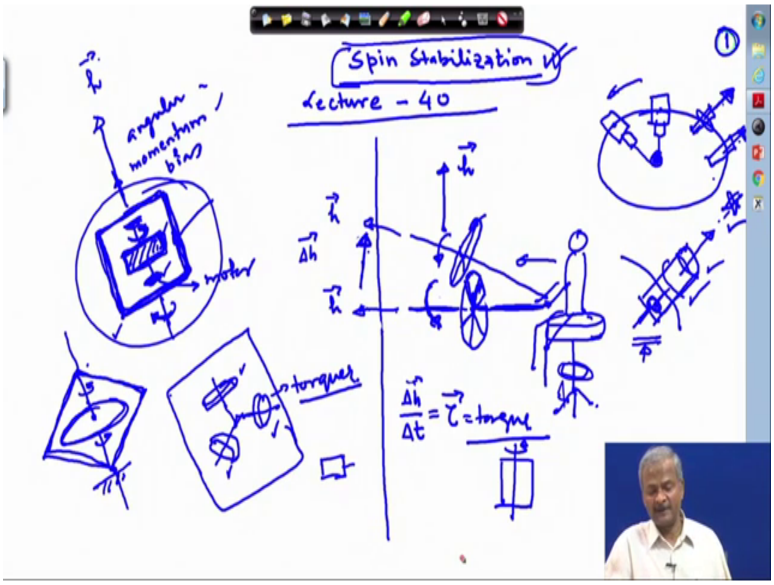
Satellite Attitude Dynamics and Control

Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture - 40
Spin Stabilization (Contd.)

Welcome to the lecture number 40. So, we have been discussing about this Spin Stabilization. So, you may be wondering why we are discussing about the spin stabilization. So, later on we will see once we have start with the reaction we will send the control moment gyros. So, at that time you will see that though the reaction will it is used to actuate the satellite, but if suppose I have a satellite.

(Refer Slide Time: 00:45)



Let us start because we need the this concept so, we will discuss it here. If this is a satellite and inside this, I have a wheel mounted on one axis. So, it is rotating about this axis. So, if this wheel is rotating on this axis. So, what you are doing that you are giving it momentum bias, angular momentum bias. So, from outside the satellite may not be rotating ok. It is a possible that from outside the satellite is not rotating and you have already said the wheel rotating before putting it into the orbit.

So, whatever the direction of the initial direction of the satellite it will be maintained until unless some disturbance comes into picture. So, the same thing we will look. So, one gyros through the YouTube video and if you want you can go and look yourself in those videos. So, there you will find that once a spinning wheel it is a put in a box. So, there is a box in which it will be inserted and then it is left on this corner. So, this box remains in this position itself. So, you can understand that the satellite behaves in the same way. So, you can consider the box to be the satellite. And there is a wheel rotating inside ok, which you are not able to see what is be inside there, but because of this wheel this box will be able to maintain this position.

So, the same thing whatever we have been discussing about this spin stabilization. So, here this refers to the case where the your spinning velocity may be constant. So, the any other changes are not taking place, but there can be case like this wheel is spinning here. And if I further increase the angular speed means we accelerate the we give it angular acceleration to this wheel. So, you will see that because the, this system is free from the external torque, this is my satellite which is free from the external torque. So, the total angular momentum must be conserved. Here if we are talking it means there is a motor here. Suppose in this place there is a motor and this motor is a root once you pass electricity through this motor. So, this will accelerate you can increase the voltage input.

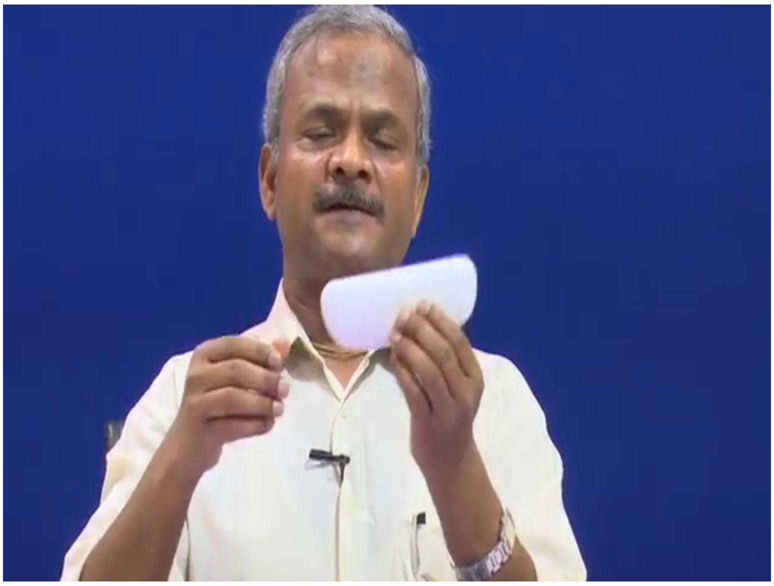
So, that the wheel goes at a faster speed. So, once it accelerates so, you can see from outside we are not applying on this any other torque ok. So, the angular momentum of this system this plus this must be conserved as a whole. So, this implies that if it is accelerates here in this direction means the anticlockwise. So, this will the satellite the outer part of the satellite it will go in a opposite way means it will go this way. So, it will start rotating here in this way.

So, this way by actuating a motor inside (Refer Time: 04:13) by putting input signal to the motor and we can make the wheel accelerate. And thereby we can make the satellite rotate about this axis. So, we can similarly if we have on other axis say if I have a bigger box. And I have 3 wheels mounted here, one wheel here, one wheel along this axis and another wheel on this axis. So, they are perpendicular to each other on 3 different axis ok. So, by moving these 3 wheels it will be able to control the angle orientation, angular velocity of the spacecraft.

So, in that context we are using this as a torquer inside ok. So, it acts as a torquer, but if it is moving at a constant speed, if it is speed is maintained then it will try to point in a particular direction and this is what we are studying here in this particular topic. So, spin stabilization it refers to the case where the satellite is set into the motion along one of the axis. One of the axis and then there after the satellite keeps rotating along that axis. So, it may be that because of the internal friction your the wheel may start dyeing out. So, we would like to maintain at the same angular velocity. So, that can be done; obviously, that is a different issue it is of circuit issue. Basically electrical engineering or either electronics issue whatever it can be said.

The main thing is here the dynamics part that if you are there is a angular momentum bias inside. So, if I have a angular momentum vector here. So, until unless I apply an external force on this your external torque, it is not going to change its orientation. So, the same thing you can consider through say if we have a rotating wheel and somebody is handling it here a person is holding it. And he is sitting on a chair which is a revolving one. So, he is sitting on a chair and it is a revolving one. Then let us say that it is a rotating like this since from his side. So, it is something like this is or we take this.

(Refer Slide Time: 07:13)



So, suppose this is the axis. So, around this axis it is rotating like this ok. So, it will rotate like this so, there is a wheel or you can think of like a you can think of life cycle wheel. This is a suppose this is a cycle wheel and here this is your axel ok and this wheel is rotating ok. It is a rotating in the anticlockwise direction. So, if the we look from the this person sitting on this revolving chair point of the view. So, wheel change the direction it will look something like this ok. He will look from the person side from this side. So, it will be revolving in this direction and if this person he applies a torque on this axel and tries to change it is position from this position to this position ok. So, this wheel comes from this position to this position ok.

So, here in this case you can see that this is the angular momentum vector. Here the wheel speed is remaining same. So, this angular momentum vector goes here in this place. So, there is a net change here in this direction ok. Little bit inclined because this if we show this as the magnitude. So, it will appear like this. So, it is a little bit inclined it goes like this. So, and this is your delta h ok. We know that this delta h by delta t this is nothing but our torque ok.

This implies that the change of this vector from this place to this place it implies that there is a torque applied along this direction. So, as a result of this now I am not going to discuss the whole details, but as a result of this person will rotate on this axis ok. So, here a torque is so, he has starts this person has starts rotating about this one ok. So, why he is rotating because, what he is doing that he is applying torque to this wheel ok. And as a reaction of this because this is free to rotate. So, this one is rotating. So, in which direction it will rotate and other things we will discuss it in the once we are start the.

Sir flash [FL] off [FL] flash on [FL]..

How much lecture?

Lecture 10.

So we will discuss this details later on. So, we go to what we have been discussing earlier. So, what is whatever we are discussing it is a purely relevant to the satellite because, you are not going to actuate the satellite continuously in the orbit. So, what is required that if the satellite is going here in this orbit. So, it should be pointing in a particular direction initially or it may be that it is always pointing towards the earth ok.

So, the camera is there which is always pointing towards the earth. So, in that case it is required that it rotates at a constant angular velocity about this axis ok. Or it may be required that all the time the satellite is pointing in a particular direction ok which is in the case of the Hubble telescope you know that it is a used for measuring the location of the stars. So, this is industrial orientation and while this is the earth pointing satellite.

So, both the jobs can be done through the spinner stabilization. So, say if here in this case if I have a wheel which rotates such that it is angular momentum vector is directed along this direction. So, it will try to maintain it is orientation. And we want to a study here whether this kind of configuration like here my Hubble telescope in the orbit. And there is a pressure here through which the star at a distance is being observed.

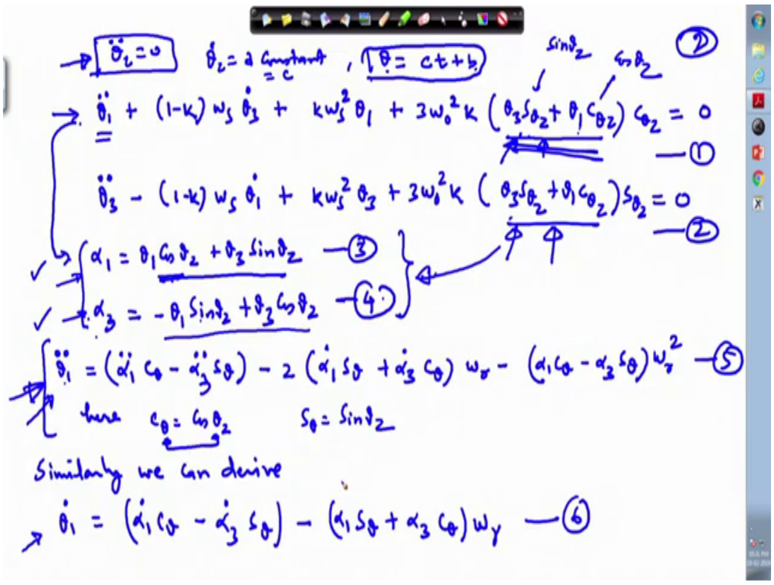
So, whether it will maintain this orientation throughout the orbit or not? So, this is what we are trying to study. So, we have written the Euler's dynamical equation and then we are trying to get into the get through the dynamics that whether this kind of rotational configuration where this suppose this is rotating here. Or in the case of the Hubble telescope we cannot afford to rotate the whole telescope itself, but here inside we can put some wheel which is rotating and then it is a trying to maintain disorientation this is possible.

Or either seem some very precise active control system just like the reaction wheels or like your the small thrusters are there which are micro thrusters available. So, this can be used to continuously maintain a very precise direction ok. So, that becomes a part of control. Once you are using thruster continuously and trying to maintain the orientation, but if we are trying to put a momentum bias just like we were discussing here. That this wheel is rotating here and therefore, this wheel always points here this direction until unless it is a disturb.

So, that kind of configuration we can achieve that is called the stabilization part, but here it is a because it is a coming because of momentum bias. It is not because of the external torque being applied ok. If you are the internal torque itself. So, that we are we have the reaction wheels inside which is being rotated and it is trying to maintain this orientation. So, that is part of the controls that you point it in a particular direction, but here we are not discussing at this stage any matter about the controls.

We are discussing about just about this dynamic system where one wheel is rotating inside or itself this is a satellite which is rotating on it is axis. So, whether it is a stable or not whether it will maintain this orientation or not. This is the topic we have been working with. So, in this context we work with the Euler's difference Euler's dynamical equation and there we saw that.

(Refer Slide Time: 15:03)

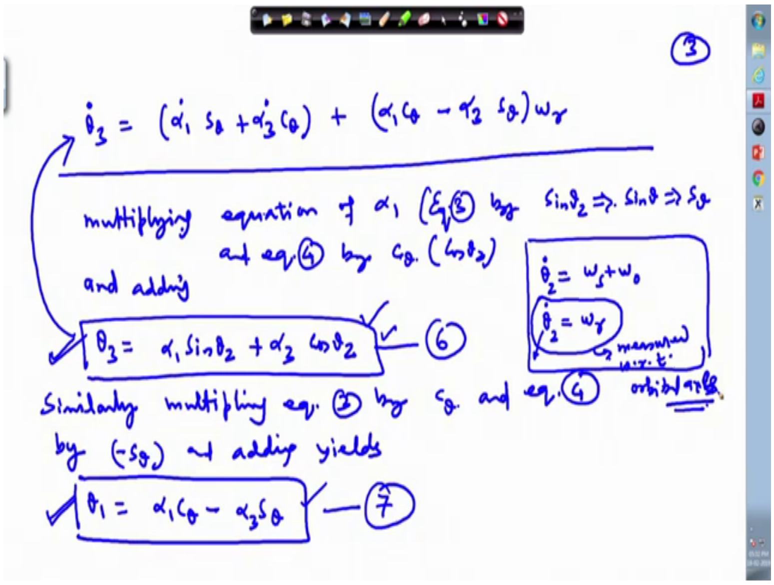


We derive this equation theta 1 double dot plus 1 minus k c theta 2 times. And another equation that we derived this equal to 0 and theta 2 double dot we got this equation equal to 0, means theta 2 dot this will be a constant. And therefore, theta this will let us say this constant is c. So, this will evolve bit time. So, ct plus let us say this we can write it in this format ok. So, this says that here the theta will continuously grow with time.

So, we will discuss about this issue let us first develop this. So, in this context, then we root alpha 1 equal to theta 1 cos theta 2 plus theta 3 times sin theta 2. So, basically this part we are taking from this place theta 3 sin theta 2 s theta 2 is nothing but your sin theta 2 and similarly cos this is cos theta 2. And theta 1 times cos theta 2 this part is written here in this place. And alpha 3 then we wrote as minus theta 1 sin theta 2 plus theta 3 cos theta 2.

So, we utilize this information to get theta 1 double dot this particular one ok. In terms of alpha and rate of change of alpha so, we have derived using this. So, follow from here last lecture here c theta is nothing but cos theta 2. This theta we have replaced by unnecessarily; we are not curing this 2 notation. We understand that this is the pitch angle we are using here. Similarly, s theta is sin theta 2 ok. So, this we have derived in the last class. So, the way we have derived similarly we can derive theta 3 dot ok. So, for this we need to work out using this.

(Refer Slide Time: 19:55)



So, theta 3 dot can write as alpha 1 s theta. So, what we have done that alpha 1 we have written this equation alpha 1 and alpha 3. So, we differentiate at this ok. And after differentiating this we are trying to get this theta 1 dot and theta 2 dot volt. So, I will write the previous step. So, that there is a continuity multiplying equation of alpha 1 which is our equation number 3, which is equation 3 by sin theta 2. And this we are writing as sin theta means we have written this as s theta in the short notation.

So, multiplying equation by 3 by sin theta 2 and equation 4 by c theta means the this is cos theta 2. And adding the tilde it is the theta 3 equal to alpha 1 sin theta 2 plus alpha 3 cos theta 2. Similarly, equation 3 by c theta and equation 4 by minus s theta and adding yields theta 1 equal to. Ok there after what we have done that we were interested in see if the equation we have this is bit involved equation.

So, what we are interested in that we want to replace this theta 1 double dot and theta 3 double dot and here theta 3 double dot theta 1 dot ok; all this quantities theta 1 and theta 3 here so, in terms of alpha 1 and alpha 3. Why we are doing because this is here in this place. These are not constant terms ok. These are the periodic terms it is the coefficient it is a time dependent term which is a periodic term basically. And solving in this for format it is a formidable.

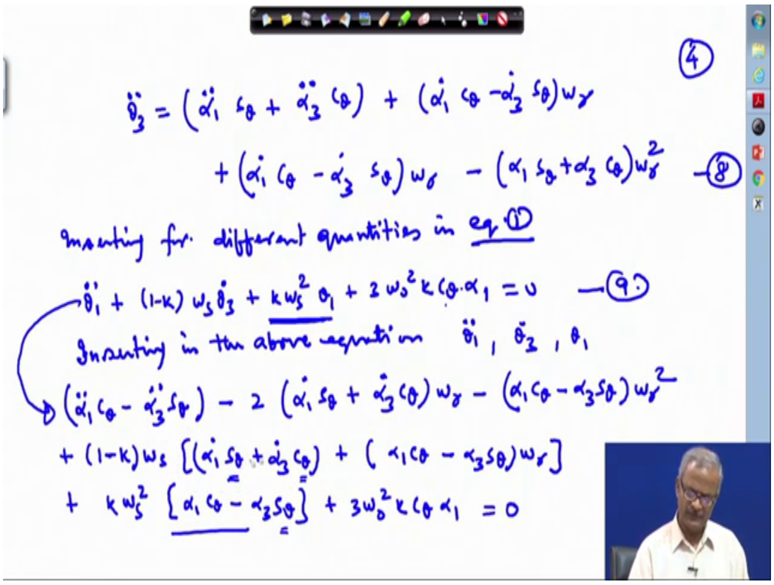
So, it is a better to reduce it in the format where is the coefficients they all appears in the time independent format. So, this manipulation so, this assumption we are doing to get rid of this periodic coefficients. So, for this we are assuming this that alpha we are defining one variable alpha 1 another variable alpha 3 which equal to theta 1 cos. This part basically, this part has been copied here in this place ok.

And alpha 3 another variable has been defined like this way if just by changing this to with minus sign ok. And these two equations then we are using to get theta 3 and theta 1 and thereafter we are differentiating these two equations. So, by differentiating and we have got this equation last time ok. Theta 1 double dot and similarly theta 1 dot also we got. So, I will just make a note of this here in this place ok. So, the theta 1 dot we have got as alpha 1 dot c theta though we are written afterwards. So, this equation is differentiated to get this equation and that we have already done by arranging the terms and all other things.

So, when did not repeat all these things, but for the continuation of the lecture, I have to reproduce all this things here. This difficulty we face because of their equations and we move once the lecture ends for a particular day. So, we have to recall all the things so, that we can proceed ok. So, first we got this from there we are getting this theta 1 dot. And from there then by differentiating this and doing all the replacement or the instructions we are getting this theta 1 double dot. In the same way by differentiating this theta 3 equation we get this equation.

So, if you differentiate it we will get this equation ok. And where we have used this theta dot we have written as omega s plus omega 0. And so, theta dot is nothing but your theta 2. And so, theta 2 dot this is nothing but your omega r. Because this is the Euler angle and this is being measured with respect to the orbital axis, with respect to the orbital axis ok. And this we have I have explained you in the last lecture.

(Refer Slide Time: 28:43)



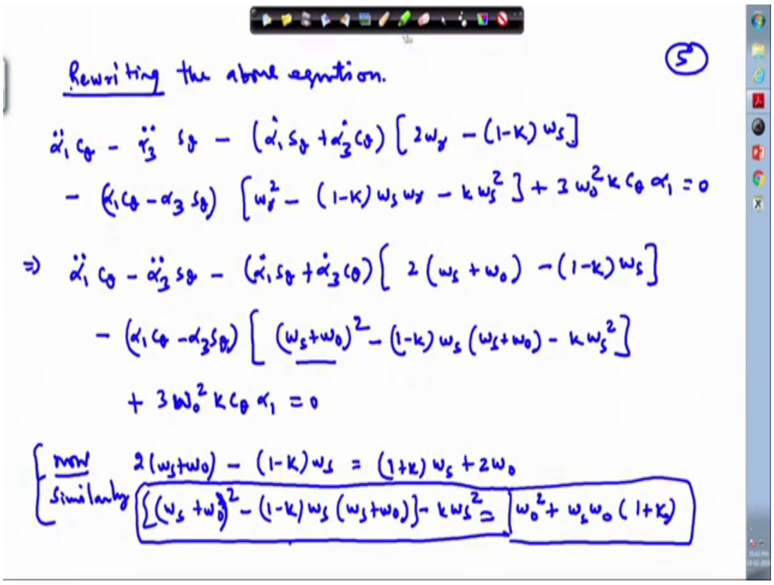
So, theta 3 dot we have written if we differentiate that. So, we get theta 3 double dot you can check all these equations because it is a time taking. So, I am doing it quickly. So, we are using this equation this equation is being differentiated. This is equation number 8 ok. Now we have got this theta 3 dot also and we have got also theta 3 theta 3 double dot and theta 3 dot. So, we need to insert these values here like theta 3 dot is here ok.

And theta 1 dot is here theta 3 dot is here so, theta 1 double dot. So, all these quantities we have derived earlier ok. So, these quantities need to be inserted here in this place. And we need to rearrange this equation. So, rearrange these two equations. So, if he rearrange these two equations. So, we get it in a simpler format, where the coefficients are independent of time ok. Means they turn out to be the constant coefficients and therefore, those equations can be solved easily the stability of the system can be described in a easy manner.

So now insert all this things. So, we take the first equation here. We have theta 1 double dot we take this equation the first equation ok. So, inserting for theta 1 double dot theta 3 dot and theta 1 and all this parameters here in equation 1 so, equation 1, I am reproducing here. So, this part is your theta 3 plus sin theta 2 theta 1 times cos theta 2. So, this we are writing as alpha 1. So, alpha and this c theta 2 we are writing as c theta this part. So, here c theta we have written and then we are writing here alpha 1. So, this is equal to 0. So, this looks little shorter than the previous equation. So, from different equations we need to insert all these values. So, theta 1 double dot this quantity then becomes alpha 1 double dot c theta. This equation is bit long all these things are tedious not difficult, but tedious.

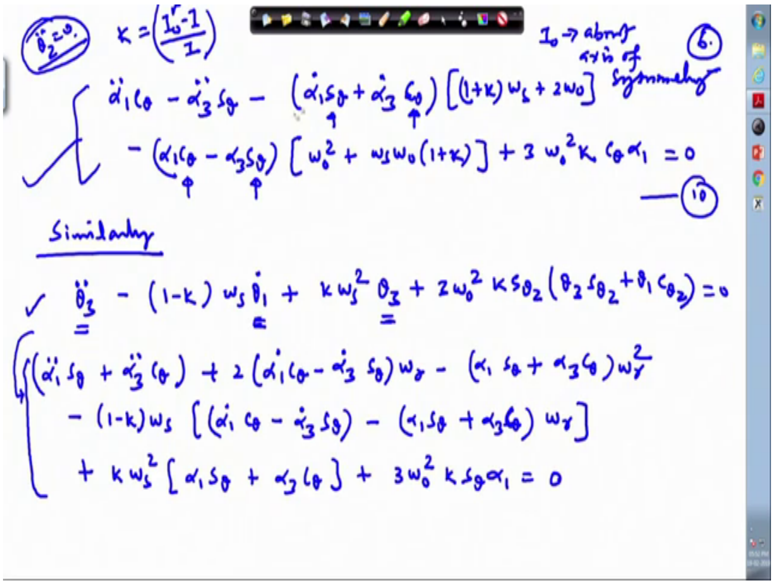
This is this particular term it is a written here in this place this equal to 0. So, this has got converted here into this format. So, still we need to rearrange it because you can see that s theta c theta all these terms are here. So, still it is these are the periodic terms. So, we need to eliminate. So, we need to rearrange this equation rewriting the above equation.

(Refer Slide Time: 35:03)



We are rearranging the above equation; omega r being replaced by omega s plus omega 0. So, these terms we need to rearrange it. So, by rearranging we can see that this gets reduced to. So, expand it and check this term. So, I am writing the final result here in this place. These are the few shortcuts I have taken ok. So, we can insert these values here. So, this gets then simplified.

(Refer Slide Time: 39:05)

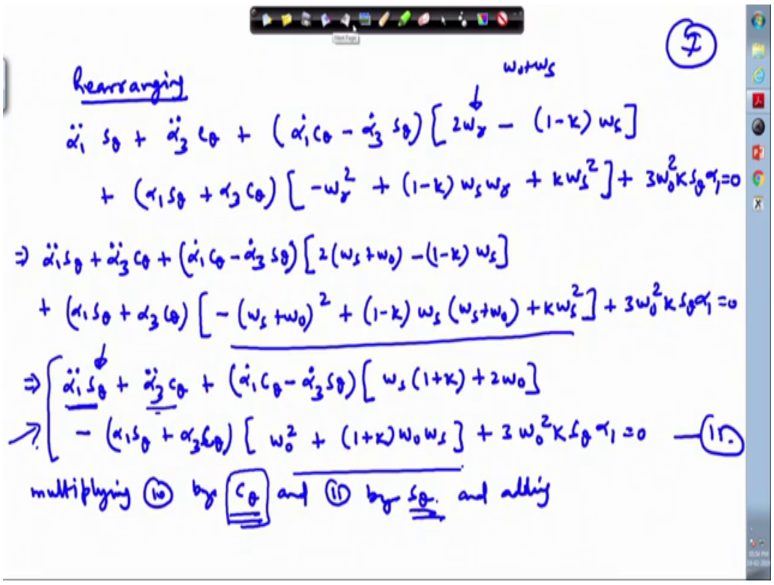


This our equation number 10. Similarly, the equation for theta 3 dot, it can be reduced. So, still you can see that, this is a sine and cosine dependent terms ok. So, still we have not got rid of these terms. So, this is the equation for theta 3 dot that we have written earlier. See if here we have taken the case of the satellite which is symmetric about one of the axis is called the inertial symmetry.

So, because of that your equation is getting simplified. Otherwise if we will look for the 3 axis satellite because in that case you are theta 2 double dot this has got reduced to 0 if you do not assume that symmetry. So, this will not be equal to 0. So, that will put another problem ok. So, the equation it becomes formidable. So, in this place wherever the k is appearing this k we have assumed it to be i 0 minus i divided by i. Where i 0 is the moment of inertia about the i 0 this is about the axis of symmetry ok.

So, theta 1 double dot we have already reduced now we are working with theta 3 double dot. So, inserting for already we have got theta 3 double dot theta 1 dot theta 3 etcetera. So, insert all this things here. We cannot avoid this mathematics alpha 3 this is c theta. So, instead we are inserting these values here. You can develop the whole thing to do it yourself and then match it. It requires little patience, but you can do it.

(Refer Slide Time: 44:33)

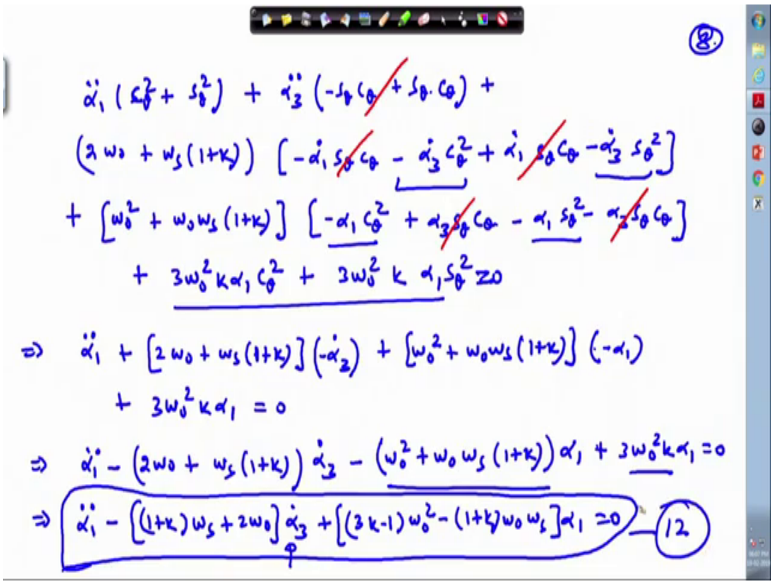


So, this equation has been written here ok. Now we need to reduce it. So, these terms can be rearranged. Now we need to replace this omega r by omega 0 plus omega s. If you are genuinely interested in learning this subject, then you must derived all these things. Because at one stage or others stage I have derived all these equations ok. So, this one we have reduced and written here in this place.

So, do this part yourself ok. So, multiplying equation 10 the previous equation, this equation and equation 11 this particular one by c theta and s theta and adding. So, you can see that there are the terms. So, if we multiply like the equation 11 this equation by s theta. So, we get here is s square theta term. This will be s square theta term and your alpha 3 times c theta times s theta if you give on the previous page.

So, the previous equation we are multiplying by this c theta. So, here this we are multiplying by c theta so, alpha 1 double dot c square theta. And here in this place we are getting alpha one double dot sin square theta. So, this term will add up and we get alpha 1 double dot. And we get rid of this sin theta term here. Similarly, this term is here alpha 3 double dot this comes with a minus sign alpha 3 double dot and this comes with a plus sign. So, this term will get eliminated while you add. So, there why we will get terms which are independent of periodic or time dependent coefficients.

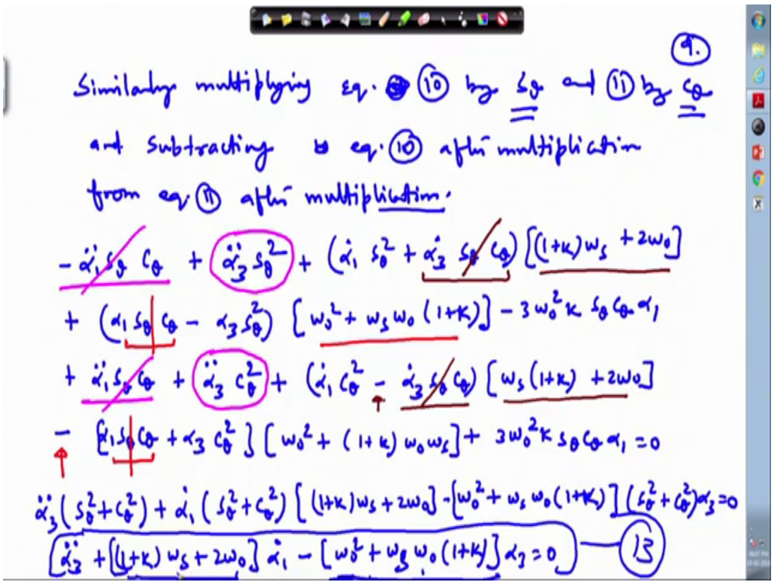
(Refer Slide Time: 50:01)



There is all the terms we need to add. So, some of the terms will drop out. These term drops out ok. Then we have the term here this term and this term drops out and here this term and this term drops out so, this equal to 0. So, this get simplified to alpha 1 double alpha 1 double dot plus 2 omega 0 1 plus k. Here this term and this term will add up this gives is alpha 3 dot this is with minus sign ok. Similarly, here this term and this term adds to give minus alpha 1 ok. And this 2 terms add together alpha 1 this equal to 0. So, you can see that in this equation, we have got rid of all the sin theta and cosine theta terms.

So therefore, here let us first write this equation. Rewrite it. So, minus 2 omega 0 alpha 3 dot omega 0 alpha 1 plus 3 omega 0 square k times, alpha 1 this equal to 0 this terms and this term because alpha 1 is common. So, you combine it. So, this is the equation in alpha 1. Similarly we will, but here you can see that alpha 3 dot it is appearing. So, still it is a coupled means the roll and the yaw they are remaining coupled they are not got disassociated.

(Refer Slide Time: 55:17)



So, equation 10 we have multiplied by this is equation 10 and this is equation 11 by c theta and s theta. Now, what we are doing that we are doing it by s theta and c theta. So, there we have added here we have to subtract. I am subtracting equation 10 after multiplication from equation 11 after multiplication. So, basically the we have equation 10 here. So, this equation 10 we have to multiply and equation 11 we have to multiply. So, we have to subtract these equations. So, we will put a minus sign before this. This is our equation number 11 and this is our equation number this particular equation.

So, we are going to multiply this particular equation which is equation number 10. So, for equation number 10 once we multiply. So, alpha double one dot s theta we are subtracting. So, we are inputting a sign right now here. And then we need to simplify this. So, whichever the terms that dropout we need to eliminate them. So, we can say that this term this term they cancel each other ok. Then what else we have here. These two terms will add up this term and this term they will add up ok. So, this way we have to look for the terms which can cancel out and which will add up. So, here we have this term here alpha 3 dot multiplied by this quantity and the same quantity of yours here with a minus sign here ok.

So, this term and this term will drop out ok. They will cancel each other. Similarly, we have a term here which is multiplied by this alpha 1 s theta c theta. And we look for this term here this is the term. So, here you see that before this there is a minus sign. So, this term and this term we will drop out this term we will drop out. Therefore, this equation can be simplified ok. So, if we add and simplify this equations simplify then. So, this gets reduced into this format alpha 3 double dot s theta s square plus c theta s square ok. We have got this 2 differential equations and this is differential equations can be solved. Now because you see here this term is a completely constant this term is a completely constant. Omega 0 is a constant a s spin rate your taking this as a constant and omega 0 is the orbital angular frequency or the orbital frequency.

So, of the coefficient now in this equation they turn out to be constant. Here also all the coefficient this and all these terms are constant. And therefore, the simplification is possible ok. So, this is already got simplified, now we need to work with these equations and look into the dynamics of the system how the system will behave. Whether it is a we are we are interested in looking for stability of the system. So, through these two equations we are going to look into the stability now. So, we will continue here in the next lecture.

Thank you very much.