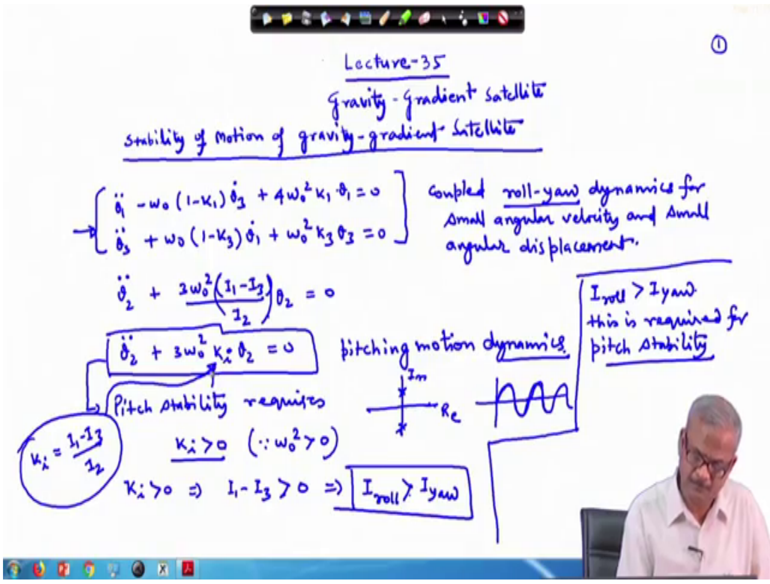
Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture – 35
Gravity Gradient Satellite (Contd.)

Welcome to the lecture number 35, we have been discussing about the Gravity Gradient Satellites. So, we will continue with that. So, last time we derived the motion of the gravity gradient satellite for a small angular displacement and a small angular velocity. So, in that context we were looking into the stability of the related system or the stability of the motion of the gravity gradient satellite. So, will continue with that.

(Refer Slide Time: 00:46)



So, if you remember we have derived this relationship therefore, a small angular velocity in a small angular displacement we have written just two equations, which is for the roll yaw and the yaw dynamics which are coupled together. And thereafter we have written the equation for the pitching motion which was theta 2 double dot plus 3 omega 0 square I 1 minus I 3 divided by I 2 times theta 2 this equal to 0. And this particular part, we have written as theta double dot plus 3 omega 0 square, the quantity which is present here this we have written as K I; K I times theta 2 this equal to 0.

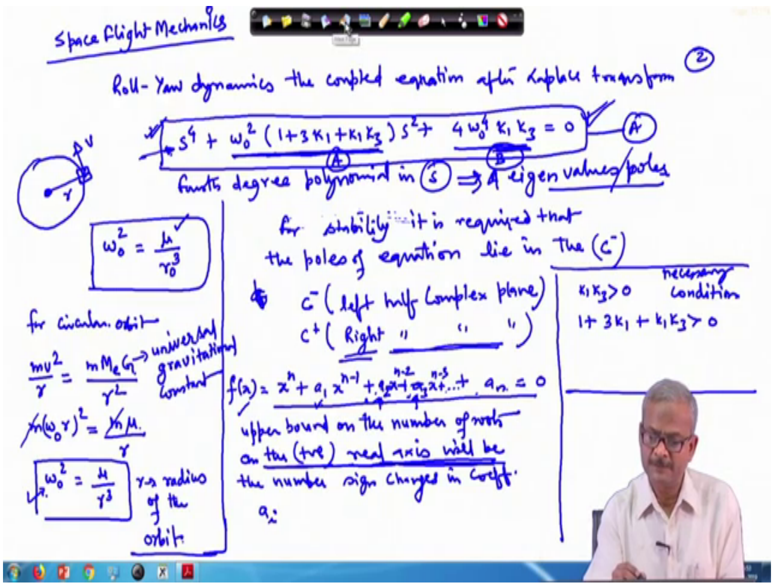
So, this is the pitch dynamics, pitching motion. Now, the stability of the motion for the pitching; so, the pitch stability requires that K I should be greater than 0 because omega

0 square, this way already greater than 0. And; obviously, in that case it is a very obvious that this motion is of the same format as the HM the Simple Harmonic Motion ok. So, if K I is greater than 0 then the pitch motion is going to be stable, means in this case you are going to get two poles on the imaginary axes and therefore, the motion will be continuous if it is a poles on the imaginary axis.

This implies that we have two poles like this ok, and if this implies that this motion will keep continuing like this, say if I have a simple harmonic motion so in that the amplitude keeps varying like this. So, will return back to all these things later on, this also implies that K I greater than 0 this implies that I 1 minus I 3 this would be greater than 0 and this implies that I 1 is nothing, but your I roll as I have discussed earlier also this is greater than I yaw. So, I roll greater than I yaw I roll greater than I yaw the moment of inertia along the roll axis should be greater than moment of inertia along yaw axis.

So, this is a this is required for pitch stability ok, then we took these two equations and converted in terms of Laplace using Laplace transform we converted it and thereafter we looked into the corresponding equation. So, this equation we have already reduced.

(Refer Slide Time: 05:57)



And the reduced equation for the for the roll yaw dynamics, the coupled equation after Laplace transform it appeared like S to the power 4 3 K 1 plus K 1 K 3 this equal to 0.

So, this is a fourth degree equation. So, fourth degree polynomial or the quadratic polynomial in S and therefore, this will have 4 roots, this implies 4 eigenvalues or the 4 eigenvalues, eigenvalues and 4 are the same thing for eigenvalues ok. So, till this extent we have been working and one more thing that we were discussing perhaps I have not explained you was the omega 0 term which we have replaced by mu by omega 0 square equal to mu by r 0 cube, this is the term which was appearing in the gravity gradient torque which appears on the right hand side of the Euler's equation.

So, this part perhaps I did not discuss. So, let us first finish this then will return back to this, I have another lecture series on space flight mechanics if you go in that and look into the topic. So, for circular orbit or let me derive it here itself, for circular orbit it we can write the equation of motion like this, this is basically of satellite there that is the earth centre of the earth there is a satellite here. So, this is the centre of mass and its moving with velocity v and this is a circular orbit. So, the necessary centripetal force or the acceleration is provided by the gravitational force.

So, m times M earth times G divided by r square. So, this is the equation ok. So, from here what we get that m v is nothing, but because this a circular orbit. So, the angular velocity will be constant and we can write this as the omega times r square and then this quantity is here m times m times G we write this as mu The quantity that mu appears here in this place this is nothing, but m earth times g where G is the universal gravitational constant, universal gravitational constant ok. So, then this gets reduced to here m mu by r m we can cancel out here and therefore, this gets reduced to omega 0 square is equal to mu by r cube.

Where r is the radius of the orbit, this is r radius of the orbit if you want to learn in detail something so look into the lecture notes on a space flight mechanics on NPTEL. So, this omega 0 which is appearing here this is nothing, but the quantity which is given here in this place ok. So, as we were discussing last time that this is a fourth degree polynomial or the quartic polynomial.

So, for poles what we are looking for that the system should be a stable ok; so, let state it like this. For stability; stability means here dynamic stability not the static stability for, stability it is required that that the poles of equation a lie in the we write it as c minus this call the left half complex plane left half complex plane ok. Because S we are obtaining it from the Laplace transform and C plus we will write right as the right half complex plane ok. So, for a stability its required that the poles of the equation lie in the C minus ok. Now if we what we were discussing that from the Descartes rule that if we have a polynomial say x to the power n plus a one times x to the power n minus one and so on.

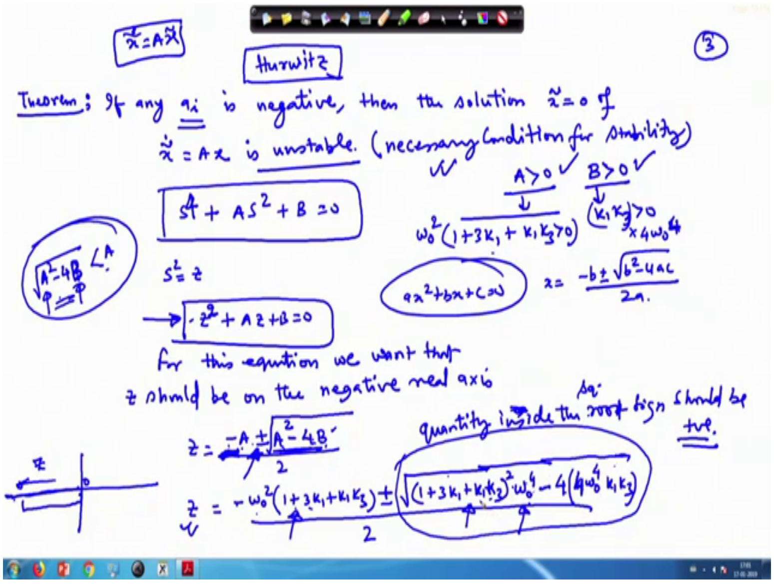
This is a polynomial of order n and for this polynomial the roots will lie in the right half complex plane, means if the poles are in the right half complex plane the system will be unstable and if the poles are in the left half complex plane system will be stable. So, this is the basic fundamental. So, we are trying to get the poles for this, as you know that from the Descartes rule so as many sign changes that takes place in this equation ok, that will be the upper bound on the. So, upper bound on the number of roots on the positive side, positive real axis will be the number of sign changes in coefficients a i. So, if say this is positive thereafter the next coefficient a two turns out to be negative.

So, will count this as the one sign change and suppose the next coefficient a 3 is positive. So, this is positive here it is negative and then this turns out to be positive. So, from here to here this is one sign change and from here to here again there is one sign change ok. So, a 1 and let us write it here a 2 x to the power n minus 3 plus a 3 x to the power n minus 3 and plus like this. So, here if this is positive, if this becomes negative and a 3 becomes again positive means we have total two sign changes. So, it says that the roots on in the and thereafter assume that all other coefficients are a i error, they are positive.

So, we will have at maximum two poles on the real axis on the positive side ok, we cannot have more than that and what we want here? We want that there should not be any poles in the type of complex plane this is our requirement. So, this requires that all this coefficients here, although the some of the coefficients are absent like the S cube coefficients are absent and then S coefficient is also absent so, but there is no sign change ok. So, here we count this. So, what is the requirement that these coefficients which I am underlining here this must be positive.

So, that there will be no sign change and no poles will lie in the right half complex plane and therefore, we stated that K 1 times K 3 this should be greater than 0, this is one requirement that poles are not in the right hand plane. So, K 1 K 3 this should be greater than 0 another one we have written as one time 1 plus 3 K 1 plus K 1 K 3, this should also be greater than 0. So, suppose these two conditions are satisfied. So, this ensure that this says that this is necessary condition; this is the necessary condition that no of poles are lying in the on the real axis on the positive side.

(Refer Slide Time: 17:01)



So, we can state a theorem on this, if any and if you look here in this particular this particular equation. So, this is the Laplace transform we have applied to a linear system ok. So, the linear system it can always be represented in the format h dot equal to A x, you can always represent it in this format we had x these are vectors. So, if any a i is negative then the solution x tilde this equal to 0 of x tilde dot equal to A x is unstable.

So, for this it is required and this is the necessary condition, necessary condition for stability and there is a related theorem for the sufficient condition, but and this is due to the Hurwitz ok, but that particular theorem is not applicable here in this case. So, therefore, we are not going into the next theorem, now what will look for that if we have this problem. So, let us write this quantity as A and the this quantity as B ok. So, these equations can be written as S square S to the power 4 A times S square plus B this equal to 0. So, this the quartic polynomial.

Now this one if this condition is satisfied. So, we know that here its required that a should be greater than 0 and b should be greater than 0 and this is implying that K 1 K 3 this should be greater than 0 and this is applying that 1 plus 3 K 1 plus K 1 K 3 this should be greater than 0, this is what we have written here in this place ok, but to explore further let us assume that we write S square equal to z. So, the above equation can be written as z square plus A times z plus B this equal to 0; now rather than enquiring for the imaginary or the complex root just look into this particular one.

For this equation we want that z should be on the negative real axis ok. So, for this requirement to be fulfilled what is required let us write the roots for this, this will be minus A plus minus B square minus plus minus A here. So, I will ok, I will continue with this notation I should have written here B and this could have been written C. So, this is also fine. So, minus a plus here then it will be a square this will be a square minus 4 a c. So, here this coefficient and this coefficient multiplied here.

So, here in this case the there is no coefficient, as you know for the quadratic equation in the quadratic a x square plus b x plus C equal to 0 once you solve it. So, how do we write for this solution, x equal to minus b plus minus b square minus 4 a c under root divided by 2 A. So, the same thing we are applying here. So, this becomes under root this and then 4 b divided by 2 ok. So, from here then A is; obviously, the quantity this quantity times this omega 0 square. So, this is minus omega 0 square times 1 plus 3 K 1 plus K 1 K 3 then plus minus b square. So, again 1 plus 3 K 1 plus K 1 K 3, whole square minus 4 times B which is.

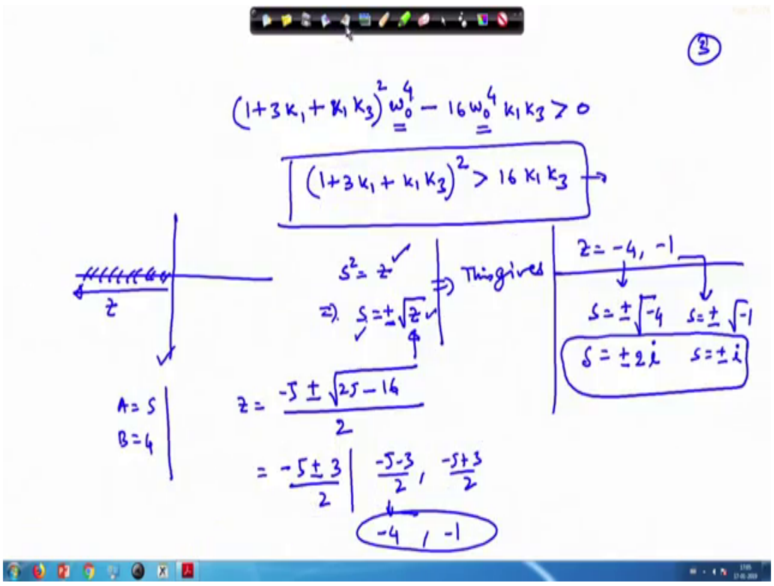
B here in this case is 4 omega 0 to the power 4 K 1 K 3. So, we have we have not written the part omega 0 square. So, will add here omega 0 square and with this we have this multiplied by 4 omega 0 square omega to the power 4, this is 4 omega to the power 4 ok. So, 4 omega to the power 4 here also we should mention it, this part then becomes omega to the power 4 because this omega 0 square is there minus 4 times 4 times the quantity which is present here, this is K 1 K 3 times 4 times 4 omega 0 to the power 4 4 times 4 K 1 K 3 and this total under root divided by 2.

Now, looking into this equation as you knows this quantity it also interfere in this place, this particular part is also present here ok. So, when the roots will be this z will be on the negative real axis here when it will lie here in this range on this side, when the z is going to lie on this side? This the question. So, far this to be negative this z to be negative it will be required that this quantity should be the quantity which is present here this quantity should be greater the quantity inside the square root sign. So, the quantity inside the square root sign should be positive.

Quantity inside the square root sign square root sign should be positive why it is a very obvious if you look here in this place this is minus A, if this quantity is positive then that will happen A square is a we have written this is a positive quantity and from there B is also a positive quantity because of this requirements. So, if this quantity, this is positive and this is positive and we if this quantity becomes positive. So, obviously, this A square minus 4 b under root this will be less than A ok, this quantity will be less than A which is very obvious because a and b both are positive.

So, then a quantity which is smaller than B if you add to that so still this quantity will remain negative this minus a if we add this is a quantity which less than A and to this we are adding minus A. So, that will remain negative and this minus A and here the minus sign this particular sign. So, that again it will make it negative. So, it ensures that if this quantity ok, which also we call it the discriminate; so, if this is positive then we are going to get the z in the on the left hand side on the real axis ok.

(Refer Slide Time: 27:19)



So, from here what we get hat 1 plus 3 K 1 plus K 1 times K 3 omega 0 to the power 4 minus 4 into 4 so that makes it 16. So, will right here directly as 16, 16 omega 0 to the power 4 K 1 times K 3 this would be greater than 0 and omega 0 they are positive quantity so we can eliminate it. So, this gets reduced to this is a square here 1 plus 3 K 1 plus K 1 times K 3 square this should be greater than 16 K 1 K 3.

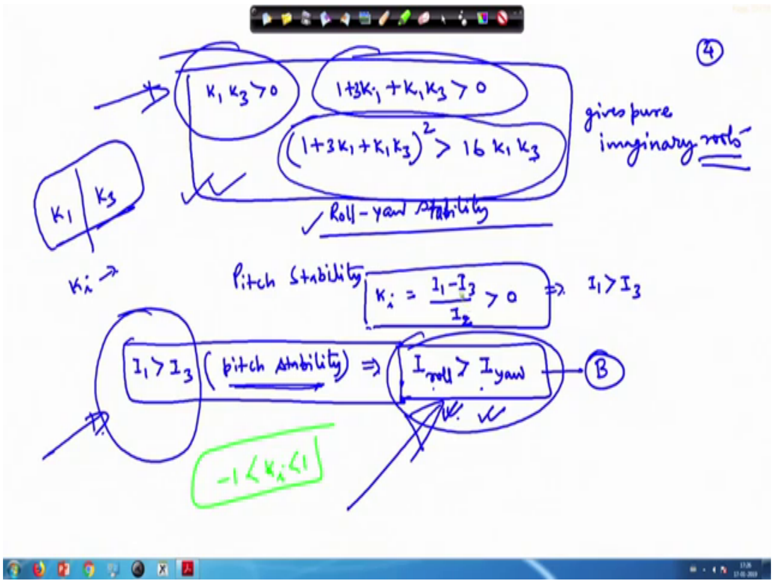
So, this was the requirement we have put and if this is ensured then what we observed that z is lying on the negative side on this side ok. So, with this we have written S square equal to z and this implies then that the S will be z under root and if z is lying on the real axis here on the left hand side, so S is going to be purely imaginary. So, we will have here plus minus z ok. So, you have two values of z from this equation and then coming here in this place S will be equal to plus minus z. So, two values we need to insert here and the correspondingly we get 4 imaginary roots.

So, this (Refer Time: 28:59) you, this gives we can take one example maybe and look into this, let us say A equal to 5 and B equal to 4 A equal to 5 and B equal to 4 just for example, sake ok. So, z will be minus 5 plus minus 25 minus 16 divided by 2. So, this is minus 5 plus minus here this is 9 ah. So, 9 this becomes 3 divided by 2. So, we will have 2 roots here minus 5 minus 3 divided by 2 and minus 5 plus 3 divided by 2. So, that gives me minus 8 divided by 2 is minus 4 and this gives me minus 2 divided by 2 equal to minus 1.

So, these are the two roots, for the. So, z equal to minus 4 and minus 1. So, correspondingly we get for this the S will be plus minus 4 under root and corresponding to this we get S equal to plus minus minus 1 under root. So, this is minus plus minus this is 2 i and this gives you plus minus i. So, these are the 4 purely imaginary roots as I told you in the last lecture.

. So, here it so happens in this case that we get purely imaginary roots for this particular equation, provided these coefficient coefficients this coefficient, and this one they are positive and moreover there another requirement is that this condition should be satisfied. So, we have total 3 conditions to be satisfied.

(Refer Slide Time: 31:13)



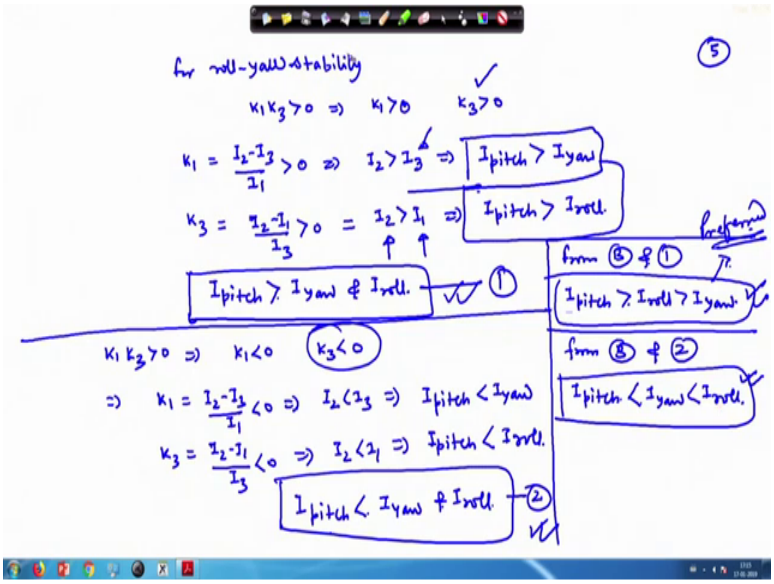
That K 1 K 3 should be greater than 0, 1 plus 3 K 1 plus K 1 K 3 this should be greater than 0 and also 1 plus 3 K 1 plus K 1 K 3 square this should be greater than 16 K 1 K 3.

So this pure imaginary roots; so, we have total 4 imaginary roots now we can analyze the system stability from this, what kind of under what circumstances what if the value of K 1 is something, if the value is of K 3 is something. So, how the roll yaw dynamics will look like. Together with we have to also look for the K I, the K I we have defined for the pitch dynamics. So, if we go back this K I which is entering here this we have also defined earlier and the symbol for this that we have used K i equal to I 1 minus I 3 divided by I 2.

So, this is the quantity which enters here in this place and this should also be positive. So, this is for roll yaw stability which are coupled together, these two dynamic dynamics along these to axis roll and yaw axis is coupled together; while for the pitch stability pitch stability its required that K I which we have defined as I 1 minus I 3 divided by I 2 it should be greater than 0. And this tells that I q should be greater than I 3. So, I 1 greater than I 3 for pitch stability and this implies that moment of inertia along the roll axis should be greater than moment of inertia along the yaw axis for the pitch stability this is your requirement.

In the same way we can discuss about the stability along the stability for the roll and the yaw. So, next we take the stability for roll and yaw.

(Refer Slide Time: 35:06)



For roll yaw stability K 1 K 3 greater than 0 this implies that K 1 should be greater than 0 and K 3 should be greater than 0, and how we have defined this K 1 and K 3 we have to go back and look into those equations ok. So, the K 1 we have defined as I 2 minus I 3 divided by I 1. So, the if this is greater than 0 this implies I 2 will be greater than I 3, and this says that I pitch should be greater than I yaw. Similarly the K 3 equal to I 2 minus I 1 divided by I 3, this should be greater than 0 according to this.

So, this implies I 2 will be greater than I one and this says that I pitch should be greater than I roll because, along this axis one axis you we have roll along the 2 axis we have pitch along the 3 axis we have yaw. So, taken in together these two things are state that I pitch should be greater than I yaw and I roll ok, I will name this as say 1 this is the one condition. The other condition we can get in the same way K 1 K 3 greater than 0 this also implies that K 1 can be less than 0 and K 3 can be less than 0. So, this implies that K 1 equal to I 2 minus I 3 divided by I 1 this should be less than 0 and therefore, I 2 will be less than I 3 and this implies I pitch should be less than I yaw.

In the same way the K 3 this part K 3 less than 0. So, K 3 is here I 2 minus I 1 divided by I 3 this is less than 0 this implies. So, this is I pitch should be less than I roll ok. So, this two can be summarized together like I pitch this should be less than I yaw and I roll. So, let us name this as the 2nd one. So, what we see that either I pitch should be greater than I roll here or either I pitch should be less than I roll and yaw, and further pitch stability we have only one condition that I roll should be greater than yaw ok. So, now, we utilize this.

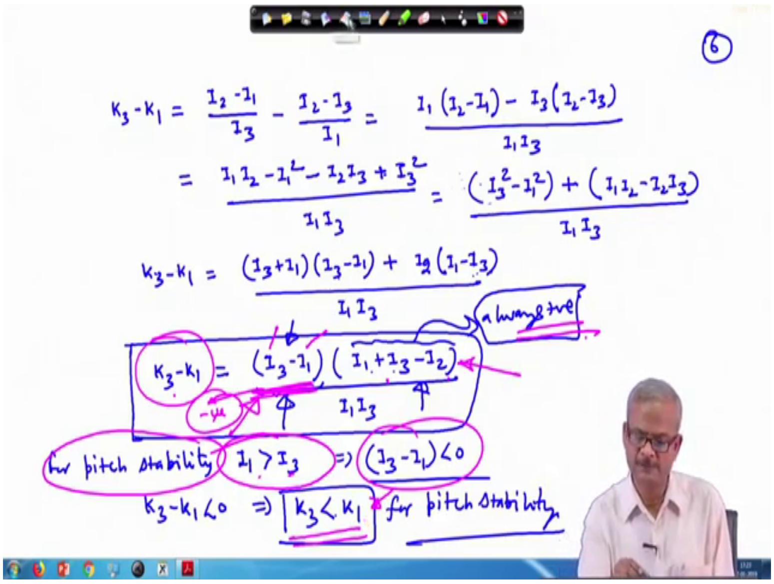
So, I will write here in this portion. So, if this is 1 2 and here I will write this as maybe say A earlier I have written perhaps. So, I will name this as B. So, from B and 1 what we get that I pitch should be greater than, I roll has to be greater than I yaw for satisfy for the satisfaction of pitch stability for ensuring pitch stability.

So, and here I pitch is there is no pitch and pitch here just check it. So, I pitch should be greater than I roll and this should be greater than I yaw. So, this is what we conclude using this one and this one, on the other hand from B and 2 this one, ok. So, from here we conclude that I pitch should be less than, what is the another condition? That I roll has to be greater than I yaw ok.

If you look in to this I roll has to be greater than I yaw. So, this implies that I yaw should come like this and here I roll. So, this is one and this is one. So, these are the two conditions on the moment of inertia we are getting here in this place and out of this, this one is preferred for the reason which I am going to explain afterwards ok. So, that says that the I pitch should be greater than I roll and I yaw then your pitch stability more over roll and yaw stability will be ensured and even if this condition is satisfied then also it tells that the pitch and the yaw roll stability will be satisfied.

But we have to wait further ok, because we have just taken this condition we have just concluded based on this condition and this condition ok. Other things we need to conclude what is remaining this part, this part is remaining and this part is also remaining this two we need to explore before we can tell something else. So, to explore further let us look into some of the things which will be required for further analysis.

(Refer Slide Time: 42:41)



Let us write K 3 minus K 1 this quantity K 3 is I 2 minus I 1 divided by I 3 K 3 I 2 minus I 1 divided by I 3 from this part and then K 1 we have to put here, K 2 minus I 3 divided by I 1. So, this is I 2 minus I 3 divided by I 1. So, I 2 minus I 3 divided by I 1.

Student: (Refer Time: 43:33).

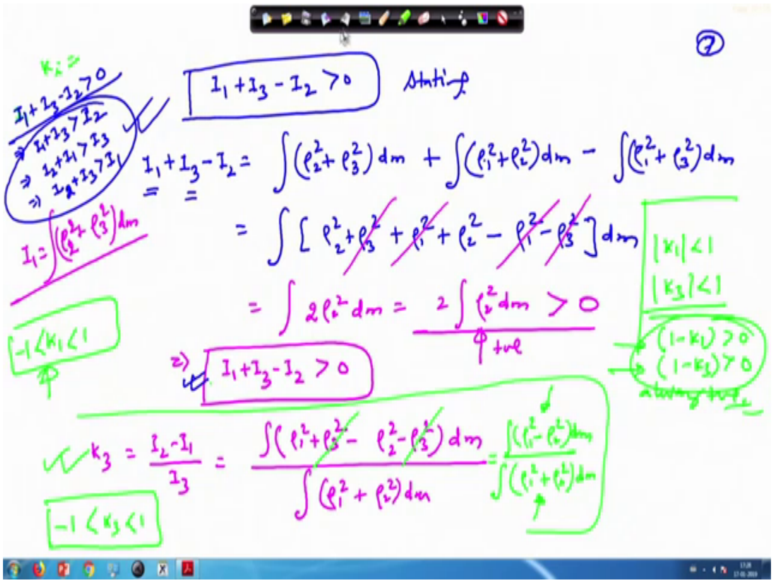
Thus rearranging this particular equation thus we have K 3 minus K 3 minus K 1 this gets reduced to I 3 plus I 1 minus I 2 we can take it outside times.

Student: (Refer Time: 45:03).

So, we can rewrite this as here we are taking exchanging this side minus sign we are taking outside and writing it like this and taking the common I 3 minus I 1. Now go back and look for the what was required for pitch stability, for pitch stability was record at I 1 should be greater than I 3 for pitch stability.

So, for pitch stability I 1 should be greater than I 3. So, if that happens so implies that the quantity here, this quantity will be negative if I 1 gets becomes greater than I 3. So, this implies I 3 minus I 1 will be less than 0 ok. So, this quantity becomes negative and I will show that this quantity this is always positive, always positive ok. So, this implies that K 3 minus K 1 because this quantity is negative and this is always positive. So, this K 3 minus K 1 will be less than 0 and this implies that K 3 should be less than K 1 for pitch stability ok, this is the requirement. So, one requirement we have got here which is K 3 should be less than K 1 ok. Now, let us look into this part I 1 plus I 3 minus I 2 why this is always positive.

(Refer Slide Time: 47:11)



I 1 plus I 3 minus I 2 this is greater than 0 this is what I am stating. Now I 1 plus I 3 minus I 2 we can write as rho 2 square plus rho 3 square d m, this is the inertia and this is how inertia we have defined then I 3 equal to rho 1 square plus rho 2 square d m and then minus I 2. So, this is rho 1 square plus rho 3 square d m. So, we combine together so this gets reduced rho 2 square rho 3 square rho 1 square plus rho 2 square minus rho 1 square minus rho 3 square d m. And there after this rho 1 square rho 1 square this cancels out rho 3 square rho 3 square cancels out and what we get here rho 2 square 2 d m is 2 rho 2 square d m.

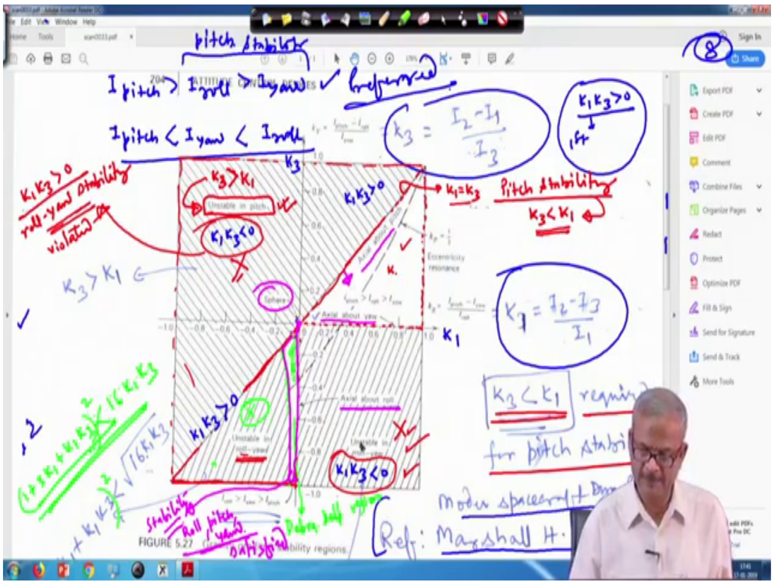
This quantity is always positive and therefore, this quantity will be always greater than 0. So, while discussing in this place we have stated that this quantity is always positive therefore, this is positive and this quantity will be negative from the requirement that for the pitch stability I 1 should be greater than I 3. So, I 1 greater than I 3 this implies this quantity is going to be negative. So, you this is negative so automatically this K 3 minus K 1 that gets negative and therefore, we get this and this implies this that K 3 should be less than K 1. So, this is another conclusion we have got ok, moreover if we look into the some more think I will state in this figure.

This here K 3 so, I 2 minus I 1 divided by K 3 is I 2 minus I 1 divided by I 3. So, if the same way this equation if you use this kind of equations so, you can write it I 2 will be rho 1 square plus rho 3 square and I 1 will be, and then rho 1 will be. So, this I 1 is rho 2 square plus rho 3 square d m ok. So, this we need to insert here. So, this is rho 2 square minus rho 3 square this is d m and I 3; obviously, you have rho 1 square plus rho 2 square d m and if we look into this quantity. So, rho 3 and rho 3 it cancels out and this gives you rho 1 square minus rho 2 square divided by rho 1 square plus rho 2 square d m.

So, what it says this quantity is always positive, this can be positive or negative, but one thing is ensured that K 3 will be less than 1 and also this will lie between minus 1 and 1. The same way if you look for K 1; so, K 1 also its a going to lie between 1 and minus 1 you can use the same line of deduction what if I have done here in this place and so that K 1 is also like that. And in the same way the K I, if you are looking for the K I quantity K I we have written here in this place ok, along the same line K I will also lie between minus 1 to 1 because this is the property a simple thing. So, this implies, this implies that K 1 magnitude this will be less than 1 and also the K 3 magnitude this will be less than 1 ok.

And therefore, wherever you have got quantitative line 1 minus K 1 and 1 minus K 3 this will always positive always positive ok, this as appeared this two terms have appeared in the roll yaw dynamics. So, this information so whatever we have derived here we can utilize it for our purpose. This part this also says I 1 plus say, the I 1 plus I 3 minus I 2 this is greater than 0. So, this also implies that I 1 plus I 3 will be greater than I 2, along the same line also you will have I 2 plus I 1 this will be greater than I 3 and also I 1 plus I 2 plus I 3 this will be greater than I 1 ok. Very easy to reduce using this kind of information ok, that you can check just work like this and you will get all these results.

(Refer Slide Time: 54:01)



So, now we are almost ready to explain the stability of the system. So, what we have looked till now that I pitch should be greater than I roll and I yaw this requirement comes this particular part comes from the pitch stability or either I pitch should be less than I yaw and I roll, while we are preferring this.

This is preferred one, and we have to get into this to explain the whole thing. Now we go into a figure for explaining this part ok, now look into this figure here on this axis K 1 is plotted and on the vertical axis the K 3 is plotted where the K 3 is written here and K 1 is written here in this place and this particular figure is from the modern spacecraft dynamics and controls by marshal H Kaplan. So, if you look here into this figure. So, what are our requirement? The one requirement we have written was K 1 K 3 this should be positive, K 1 times K 3 this should be greater than 0. So, where the K 1 and K 3 will be greater than 0 it will be greater than 0 in this quadrant in the first quadrant and in the in this quadrant again if you look the K 1 and K 3 multiplication.

So, K 3 will be negative along this side while positive here; so, this becomes negative. So, here K 1 K 3 this is less than 0 here on this side we have K 1 K 3 greater than 0 on this whole sides in the third quadrant K 1 K 3 this will be greater than 0 and while here K 1 K 3 this will be less than 0. So, to satisfy this requirement what we see that the regions marked by this red cross and this red cross they are ruled out and this stability requirement is from the for our roll yaw stability ok. So, the K 1 K 3 greater than 0 it says required for the roll yaw stability.

So, this two things are not satisfied either here in this domain or either here in this domain. So, this two are ruled out thereafter for the pitch stability; for pitch stability also we have observed that K 3 should be less than K 1 as it is written here is required that pitch stability K 3 should be less than K 1. So, where the K 3 is less than K 1 if we look for this; so, this line this is K 1 equal to K 3 this just at 45 degree ok, one here in this place. So, on this line your K 1 equal to K 3. So, below this here in this region you will have the for the pitch stability if you look here for this region. So, this is written here on a stable in pitch. So, in this region above this line, above the line which is drawn here like this ok.

It is a going from this place to this place ok. So, for this line you will have K 1 greater than K 3 greater than K 1. So, while the pitch stability is required that K 3 should be less than K 1 and therefore, this becomes unstable in pitch ok; so, for this region this is unstable in pitch. And this is applicable for all the points lying above this line ok. So, for this whole region which is as like this for this upper region, for region which is belonging to this portion here this whole thing and this whole portion this is pitch unstable ok, because this condition is not satisfied K 3 greater than K 1 it will not be satisfied above this.

And therefore, it has been has like this when written this is a pitch stable and already we have seen that in this region also roll yaw in stability is there because K 1 K 3 is not greater than, this is basically less than 0. And therefore, you can compare this condition is violated ok, this is violated and therefore, roll yaw in stabilities also present here in this region already roll yaw in stability is there because of this condition not getting satisfied that K 1 K 3 should be K 1 K 3 multiplied together should be greater than 0. So, this is also violated and therefore, here also this is unstable.

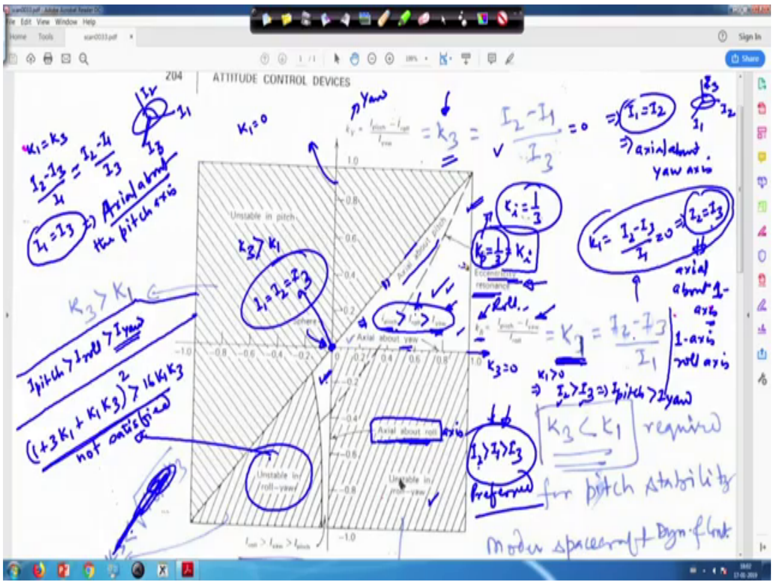
So, what remains towards this region which is the lower one, lower part from here to here this particular part all the way here this part and then this part which I am showing by redline and also this part and this whole thing here. So, this region and this region is remaining now this part we have to look into. So, again here it is written that it is a unstable in roll and yaw ok. So, for what region this is unstable we have to find out.

So, this instability if you look for this, this line which is shown here say this line above this line this condition 1 plus 3 K 1 plus K 1 times K 3 greater than 0 this will be satisfied only in this region and it is not satisfied here in this region ok. So, basically sorry we are writing here this a square this should be greater than 16 K 1 K 3, this square should be greater than K 1 K 3 here there was some term. So, here the square should be there which is missing here. So, this quantity we need to have greater than 0 which is not satisfied here in this place rather it is a violated, means in this portion this will less than 16 K 1 K 3 and this line arises by solving this particular equation.

So, on this side you have this condition violated while on this side it is satisfied and this particular region which I am marking by this green is called the Debra Delf region we do not have space here ok, for the person who solve this problem. So, after him this name is given. So, only this white portion which is remaining here this particular portion in this here a stability ensured both roll and yaw stability; raw stability yaw stability its ensured and already we have looked for that the on the upper side pitcher stability is not satisfied, but on the lower side on this side the pitcher stability is satisfied.

Therefore in this region all the roll pitch and yaw stability all of them are satisfied ok. So, this is satisfied; while here on this side your all the conditions are satisfied in this place. So, we will discuss further this particular figure in the next lecture for the time being we are stopping here because already if we have exceeded much more than half an hour supposed for this lecture ok. So, few things which are remaining we will take up in the next lecture like the axial about the pitch, here it is written axial about the yaw here it is a written axial about the roll ok, then here in this portion it is a written this sphere which is being indicated for the central point so.

(Refer Slide Time: 64:54)



So, we are taking again the old figure and looking into that because that was totally filled up by different comments. So, here in this region we have seen that this condition is not satisfied ok. So, here not satisfied and therefore, this is unstable in roll and yaw, all this things we have already discussed. Now there are certain comments it is a written here axial about the roll axis, it is a axial about the roll axis. So, what does this mean? On this axis if you look here on this axis. So, this particular axis K 1 is 0 ok. So, K 1 here this equal to 0 so, this implies K 1 is 0 if we look take up this particular part. So, this will be K 1 equal to I two minus I 3 divided by I 1 this equal to 0.

So, this implies I 2 equal to I 3 and this implies axial about the one axis. So, therefore, it is written the axial about the roll means the one axis it is a related to your roll axis ok. So, one axis this is nothing, but your roll axis. So, therefore, all the points on this axis it is a termed as the axial about the roll in the same way axial about the yaw on this axis ok. So, for that we have to set see that K 3 is 0 here on this place K 3 is 0. So, we need to set it to 0. So, if we set it to 0 this implies I 1 will be equal to I 2 and therefore, this implies this happens in the case of a say the disk, here if we take this as the I 1 and this as the I 2 and this is the I 3.

So, this says that this is the axial about yaw axis and this is what is written here axial about the yaw axis this line is written here axial about the pitch axis so this particular line. So, here K 1 equal to K 3 on this line ok. So, K 1 equal to K 3. So, K 1 is I 2 minus I 3 divided by I 1 and K 3 is this part I 2 minus I 1 divided by I 3. So, this implies I 1 will be equal to I 3 ok. Only then these two quantities on the left and right will be equal and this implies that axial about the pitch axis. So, something like this you have I 1 and we will have to name it properly.

So, we will name this as I 1, I 1 and I 3 are equal. So, this is I 1 this will name as I 2 and this will name as I 3. So, I 1 I 2 and I 3 using the right hand rule ok, then we have the quantity here K p equal to 1 by 3, this is K p which is nothing, but in our case this is K I for the K I already we have defined. This is appearing for the pitch, in the in this book this has been used for the pitch similarly K y it has been used for the yaw while we are using it for K 3, similarly here the K r is a standing for roll this is for roll.

Now, what remains that in this region already we have discussed that in this region and this region our conditions are satisfied moreover so only this region shown in white which is not has this satisfying the given conditions for the pitch roll and yaw ok. And what is shown here by this line which is a standing for and this is applicable for the eccentric orbit which we have still not discussed, we have taken the satellite to be in the circular orbit. If we take it to be the eccentric orbit so for that you will get that solution. So, we are not discussing about that ok, once we tick the eccentric orbit I will give one lecture for that. So, at that time I will take up this particular part, now here if we look here in this part. So, its written here I pitch greater than I roll and I roll is greater than I yaw.

So, already we have observed that for our stability the high pitch should be greater than I roll and I roll should be greater than I yaw or either I pitch should be less than I yaw and I yaw should be less than I roll ok. So, then this part we do not find the space here much K 3 is greater than K 1 here. So, only in this region we are getting I pitch for this particular region, greater than I roll is greater than I yaw. In this region you have already you can observe that I pitch minus I yaw because here as it is appearing this K 1 this quantity is positive here in this region ok.

And therefore, K 1 is positive means if K 1 is greater than 0. So, this implies that I 2 will be greater than I 3 means I pitch will be greater than I yaw this implies I pitch this is greater than I yaw, which is written here ok. I pitch happens to be greater than I yaw and also for the pitch stability requirement is that the, I roll should be greater than I yaw. So, and already we have seen that that condition is satisfied in this region and therefore, this particular condition is only satisfied in this region and also in this region and as I have stated earlier that this is the reason that I pitch or I 1 greater than sorry this I pitch is I 2. So, I 2 greater than I 1 greater than I 3 this is preferred as earlier I have stated you.

So, this is from this figure we can get all the results, one particular part is remaining is this part the centre at the centre your K 1 is vanishing and also K 3 is vanishing. So, in that case K 3 is vanishing means I 1 equal to I 2 and K 1 is, once the K 1 vanishes here the K 1 vanishes. So, in that case you are getting I 2 equal to I 3. So, this implies in this place you will have I 1 equal to I 2 equal to I 3, and this is the case only for this sphere ok. So, this is referring to your sphere this particular point the origin ok.

So, this may you can analyze this figure, now this figure once we have got this figure. So, while you are designing you are selecting the moment of inertia then you have to ensure for the gravity gradient satellite that your the moment of inertia once your choosing so that your K 1 and K 3 and K I all they should satisfy this particular white regions which is shown here in this figure ok. So, I hope for the time being decision of and I will give one more lecture next time which will be on the gravity gradient satellite in the eccentric orbit and wind up this particular topic. And there are much more to discuss, but only 13 lectures we have scheduled on this particular topic, 5 lectures scheduled we have already given 10 lectures; if I continue we can exceed 15 lectures. So, of the other things then we will fall sort of. So, we will stop after completing one more lecture.

Thank you very much for listening.