lambda. And this we will be get according 10 lambda. And this will turn up with lambda square minus 9 ok. So, for this we need to take the determinant.

If you now solve it, so, gets you lambda to the power 4 85 lambda s square plus 54 is equal to 0. And the values of the loads then lambda will be plus 9.18 i and minus 9.18 i the other set will be 0.80 i and minus 0.80 i. So, what we see that all this 4 Eigen values they are lying on the imaginary axis. Means as I told you that the system cannot be asymptotically stable. It is not possible because you B matrix this is not positive definite.

If it is positive definite and this is also positive definite then only we can say that the system is asymptotically stable. So, here in this case this is not positive definite, but still what I want to show that. So, this is statically not stable statically unstable, but because of this term B term which is the gyroscopic term; gyroscopic term ok. So, your Eigen values if this term B suppose that B term was not present.

So, in that case we will have only the C term. And with C term because it is a non it is a not positive definite. So, you can see that this is negative definite because this part this is 6 minus 6 the first term this is negative therefore, this is not positive definite. And therefore, only with C term your system cannot be stable and as you know that only if C is present.

So, we can write our equation x double dot minus 6 times 6 delta this equal to 0. So, you know that this unstable ok. And similarly the other part can be written as x say this is x 1 double dot and this is x 3 double dot minus 6 minus 9 x 3 tilde this is equal to 0. So, this is also unstable, but because of this term which is present this also not positive definite.

But a proper choice of the values here that can ensure that finally, your Eigen values do not lie in the right half complex plane. So, here the Eigen values are not lying in the right half complex plane. So, if this will be visible let us expand it here and then write. So, this will also give us certain insight this is lambda to the power 4 minus 15 lambda and plus 54. And from here we get plus this becomes 100 lambda square this is lambda square.

So, what we can see that this is a negative term and here this is a positive term. If it so, happens that if we choose the values of omega s and k such that these 2 comes together become negative ok; that means, the in this equation if this coefficient becomes negative ok. So, in that case your system will become unstable. So, by choosing properly the values of k1 k and omega s cap, we can ensure that the system remains stable.

So, in this case the system is statically unstable. We say it is a statically unstable because this the stiffness part is not positive definite ok, but this gyroscopic term this happens to support this case. And this pulls out of an instability this particular system.

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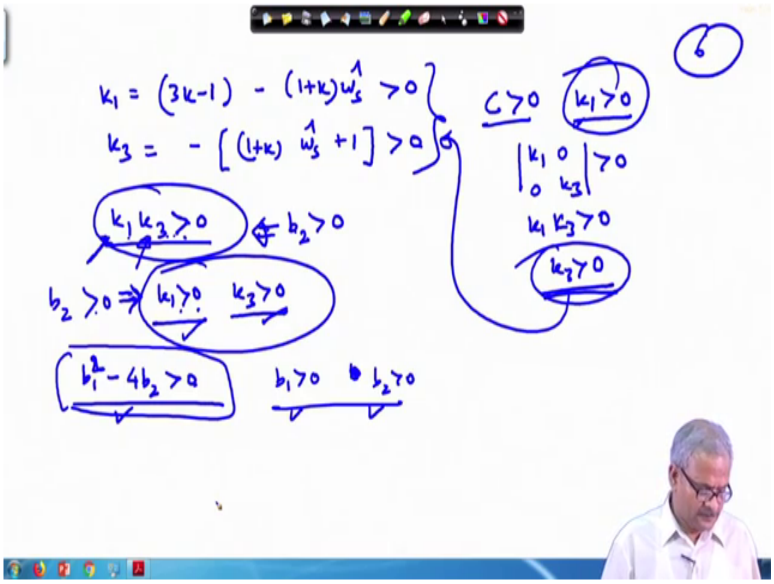

So, if now we can plot this equation. And if we plot say the k1 we have 3 k minus 1 minus 1 plus k times omega s cap this should be greater than 0 and k 3 1 plus k times omega s cap plus 1. This would also be greater than 0. Why this should be greater than 0? Because, these are the terms which appear in this matrix. This term we are writing as k1 and this term we are writing as k 2. So, k1 times k 2 this should be greater than 0.

So, what k1 times k 3 we have written here k 2 over k 3. This we have written this is k 3 term. This we are writing as k 3 k1 times this should be greater than 0. And anyways for the C matrix to be positive definite it is required that k1 should be greater than 0. And also the miner this k1 0 0 here k 3 this should also be greater than 0. So, this implies that k1 k 3 this should be greater than 0, because k1 is greater than 0. So, k 3 is also required to be greater than 0 means both these quantities need to be greater than 0.

So, one requirement is that this must be satisfied k1 times k 3 greater than 0 as we have written earlier, that this b 2 is greater than 0. This implies k1 is greater than k1 k 3 is greater than 0 k1 times k 3 and this also implies that k1 should be greater than 0 and k 3 should be greater than 0. It cannot be this can be greater than 0 even if k1 is less than 0 and k 3 is less than 0 both are negative. So, in that case also this can be greater than 0.

But that will violate the requirement here that k1 is greater than 0 and similarly k 3 greater than 0. So, therefore, that case is ruled out only this is acceptable. That k1 and k 3 both of them should be greater than 0. So, b 2 greater than 0 this implies that k1 is greater than 0 and k 3 this is greater than 0. In addition, we have b1 square minus 4 b 2 this should be greater than 0.

So, taking, this we can plot it b1 greater than 0 and b 2 greater than 0 and this part we can plot and if we plot. So, the plot will look something like this.

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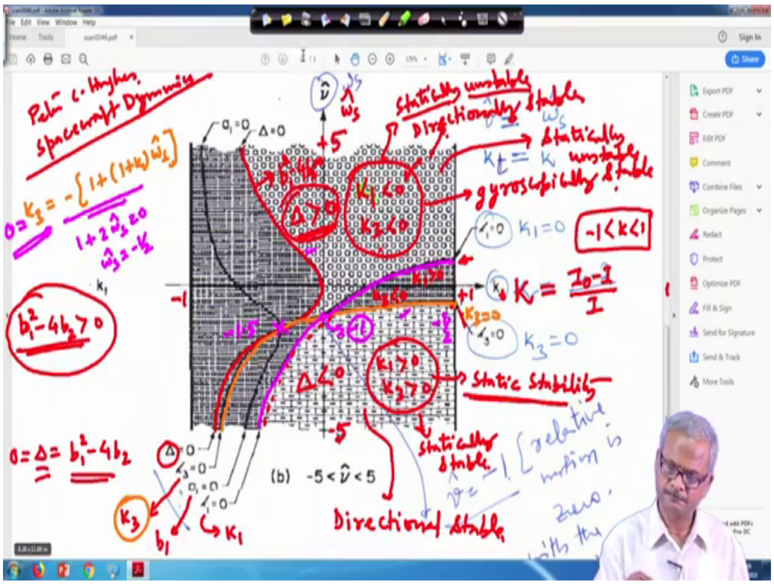

So, the here is a graph for see what we have the graph. So, this graph is plotted here. This is a figure from peter c process. And this is a book on a spacecraft dynamics, but this book is almost unreadable. So, I have taken this figure from that place and I will show you what does this mean. So, the delta is basically b1 square minus 4 b 2, this equal to delta. So, this curve here it is plotted for from the first curve it is being shown, so, this curve.

This is plotted for delta equal to set this equal to 0. And then plot it by giving the b1 and b 2 these are functions of k and omega s cap. So, here instead of new cap we have used this omega s cap. And here this is written as k t, but we are using this k term here. And k is nothing but I 0 minus I divided by I. And as we know this k will lie as from our graphite radiant discussion, this k always lies between within the range minus 1 to plus 1 ok. This small k1 slanted k1 is shown this is nothing but your k1 and this is your k 3 as we have previously discussed and this is your b 1 equal to 0.

So here it is showing k 3 this equal to 0 this curve. And this value is minus 1 by 2 it is not given here, but if you put the say here it is on the right hand side ok. And your k 3 equation if you put in that. So, k 3 equation we have written as minus 1 plus k times omega s cap. So, this point we are picking up here in this place ok and k 3 equal to 0. So, this line it is been shown that this line is k 3 equal to 0.

So, this line is running from this place and it is going like this. So, this is your k 3 equal to 0, this is shown here while. This z 1 and this line is for this line is for k1 equal to 0. And this line comes like this and both of them cut on this y axis on the omega s cap axis. So, this is their cutting point. And this particular line b1 equal to 0, this is not of interest to us. So, if you set k 3 this equal to 0 and solve this.

Here k equal to 1, we are taking it on the right hand side extreme right ok. So, if you take k equal to 1 put it here. So, this is 1 plus 2 omega s cap this equal to 0 and that implies omega s cap will be minus 1 by 2. So, this quantity here this is minus 1 by 2. And then it cuts the this axis here in this place. So, for this part k will be equal to 0. So, set k equal to 0 in this equation. And that will give you the value of how much the omega s cap will give.

So, here if you put that, so, this point it will have a value of minus 1 ok. So, this is minus 1 here ok. The point where this curve and this particular curve both of them cut here in this place this value is minus 1.5 ok. Accordingly, you can look into this. Now, so, you know that what this graphs are standing for. So, this is let us go on the top and look into this figure ok. This figure we cannot look because first we will discuss it here and then we will take that figure again on a new page. So, here if you see the now we will look into our requirement. On this side k1 k 3 k1 k 3 and here also we will have the k1 and k 3.

So, what are the values of k1 and k 3 in this region. On this line k1 is 0, on this line on this line k1 is 0, on this line we see that k 3 0. So, what about this range? So, here the range which is shown and this range this one is your b 1 square minus 4 b 2 is equal to 0; which is written as delta. So, here k 3 is less than 0 and k1 is also less than 0. While we are in this range k1 is greater than 0 k 3 is also greater than 0. In this part k1 is greater than 0 and k 3 is less than 0.

This you can just check by putting various points in this region and in this region, in this region. So, on similarly here the quantity delta this will be greater than 0 while here in this range delta will be less than 0. So, this is shown here this boundary line like this from this place.

So now we can conclude few things. This k1 greater than 0 greater k 3 greater than 0, this is a requirement for static stability. So, in this region your static stability is ensured. So, this region is statically stable. While this region what we see that k1 is less than 0 and k 3 0 both are less than 0. So, this is statically un stable, means stiffness is not there ok; however, this delta is greater than 0. And this delta greater than 0 means b1 square minus 4 b 2, this quantity is greater than 0. So, if this quantity is greater than 0. So, this region becomes gyroscoply stable. While this is statically unstable, because, k1 and k 3 both are less than 0 here in this range. And here k1 k 3 are greater than 0. So, therefore, the static stability we can say that the system is statically stability as per our earlier discussion.

So, for the Eigen values we also we must have for the gyroscopic stability b 1 square minus 4 b 2 this should be greater than 0. So, even if your system, here you can see that even if this is the static stability is not satisfied, but if this quantitie this is satisfied then the system will be gyroscopically stable. And also this will be directionally stable. So, what does mean by directionally stable? We can discuss it in the on the next page.

Similarly, this part we will have the directionally stability. While we will take it bit afterwards ok. So, we can go on the next page and look into the figure. So, this figure it is showing you that this is from minus 1 to on this side this is plus 1. So, k value is ranging from minus 1 to plus 1, while omega s cap is ranging from minus 5 plus 5 here on this side to minus 5 here on this side.

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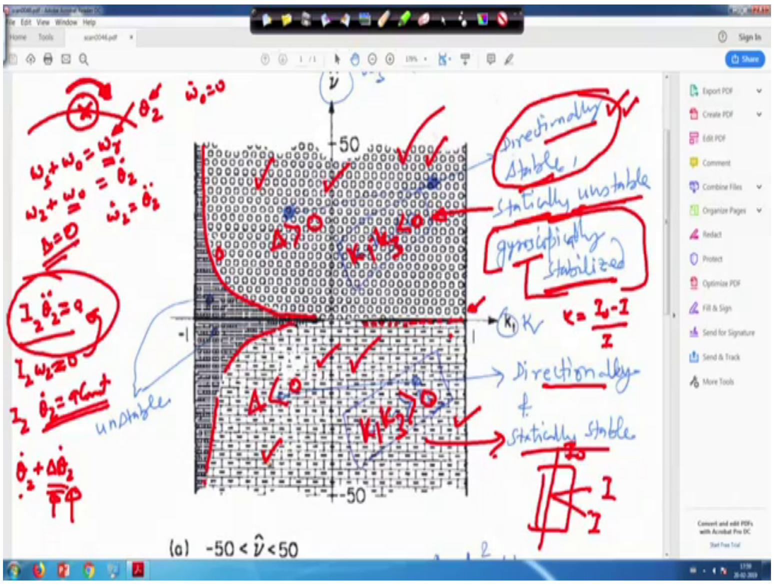

So now we take this figure. So, this figure what you are showing seeing here. So, this is here as you can see this cursor is. As you can see that nu cap or the omega s cap omega, s cap is here minus 50. And on the downside you can see that minus it is a minus 50 here in this place. The k value does not change k is from 1 to minus 1 on the left hand side. So, the figure what we have seen here in this place this is a expanded figure near origin. So, this has been expanded near origin means you can see that this minus 1 to minus 1 range, but her here on the y axis 6 it is a plus 5 to minus 5. While here it is minus 50 to plus 50 on the y axis.

So, if you expand the region near the origin or say region near the x axis, then you get the figure which is shown here in this place. So, these are the figures drawn using the same thing like we have discussed earlier. This figure these are the boundaries that we are saying here. So, while it is not visible here you can see this small portion here. So, this portion is indicating the same line. This is your delta; delta v 1 square minus this delta equal to 0 it has been plotted. While here on this side your k1 and k 3 it is plotted which is not visible. So, this region it is written directionally stability, statically stable gyroscopically statically unstable and gyroscopically stabilized.

So, k1 times k 3 this is less than 0 and therefore, it is statically unstable. While directional stability I told you that we will discuss this what does mean by directional stability? So, your satellite is rotating like this, it is a spinning. So, as we have looked into that i 2 times theta 2 double dot this quantity is 0 for i 2 times we have got in terms of this. And here omega 2 is the absolute spin rate about the 2 axis. So, 2 axis is going inside here into the page.

So, this can be written in terms of theta 2 the other term we have as you remember that omega s the spin rate this is nothing about omega 2 plus or omega s plus we have written omega s plus omega 0 equal to omega r. The relative velocity with respect to the orbital axis. And this is the quantity which is responsible for giving you theta 2. So, theta 2 is measured from this is the Euler angle which is being measured from the second axis of the orbital reference frame.

So for this part if the satellite is rotating like this. So, what we can see that, if we this equation we can write it in terms of; obviously, if we differentiate this. So, this quantity we can write us omega s is your omega 2. This is the absolute angular velocity the spin rate and plus omega 0 equal to omega r which is nothing about your theta 2 dot. So, if we differentiate this. So, another 2 dot will be equal to theta 2 double dot because omega 0 we are taking a circular orbit.

So, omega dot 0 this will be equal to 0. So, therefore, this can be replaced by this equation. And in this equation what we see that, if we do it little bit of change in theta 2 dot this is the change. So, means you are changing the angular velocity here. If you change the angular velocity here, so, because this on the right hand side you have this quantity is equal to 0. So, this implies that theta 2 dot this be will be equal to f constant ok.

So; that means, if you are checking this theta 2 dot by a small quantity. So, this quantity is not going to die out, this will remain ok. So, along the spin axis your satellite is not stable. Means it is a not stable in it is not stable in pitch ok. This is along the this should not use the along term where in this should say that this is not stable in pitch means, here if you create any disturbance that disturbance is not going to die out. As I am showing by this particular arrow.

So, this simply implies that the 3 axis stability of the system is not possible. Because of this particular equation 3 axis stability is impossible. Because on the right hand side we are getting here 0. And this we have taken for the axis symmetric case where k equal to I

0 minus I divided I, I 0 is say in the case of the cylinder, I 0 is along this direction and I I is along this direction.

So, but however, because as you can see that the (Refer Time: 65:22) is spinning means if you look here in this part. So, here say this is your satellite this is a cylinder and it is like this. So, this is the orbital plane ok. And satellite is rotating like this suppose. If the satellite is rotating like this, so, it is angular velocity vector will be along this direction ok. See here it is visible. So, angular velocity vector suppose here it is along this direction depending on the orientation; obviously. So, this is because coming because of the spinning. So, as you know that if something is spinning and I will show you one video from You Tube in the once we have start the next topic.

So, this cap if you disturb it little bit if you disturb it. So, it will start processing ok, but it will remain bounded. So, that it says that this kind of system which is rotating about one of the axis ok. So, in that case of the top, if we take the case of a top is rotating on this axis ok. And if you tilt it little bit. So, what happens the top due to gravity comes into picture. And because of that it is a continuously rotating.

So, similarly in the case of the space craft we have this in the orbit and this is rotating on this axis. So, because of this angular momentum this inertia is there. So, it is each vector it will resist to any change. So, this directional stability is ensured means it is a directionally stable it will try to point along this direction ok. So, therefore, it is written here this is directionally stable. And strictly stable unstable because of this region. And gyroscopically stabilized because in this region your delta is greater than 0. While here in this place delta is less than 0 therefore, this is gyroscope gyroscopically unstabilized. So, directionally and statically stable.

So, your k1 3 k 3 this is greater than 0 here in this place. And therefore, this is statically stable. And directional stable it is directionally stable here also in this place and here in this place also. And more and more the spin rate more the directional stability will reach. Because then it is angular momentum will be very high and changing it by even a small amount it will require a large amount of torque ok.

So, therefore, in both these regions, the system remains; system remains statically stable and sorry. In both these region system remains directionally stable here it is statically unstable, while here it is statically stable here the gyroscopic stability is ensured while here in this gyroscopic stability is not there. FL. We have discuss about the stability of the system.

Now, there are 2 correction here. Perhaps this part we missed in the while writing this square was missing here. So, I have put this square. So, note down this correction here in this place. So, we end this lecture and we will continue in the next lecture with the new topic on we have finished the spin stabilization.

So, remember that whatever we are doing it is a elementary course ok. It is not a very advanced course. We can have lecture on the satellite attitude dynamics for one more year and we will not be able to finish ok. So, we are just presenting a limited part and in that scope or keeping in view the undergraduate and the postgraduate requirements. But in this spin stabilization now, as you know that we have taken the axis symmetric case if it is not axis symmetric so; obviously, the system will be more complicated. I times theta that is what we have got that I times omega I 2 times omega 2 dot this equal to 0 this quantity will not be 0 in that case.

So, we will have a complicated system in and it will be difficult to analyse also. So, we have stopped for the stock for the time being with this and in the next lecture perhaps we are going to a start with control moment gyros we have and the reaction is. Or say that one satellite is we are having one satellite inside that one wheel is moving and using that wheel or the satellite may have 2 parts one part is stationery and other parts continuously rotating. So, we will look into the stability of that kind of system and also how the control moment can be generated using the reaction wheels or the control moment gyros. So, this is topic of our next lecture.

Thank you very much for listening.