Satellite Attitude Dynamics and Control
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Lecture - 41
Spin Stabilization (Contd.)

Welcome to the lecture 41, we have in working with the Spin Stabilization we will continue with this.

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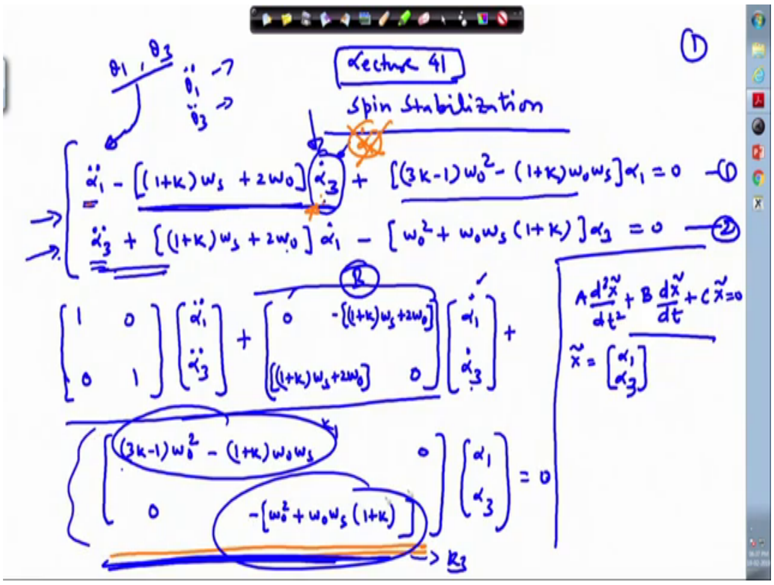

So, last time we have derived these two equations. So, what we have done that theta 1 and theta 3 we have converted in terms of alpha 1 and alpha 3. So, the difference between theta 1 and alpha 1 it is not very large, but there by we have been able to eliminate the periodic coefficients from the equations for the differential equation for which was involved in theta 1 double dot theta 3 double dot. So, these things we have eliminated.

Now, these two equations it is a quite easy to solve. Let us first write it in a matrix format so, that we can discuss few things ok. So, first term this alpha 1 alpha double dot and alpha 3 dot we can the for the first equation we can write this. You can see that if we multiply this matrix. So, it will come as alpha 1 double dot which is in the first equation similarly we should get this term. So, here we will have minus 1 plus k times omega s plus 2 omega 0 here this multiplied with this this becomes 0. So, only this term remains.

So, we recover this term here and similarly the third term we need to get here. So, for the third term we will right here in this place. So, this is 3 k minus 1 omega 0 a square minus 1 plus k omega s 0 and then we will have alpha 1 alpha 3 this equal to 0 on the right hand side. So, if we multiply we recover this particular term here, the same way we can get for this this equation. So, they can be combined together ok.

So, combining this so, already we have written for this part alpha 3 double dot this part we need to write. So, this is 1 plus k omega s plus 2 omega 0 and this entry will be 0 here, here in this part we will have 0 here as this entry and this part will be minus omega 0 square. So, if we look into this equation. So, it appears something like A times plus B times d x tilde by dt plus C times x tilde where equal to 0 where x tilde is alpha 1 alpha 3. So, as far as this equation is concerned so, this equation can be written in this format and till discuss about this particular one.

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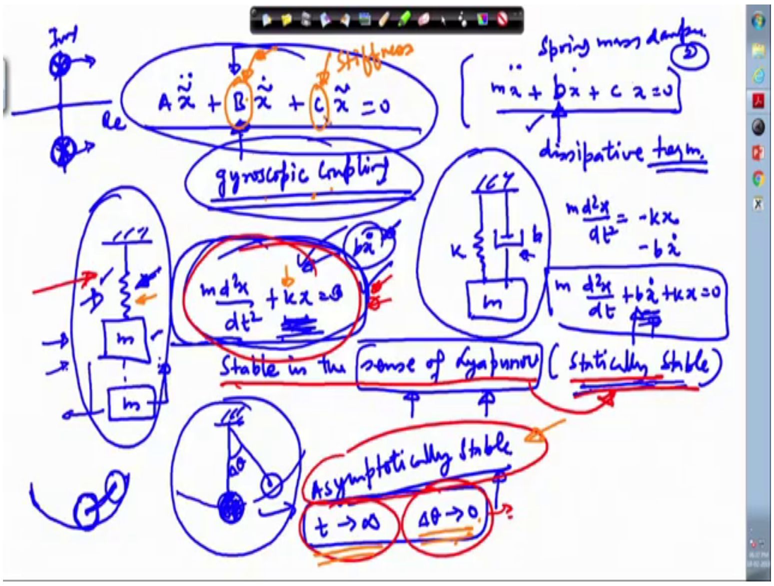

So, we write this as x tilde double dot plus B times x tilde dot plus C times x tilde this equal to 0. So, for a spring mass damper system you if you remember that for the spring mass damper system the equation of yours in this format b times c times x tilde. So, it appears in the same format, but there is a difference this term in the this is for the spring mass damper system spring mass damper. So, this b term is indeed this is the dissipative term. So, because of this the system will continuously lose its energy because of this term because, it is a dissipative term just like in this spring mass system, you have a mass hanging over and so, this is the mass.

So, for this you can write this equation this is the k the spring constant for this and damping coefficient we can write as d here ok. So, how do we write this equation? m times d square x by dt square this we write as minus kx minus b times x dot. So, this is bx dot plus kx this equal to 0. So, this tilde we should not put here tilde we are using only for the vector. So, ultimately this is this two equations they look similar, but there is the difference that this is a dissipative term while here in this case if you look here in this equation.

So, this part this is not dependent on the alpha 1 dot this is alpha 3 dot here this is not alpha 1 dot. Had it been alpha 1 dot, then we can say that this is a dissipative term if this is a plus with a plus sign if it comes with a minus sign, then it becomes opposite of that ok. But here the because of this presence of alpha 3 dot this acts like not like a if it is not a genuinely dissipative term what we call this as the this is called the gyroscopic coupling. Now, let us look first into the just spring mass system ok. Sorry if we write equation for this we write it like this. And for we know that this kind of system its a stable in the sense of Lyapunov stable in the sense of Lyapunov or we call this is statically stable.

This kind of system it is also called it is a marginally stable why we are calling this as the statically stable? Because as soon as you displays this system from this position to say some other position here it is a displaced here. So, immediately restoring force is generated. So, earlier also we have discussed that say if I have a ball like this and in there we are putting a ball. So, if I displace it here. So, immediately the restoring force is generated. So, similarly here one side displacing it; so, the restoring force is generated. So, this kind of system we say that this is statically stable system, and in a broader sense we call this is this is stable in the sense of Lyapunov, now you might have heard or you might not have heard about the Lyapunov stability analysis.

But it says that if a system let us again take for a simple case like I have a pendulum here and this pendulum if I displace it here in this place. So, over a period of time this pendulum will oscillate and it will come back to this position. So, what is happening exactly? This comes to this position over a period of time because of the presence of this particular term this is a dissipating term. So, here it continuously loses its energy and therefore, it returns back to the original equilibrium position. So, this kind of system we say this is asymptotically stable; asymptotically stable means as t tends to infinity your disturbance let us say this is the theta or delta theta this is the disturbance you have given.

So, delta theta this will tend to 0 means it will return back to the equilibrium stage ok. And equilibrium already we have defined this is defined as the stationary state of a system ok. So, this system is asymptotically stable, this system is also asymptotically stable because there is a damping term present here, but here the system we are discussing their this is the gyroscopic coupling term these is not the actual damping term. So, this type of system this is not asymptotically stable in the asymptotically a stable there is the dissipation of energy just remember ok. So, once there is no dissipation of energy which is here in this case is suppose that this is a purely spring mass system we have written there is no dissipation term present in this particular equation.

So, what happens? Once you disturb it and leave it. So, theoretically it will keep oscillating for the whole life. Assuming that there is no air resistance or there is no dissipation of heat in the spring ok. So, if there is no dissipation of heat in the spring. So, we have the or either the air resistance is not present. So, this is dissipation term is absent only these two terms are present here. So, this kind of system its call marginally a stable means the poles of the system on the real and imaginary axis if we plot. So, it will lie on the imaginary axis and this will keep oscillating for the theoretically this keeps oscillating for the whole life ok. But in reality here for this particular system it so, happens that the energy dissipates and therefore, it returns back to.

So, we need to add one particular term here b times x dot we have to add it here for making of real representation of the system though damper is not present ok; so, here if we are putting a damper here. So, this is just a modeling of this resistance ok. Energy is getting launched into this spring and then the energy is getting dissipated because of the motion which is due to the air resistance. So, all those things can be model through this b times x dot term ok, but in practical many of the systems are such that if the poles are on the imaginary axis here.

So, they absorb energy from the environment and this poles move into the right half complex plane and therefore, this kind of system mostly they become unstable. While here in this case while we are discussing this. So, this kind of system in practical sense it will be a stable because once you disturb it. So, it will oscillate and slowly-slowly energy will get dissipated in the spring in the form of it, and also that will be aerodynamic resistance because of that the motion will die out. So, it will lose its energy.

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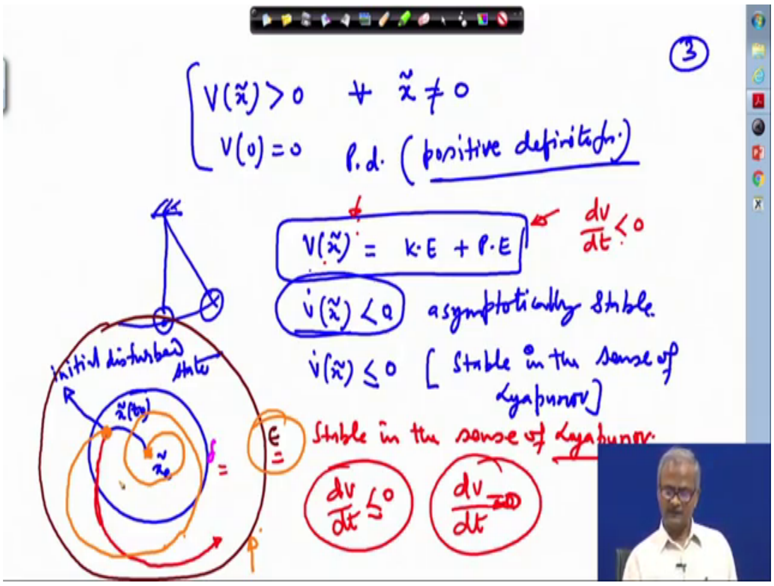

So, in the Lyapunov stability analysis we define a positive function say V x tilde, where x tilde is the state of a system ok. So, this system this is greater than 0 for all x tilde is the x tilde not equal to 0 and V 0 this equal to 0. So, basically this is a positive definite function and Lyapunov in the Lyapunov analysis. So, what is assume that if I have this pendulum system if I displace it. So, it is going to lose continuously its energy.

So, say here in this case I can write its energy V x tilde in the form of its a kinetic energy and plus potential energy ok. Now so, over a period of time; so, this will continuously decay ok. So, V dot x tilde is less than 0 so, we will say that the system is asymptotically stable. If V dot x tilde in any of the case it turns out to be less than equal to 0. So, it is a call stable in the sense of Lyapunov stable in the sense of Lyapunov. Perhaps where we may not get time to discuss about the Lyapunov stability analysis in detail, because there are many other topics are remaining, but I am giving you the idea.

So, if your equilibrium position is here x tilde e and you are disturbing this system from this position to this position ok. So, this is your x tilde t 0. So, this is your initial state once your disturbed initial disturbed state and if you leave the system; so, there after the system remains within a bound ok. So, let us say this I call this as the delta neighborhood and then I draw another neighborhood this I called this is a boundary which I called the epsilon neighborhood ok. So, if I leave it here in this place and this system it is a trajectory its remains bounded means, it does not leave this epsilon neighborhood then we say that the system is stable in the sense of Lyapunov stable in the sense of Lyapunov.

So, what we have observed that, if this is the energy function. So, energy function if the energy is getting dissipated means this should be dv by dt getting this v is getting dissipated. So, it implies this quantity must be less than 0 because it is a continuously decaying. But it so, happens that sometimes it may not decay also. So, in that case we write this as dv by dt this is less than equal to 0 ok. And if dv by dt this equal to 0 means the your energy is not decaying at all. So, whatever you wherever you are disturbed it. So, that disturbance will not die out ok.

So, it so, happens with this kind of system that if there is in the theoretical system this particular equation, that if you disturb it will keep oscillating for the whole life in theoretical sense not in the practical sense. So, this kind of system we will say that its a statically stable or statically stable or stable in the sense of here we have written it stable in the sense of Lyapunov ok. While the asymptotic stability is different it is a as t tends to infinity delta theta tends to 0 and one more condition is there for the asymptotic stability that it never leaves epsilon neighborhood bound.

So; that means, if you are looking for the asymptotic stability. So, I will show it by some other color ok. So, if the system is asymptotically stable. So, and you are leaving it here ok. So, it will never leave the epsilon neighborhood and thereafter over a period of time it will decay to the equilibrium position. So, this is the difference. So, only convergence to delta theta tends to 0, this does not say that the system is asymptotically stable. This must satisfy this condition that this trajectory remains bounded in the epsilon neighborhood if it is not bounded then it cannot be a asymptotically stable ok.

Remember this is very important, if this leaves this boundary this brown boundary this trajectory goes out of this brown boundary, then the system cannot be asymptotically stable even though the state finally, comes to the equilibrium state. So, these two things are. So, I will wind up this part here at this stage and restore the what we have been discussing about. So, we have been working with this particular system. So, here B is not a dissipative term and C is stiffness term this is the stiffness because it I just because of the springness stiffness just like here this is the stiffness term.

So, same way here we have written is the stiffness term while this is not a true damping term this is coming from gyroscopic coupling. So, this is not a true damping term and we should never be misled by this writing here written in this way that this is a damping term, because in the damping it will explicitly depend on here this would have been alpha 1 dot rather than alpha 3 dot which is not here. So, this is not a damping term rather this is or just a gyroscopic coupling ok.

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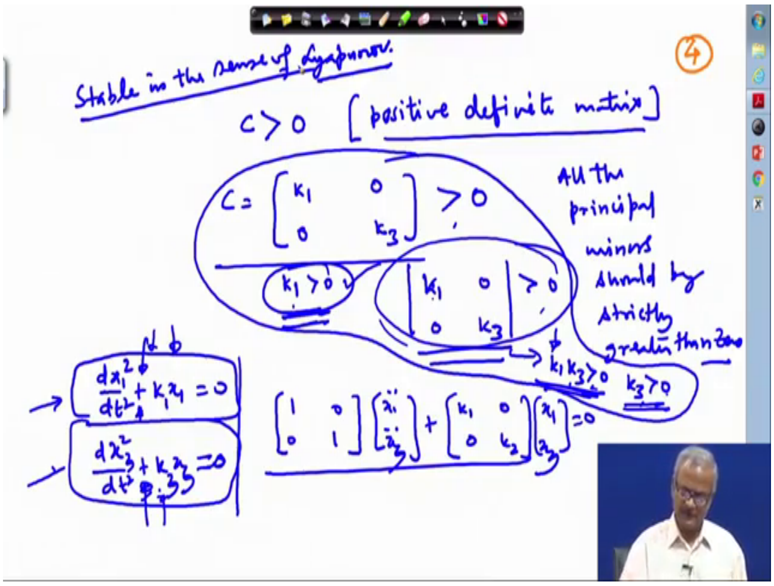

So, now we move to the other part. So, now, we will look into the stability part of this. So, for this particular equation we go back here and look here in this part. So, the for the stability what is required, that C this C is a matrix C is a matrix. So, this matrix should be greater than 0 means it is a positive definite matrix. So, your C matrix its appearing like this let us say we write this as k 1 and k 3 look here in this place this term, we can write this as the k 1 and this particular term we can write as k 3 ok. So, if we write it this way.

So, this is must be greater than 0 if we write it like this. So, this implies that this is a positive definite matrix this should be a positive definite matrix and for positive definiteness what is required that k 1 should be greater than 0 and k 1 0 0 k 3 this determinant this also should be greater than 0; that means, all the things in any matrix if you are working like this.

So, all the principal minors all the principal minors should be strictly greater than 0 greater than 0. So, if this condition is satisfied, then we say that the system is dynamically say system is statically stable. So, why we are telling that let us look through this. Let us say there are 2 spring system and we write it like this and if we will try to combine it together. So, how it will look like? 1 1 0 0 and here x 1 double dot x 2 double dot plus here we write here k 1 and this we write as k 2. So, k 1 k 2 00 x 1 x two; obviously, here in this case it is a decoupled ok, but we can get the stability notion from this place.

So, for stability what is required? This will be stable in the sense of Lyapunov as you know that the poles are lying on the imaginary axis. So, or we say this is a statically stable. So, if this sign is plus here only then this will be a stable otherwise it will diverse, if there is a minus sign here in this place. So, it will diverse similarly here there is a minus sign it will diverse. So, what is required that both this sign should be positive ok. So, that implies that not only the k 1 should be greater than 0 and k 1 k 3 is also greater than zero. So, if you look from this place this implies this k 1 k 3 is greater than 0. So, k 1 is greater than 0 k 1 is greater than 0 here and k 3 is greater than 0.

So, this will be satisfied because k 1 has to be greater than 0. So, this will be satisfied only if k 3 is also greater than 0. So, this two things (Refer Time: 23:44) I will make here the x 3 and k 3 x three. So, that it looks same ok. So, from this place its a clear that k 1 and k 3 must be greater than 0 and it will look through this part. So, here also this says that k 1 should be greater than 0 and this quantity should be greater than 0 which gets reduced into this format, which gets tells that k 3 should be greater than 0 which is indeed the thing ok.

So, if you are C matrix is positive definite then your system will be statically stable. This is one way of saying it otherwise you say that it is a better to say it is a stable in the sense of Lyapunov ok. It is a more appropriate to say that a stable in the sense of Lyapunov because it is a dynamical system statically stability it is always related to that the storing force is available, it conveys only that meaning ok. So, this system is stable in the sense of Lyapunov. So, now, we go back to this part so, here if C is greater than 0.

So, if this part this particular matrix if this is positive definite then we get the system which is stable in the sense of Lyapunov; this part we will discuss little later on. So, for this we need to define those conditions similarly we will take up the details later on let us first discuss this part. So, this is our B matrix ok. So, this is B matrix now it so, happens that even if your system is a statically stable means, this term is not positive definite. So, we will continue in the next lecture.

Thank you very much.