Satellite Attitude Dynamics and Control

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Lecture - 39
Spin Stabilization (Contd.)

Now, welcome to the lecture number 39.

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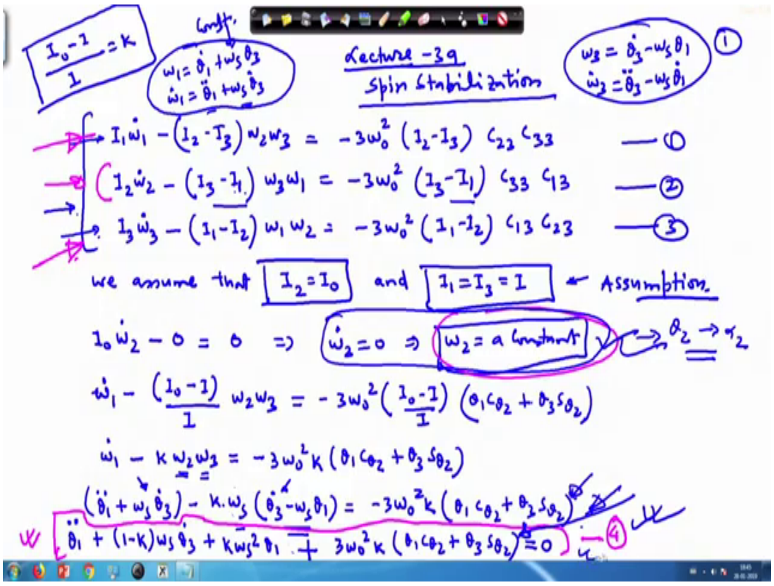

So, we have been discussing about the spin stabilization of a satellite and that too in the presence of gravity gradient so will continue with that. So, last time we have derived various relationship regarding the angles involved and the angular velocity involved. So now, we write the Euler's equation dynamical equation here. So, I 1 times omega 1 minus I 2 minus I 3 times omega 2 omega 3 this equal to minus 3 omega 0 square times I 2 minus I 3 and then c 2 c 3.

If you write it like this, we do not need to remember much, anytime you can reproduce this equation. So, these are the 3 Euler equations we have. We assume that I 2 equal to I 0 and I 1 equal to I 3. So, we are going to analyse the system under this condition. So, this is our assumption. For this case the system becomes this system equation will this is the whole dynamics it will get simplified. So, what we can observe that the second equation from this place this is I 0 times omega 2 I 3 minus I 1. So, this part becomes 0 and on the right hand side also this part this is dot here dot is missing. So, for I 0 times omega 2 dot minus omega 0 and this part is 0 this part will also become 0. So, this equal to 0 and this implies omega 2 dot this equal to 0 omega 2 this is a constant.

So, under this situation a constant angular velocity can be maintained along the 2 body axis. Rest we have to deal with the first and the second equation. The first equation we can write as omega 1 dot I 2 is I 0 minus I 3 is I divided by I. And then omega 2 omega 3 is equal to minus 3 omega 0 square and c 2 3 and c 3 3. So, we pick up c 2 3 from this place, c 2 3 is this quantity and c 3 3 is this quantity. So, we need to insert it there theta 1 c theta 2 theta 1 c theta 2 plus theta 3 s theta 2 theta 3 s theta 2.

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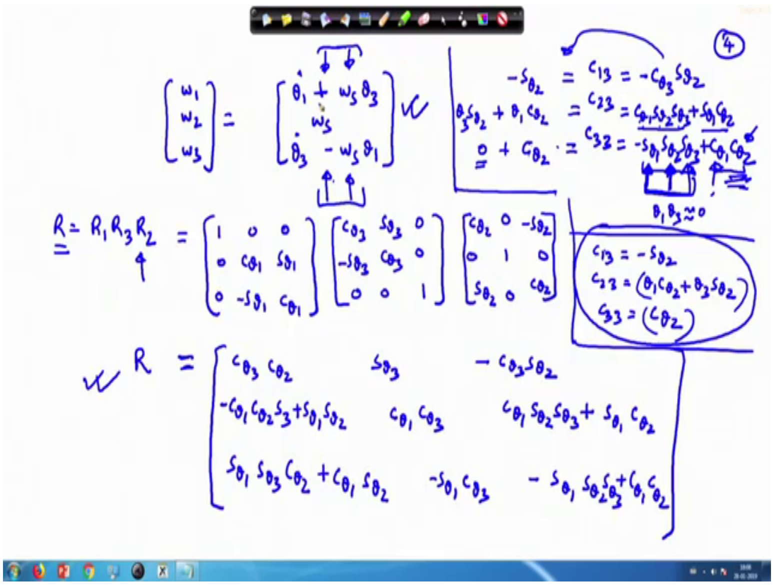

And I 0 by I 0 minus I divided by I this quantity we are going to define as I minus I 0 minus I divided by. This I this quantity will define as k. So, from this place this becomes k times omega 2 omega 3. And again omega 2 omega 3 also we need to insert here. So, omega 2 omega 3 we have already derived omega 1 omega 2 omega 3. So, this we have derived and we will have to pick up this.

So, in the next step will pick it up and put here in this place. So, minus 3 omega 0 square k times theta 1 c theta 2 plus theta 3 s theta 2 inserting the value of omega 1 here. So, omega 1 is theta 1 dot plus omega s theta 3. So, omega 1 is theta 1 dot theta 1 times theta 1 dot plus omega s theta 3 omega s theta 3. So, omega 1 dot will become theta 1 double dot this quantity is a constant this is constant. So, this will not change. So, we get here omega s times theta 3 dot.

So, dimensionally all the terms are same. So, omega 1 dot becomes theta 1 double dot plus omega s times theta 3 dot minus k. And omega 2 omega 3 we need to insert. So, omega 2 from this place will look back omega 2 is omega s. So, omega 2 is omega s and omega 3, we need to insert. So, this is your omega 1 term and this side I will write here omega 3 term. So, that we can use here omega 3 is theta 3 dot minus omega s theta 1. So, this is theta 3 dot minus omega s theta. 1 therefore, omega 3 dot it will become theta 3 double dot minus omega s by theta 1 dot.

So, we need to insert that here. So, this is theta 3 dot minus omega s times theta 1. So, omega 2 and omega 3 this is omega 2 and this is omega 3, 3 omega 0 square k theta 1 c theta 2 plus theta 3 s theta 2. We can rearrange this term to write it like this theta 1 double dot. Here this is c dot theta 3 dot is present here in this place and also here in this place. So, we can write this as 1 minus k omega s common to this. So, omega s times theta 3 dot and then plus k times omega s square theta 1. This equal to minus 3 omega 0 square k times theta 1 times c theta 2 plus theta 3 times s theta 2. And here we bring this term on the left hand side. So, also we can what we can do that we can bring this whole term on the left hand side. So, we can put a plus sign here on the right hand side we can put 0. So, this is our one equation, we have got here. And will name this as the equation number 4.

Now, this is for theta for a first equation we have work down. Now we have to work with the third equation. Second equation we do not need to work because already this has got simplified to this particular result.

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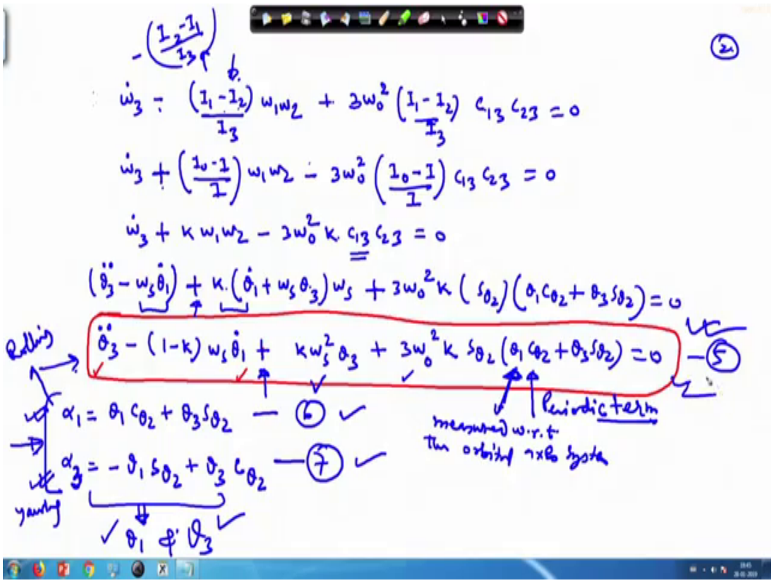

So, the third one I 3 omega 3 dot similarly we can write omega 3 dot ok. This equation we have got brought it here on the left hand side and written like that. So, omega 1 omega 2 this becomes omega 3 dot and here will take the minus sign. Now I 0 write it like this. So, this will become k and the same way here in this place we need to change. So, we will have minus sign here 3 omega 0 square. And I 2 is I 0 minus I divided by I c 1 3 c 2 3 this gets reduced to omega 3 dot. This part we have written as k. So, omega 1 omega 2 minus 3 omega 0 square, this is k c 1 3 times c 2 3 this equal to 0 on the right hand side or this as 0.

Now, insert the expression for theta 3 dot omega 3 dot. So, this is theta 3 dot minus omega s times theta 1 dot minus omega s times theta 1 dot. And then the rest of the terms we have to insert here. So, omega 1 is theta 1 dot plus omega s times theta 3. Omega s plus theta 3 and omega 2 is omega s. So, we write it like this minus 3 omega 0 square k c 1 3. So, c 1 3 now we have c 1 3 is minus s theta 2. So, this is minus s theta 2 and c 2 3 again we have to pick up. So, c 2 3 is here theta 1 c theta 2 theta 1 theta 1 c theta 2 plus theta 3 s theta 2 theta 3 s theta 2.

So, this minus sign that will make it plus. So, I will remove this sign from here. And I will put it plus sign here in this place. And this quantity will be equal to 0. So, this can be broken up and it can be rewritten like. Now here we see that this term and this term both are present here in this place. So, this is 1 plus k times omega s times theta 1 dot and then minus k times omega s, s square theta 3 plus 3 omega 0 square k this equal to 0. Here we need to put a plus sign while we are working with this I 1 minus I 2. So, what you have done that this part I forgot to change the sign here, is I 2 we are writing on this side I 1 and divided by I 3. So, this will come with a minus sign. So, this minus then get plus. The minus sign of this and minus sign this that makes it plus. So, will have a plus sign here and therefore, we need to put a minus sign here in this place. So, this becomes a plus.

So, if we take a minus sign out of this. So, this is 1 minus k. Omega s times theta 1 dot and then this particular term omega s square here this is omega 0. This part is omega 0. Which is the orbital angular velocity. So, this is our another equation that we have got theta 3 double dot 1 minus k omega s times theta 1 dot and then. So, one more sign change here. We need to have a plus because this we are made it plus. So, here also there will be a plus sign. So, this is the plus sign here. So, this is k omega s square theta 3 and then 3 the other one it is already the plus. So, this is ok, now these equations reveal number. So, this is the 4th equation. This I will write as this is the equation number 5.

Now, this 2 equations though looking simple, but the problem is that here in this place and also here in this place this particular term this one these are function of sin and cosine means these are the periodic terms. So, this term this is a periodic term. And we need to simplify this equation. So, that we are able to work with this. Without that proceeding with this equation it will be tough ok. So, in both this places the because of this particular the coefficient which is appearing which is periodic term, we get in trouble. So, to get out of the trouble we need to assume something and assuming means we need to do some transformation here ok. With appropriate transformation will be able to put the equation 4 and 5 in a proper format and their by will be able to work with them.

So, far let us assume that alpha 1 equal to whatever the coefficient is appearing here theta 1 times c theta 2 plus theta 3 times s theta 2. So, this is our equation number 6 and similarly alpha 2 we assume as minus theta 1 times s theta 2 plus theta 3 times c theta 2. And very soon it will be clear that indeed this helps ok.

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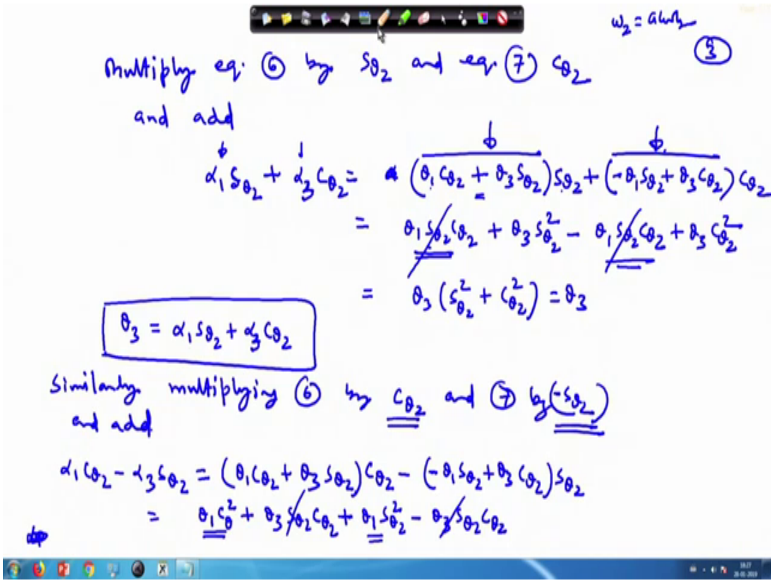

Now, once we have assume this. So, we can solve it in terms of theta 1 and theta 3. So, so from here to our objective first is to get theta 1 and theta 3. So, once we get this theta 1 and theta 3. So, will in terms of alpha 1 and alpha 2. So, will insert in this equation and in the previous equation here in this place ok.

And by inserting will be able to simplify this equations quite a bit. So, multiply equation 6 s theta 2 and equation 7 by c theta 2 and add. So, if we do that so we have alpha 1 c theta 2, alpha 1 times this we have to multiply by s theta 2 and plus the other one, alpha 2 times c theta 2. So, this will be equal to alpha 1 value we have to insert here in this place this alpha 1 what we have assumed.

So, alpha 1 is theta 1 c theta 2. So, this is theta 1 c theta 2 plus theta 3 s theta 2 s theta 2 multiplied by s theta 2 and plus this alpha 2. So, alpha 2 is minus. So, here this particular part is change. Minus theta 1 times s theta 2 and plus theta 3 this remain same and s becomes c theta 2. And this multiplied by c theta 2. So, we can check here that this quantity is s theta 2 c theta 2 plus theta 3 s theta 2 s square. And from this place we get minus theta 1 s theta 2 times c theta 2 and plus theta 3 times c theta 2 square.

Here we happen this term is plus because here this is plus. So, this is plus now we can see that one this particular term and this term get cancel each other. So, this 2 terms they cancel each other and therefore, this is theta 3 times s theta 2 a square plus c theta 2 a square this is theta 3. So, theta 3 we have got in terms of alpha 1 s theta 2 plus alpha 2 c theta 2. This is we have written here alpha 2 I will change it to the notation alpha 3; I will change it to notation alpha 3. So, this is alpha 2. Here will write in terms of alpha 3. So, theta 3 is and reason behind this is simple because we have already observed that your omega 2 this turns out to be a constant going back here in this place.

This term is a constant. So, this does not change. So, we will directly associate the angle theta 2 with alpha 2 ok. So, alpha 2 will reserve for that. And alpha 3 we will keep for this is the rolling motion this is associated with the rolling motion. And this is associated with the yawing motion. This is associated with rolling. And therefore, it is appropriate to keep the terms alpha 1 alpha 3 here in this place.

So in the same way if we multiply 6 by c theta 2 and 7 by s theta 2, with minus sign minus s theta 2 and add so, 6 and 7 this 2 equations we are multiplying it by c theta 2 and the other by minus s theta 2 and add. So, in that case this will be alpha 1 times c theta 2 minus alpha 3 times s theta 2. This will be equal to now alpha 1 we have to pick up. So, alpha 1 already it is available here in this place. So, we insert that theta 1 c theta 2 plus theta 3 times c theta 2 and minus alpha 3 alpha 3 is this term so, minus theta 1 s theta 2 plus theta 3 c theta 2 and multiplied by s theta 2.

As a expanding this fields, c theta 2 a square plus theta 3 s theta 2 c theta 2 minus that gets plus. So, for this is theta 1 times s theta 2 a square and here plus minus this gets a minus sign theta 3 s theta 2 times c theta 2. So, this term and this term they drop out living us with alpha this 2 terms. So, this 2 terms added together we do not have a space here and go on the next page.

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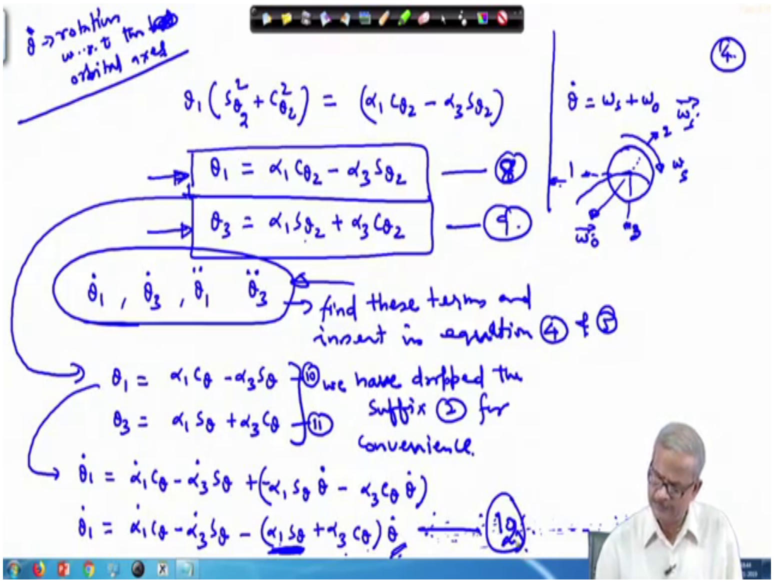

So, we are here theta 1 square c theta square theta 1 time theta 1 times c theta square theta 1 times s theta square. So, that makes it theta 1 ok. So, theta 1 times s theta a square s theta. This is c theta 2 ok. Theta 1 times s theta 2 a square plus c theta 2 a square and on the right hand side then there after we can pick up this term and right here alpha 1 c theta 2 minus alpha 3 s theta 2. So, the theta term one becomes alpha 1 c theta 2 minus alpha 3 s theta 2. So, finally, we conclude all the results here. To theta 3 what we have got here in this place this is alpha 1 s theta 2 alpha 1 s theta 2 minus this is plus alpha 3 c theta 2 plus alpha 3 c theta 2. And till now we have use a notation till 7.

So, this will right as equation number 8 and this as the equation number 9. Now from here onwards this derivation is very stretching ok, but it is a good to have a practice of this so, that you understand the whole process how the things are working. Now there after we need to get alpha 1 theta 1 dot and theta 3 dot theta 1 double dot and theta 3 double dot. And this terms then need to be inserted into the expression we have got here in equation 5 and equation 4.

So, find this terms and insert in equations 4 and 5. And we have to do it patiently because it is a quiet time taking work ok. So, theta 1 is alpha 1 c theta 2 and alpha 3 s theta 2 alpha 1 s theta 2 and alpha 3 c theta. Now let us re write this so that we do not have to carry this tag 2 ok. So, will write this as theta 1 equal to alpha 1 c theta minus alpha 3 s theta and theta 3 this equal to alpha 1 s theta plus alpha 3 c theta so, we are dropping we have this I will write as 9 this is 10. And this is a equation number 11. We have dropped the suffix 2 for convenience.

There after we can get derivative of this. So, if we take derivative of this. So, theta 1 dot then will be alpha 1 dot times c theta and minus alpha 3 dot times s theta. And the other term then we have to differentiate alpha 1 times c theta will be then s theta times theta dot. And minus sign will appear here in this place. Ok and there is a minus sign already with this. So, this is alpha 3 s theta will become c theta times theta dot. So, remember that the theta 1 theta 2 this angles and theta 3 they are being measured with respect to; measured with respect to the orbital axis system.

So, therefore, appropriately we have to put here theta dot. And in the beginning itself we were discussing about this. So, your theta dot quantity we have to write here. So, theta dot will be omega s plus omega 0. Why? Because your this is the satellite and inside going this is your 2 axis, downward your one axis and this is and this is one and this is the 3 axis. So, rotation the omega s we have shown it like this ok. And theta dot is the rotation with respect to the theta dot here this is the rotation with respect to the orbital axis. So we will write it on the next page.

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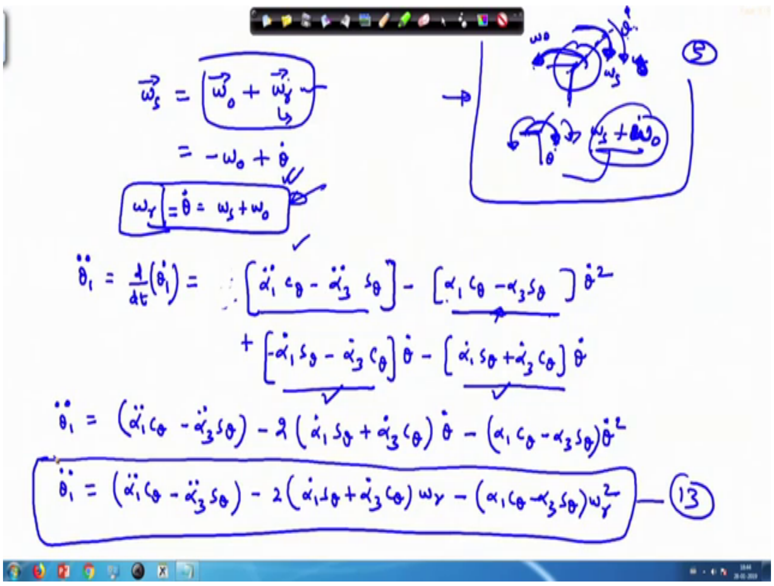

So, we will have omega s this is the absolute angular velocity. This will be equal to omega orbital angular velocity. And with respect to this then you have omega r is the relative angular velocity which is with respect to the orbital axis system. And omega s and omega 0; omega s and omega 0 so, your because the velocity of the satellite is along this direction. So, omega 0 is along this direction ok. So, m omega 0 is here while your omega s it is going here in this direction. So, we can see that they are opposite to each other ok. And therefore, while we right here so we convert it in the same frame. So, it becomes omega 0 with minus sign. So, this goes along if we put a minus sign here. So, that becomes along the 2 direction and plus omega r.

So, omega r is nothing but your theta dot; if we are looking only for the pitching one. So, theta dot is with respect to the orbital frame. So, theta dot along the 2 direction it is a theta dot. So, now the theta dot we have an omega 0, we have chosen along this direction this is the omega 0 direction and omega s is along this direction. And we are measuring with this frame is the orbital frame is rotating this way and with respect to this we are measuring theta dot. So; obviously, a it will add this and also theta dot if we right here. So, you can see that theta dot it will be equal to omega s plus omega 0 because you are measuring theta dot with respect to the orbital axis ok.

So, something is rotating like this and then you are measuring with respect to that ok. So, if we have inertial frame. So, with respect to the inertial frame only we have omega s ok. And what now we are measuring with a frame which is rotating here in this direction. So, p (Refer Time: 34:22) if this is rotating here in this directions so; obviously, you can see that omega s plus theta dot it will add and this will. So, omega s plus omega 0 this will add up and this will show you what is the rotation rate with respect to this access. So, this becomes your theta dot. So, this is exactly what has been written you can check it yourself ok.

These are the things we have discuss a lot write in the beginning. And it is a pretty time taking this issues explaining here especially through this computer. In the classroom itself very easy to discuss all this thing, but on the computer it becomes little difficult because it takes a lot of time ok. So, what we have got here this theta dot equal to omega s plus omega 0. Now this is the theta dot term which is appearing here in this place. So, we can write here theta 1 dot this equal to alpha 1 dot c theta minus alpha 3 dot s theta and here this minus sign is there. So, minus sign we can take it outside alpha 1 s theta plus alpha 3 c theta times theta dot.

Alpha 3 dot s theta minus alpha 1 s theta plus alpha 3 c theta. And theta dot is on this side whatever the value we have. So, we need to put it here as a theta dot ok so, in the same term this is not required here.

In the same way once we have got this theta dot term ok. Now theta 1 dot we have got here. Similarly, theta 1 double dot we can compute so, theta 1 double dot theta 1 dot. So, this will be equal to d by d t and theta 1 dot we have to pick up from the previous page. So, that becomes this quantity again we have to pick up and right here. So, directly we are differentiating the first term. So, we remove this part here. And then the other part, this particular part we have to take the derivative. And also c theta and s theta they need to be differentiated.

So, theta dot we can write this as omega r this quantity we can write as omega r so that we may be comfortable with writing. So, we are picking up this term alpha 1 and s theta differentiated we are taking up this term. So, alpha 1 times s theta differentiated right that becomes c theta. So, alpha 1 times c theta and theta dot will appear. So, theta dot will take it outside ok. And this term alpha 3 c theta. So, that becomes alpha 3 times going back here. We have first differentiated alpha 1 dot term. So, this term is the other term we are differentiating first this part s theta and c theta. There is no problem you can differentiate in any order and write it.

So, alpha 3 c theta so that becomes alpha 3 s theta times theta dot and already this theta dot was present. So, that becomes theta dot s square. Then the corresponding term for this we have to take care of alpha dot and c theta is there. So, this becomes minus s theta and then minus alpha 3 dot this be differentiated that becomes c theta times theta dot ok. Similarly, here in this place this is with minus sign here. So, this is minus alpha 1 dot times s theta. And then alpha 3 times alpha 3 dot times c theta plus alpha 3 dot times c theta times theta dot.

So, these are the terms, we have got here and if we replace this theta dot by omega r. It will be like we do not have to carry this square and order terms. It becomes little easy to work with. So, theta 1 double dot this will be alpha 1 double dot c theta minus alpha 3 double dot s theta. You can see this term alpha 1 dot is a here. And alpha 1 dot alpha 3 dot is here. So, here this minus it can be taken outside so this becomes minus this term and this term together this becomes 2 times alpha 1 dot s theta plus alpha 3 dot c theta times theta dot. And then this term this particular term we have to consider here. So, this is alpha 1 c theta minus alpha 3 s theta times theta theta dot square ok.

So, instead of writing it like this we write here as alpha 1 double dot c theta minus alpha 3 double dot s theta minus 2 alpha 3 dot c theta times omega r minus alpha 1 c theta minus alpha 3 s theta times omega r square. So, till now we have got here all this things. So, this will number as theta 1 dot this we number as 10 no, 10 11 already its. So, here so this we number as 12. And this equation we number as equation number 13 ok.

So, the same way theta 3 dot can be computed and once we have done this, then we can progress further by inserting all these terms in the equation that we are writing here this equation and this equation. So, we can insert here those terms in this equations. And we will get the result. So, we stop here this lecture we will continue in the next lecture.

Thank you very much.