Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture-34
Gravity Gradient Satellite (Contd.)

So, we have been discussing about the Gravity Gradient Satellite. So, we will continue with that.

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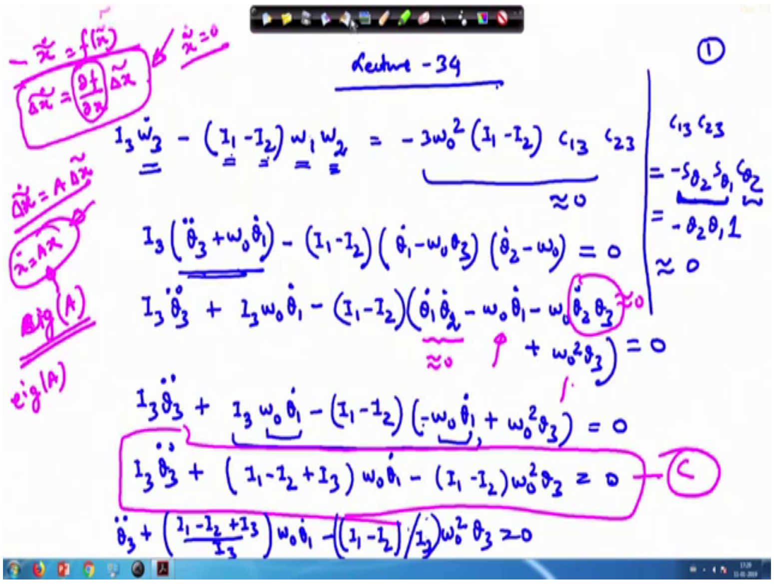

So, if you remember in the last lecture, we have derived equations for the first Euler dynamical equation in terms of Euler angles and the second Euler dynamical equation in terms of Euler angles.

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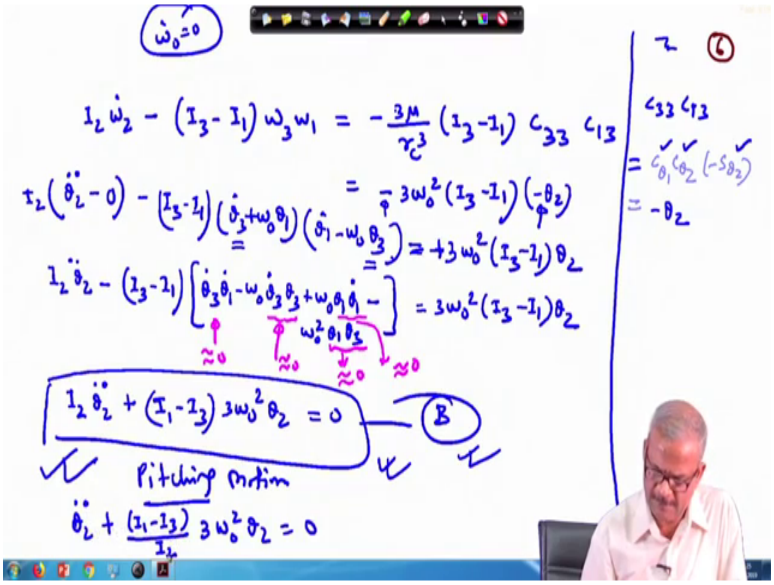

So, those equations we have written like this, this one and this one.

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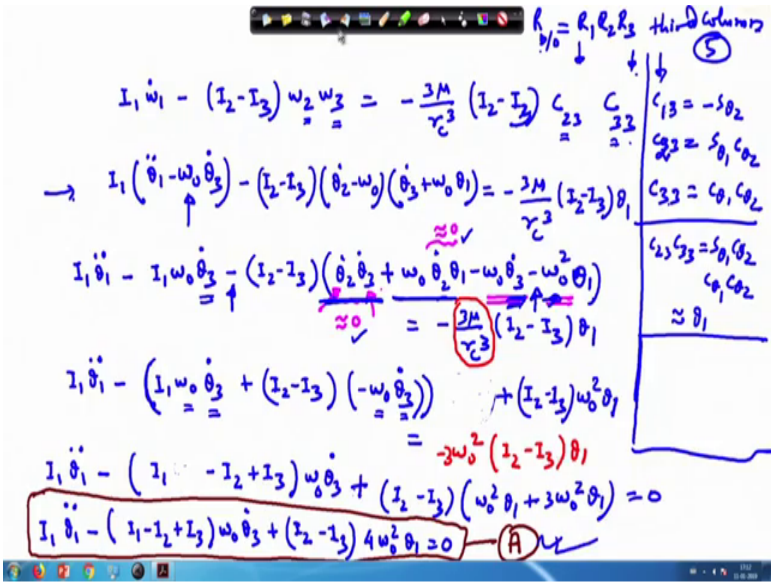

So, we will refer back to this equation again. So, we go to the 3rd equation which is I 3 times omega 3 dot minus omega 1 times omega 3 omega 1 times omega 2, the same term as related omega 1 appears here in this place.

This equal to on the right hand side 3 omega 0 square then I 1 minus I 2 times C 1 C 2 and C 3. So, C 13 times C 23 this we have to insert. So, go back here in this place C 1 3 is minus S theta 2. So, this is minus S theta 2 and C 23 C 23 is S theta 1 C theta 2 S theta

1 C theta 2 S theta 1 C theta 2. So, if you look here in this quantity this quantity is of second order which will be something like minus theta 2 times theta 1 and this is 1. So, this will be 0.

So, this sum on the right hand side it drops out. So, this is equal to 0 for the small angular perturbation and then simply we have to insert here omega 3 dot. So, omega 3 dot again we have to go back and find out those values.

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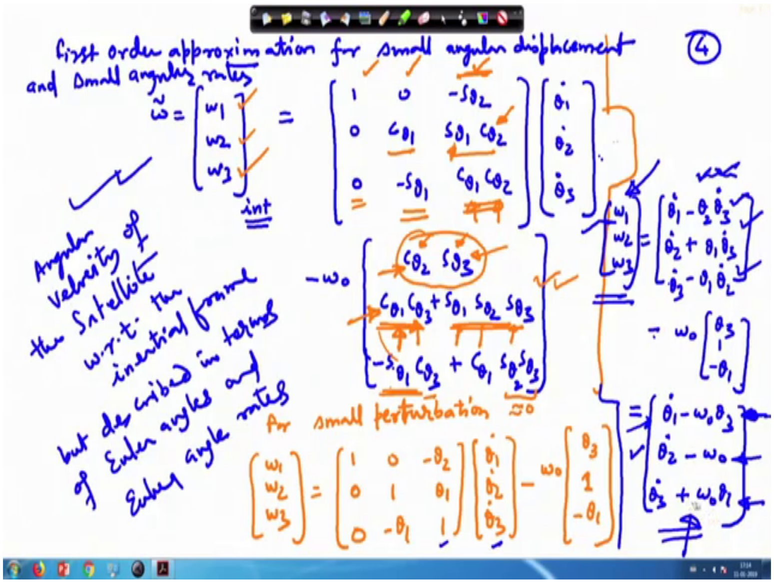

Omega 3 dot omega 3 is here theta 3 times plus omega 0 theta 1. So, theta 3 dot plus omega 0 theta 1 minus I 1 minus I 2 times omega one omega two. So, omega 1 and omega 2 again we have to pick up from this place omega 1 is theta 1 dot omega 0 theta 3.

Theta 1 dot minus omega 0 theta 3 and omega 2 dot is theta 2 dot minus omega 0 this is theta 2 dot minus omega 0 this is your omega 2. So, the right hand side is set to 0, and here because we are taking the derivative of this we are writing as omega 3 dot. So, we have to put a dot here we have to put a dot here; I 3 times omega 0 times theta 1 dot I 1 minus I 2 and expand this term as earlier we have done theta 2 dot minus omega 0 times theta 1 dot minus omega 0 theta 2 dot times theta 3.

And last term will be minus plus omega 0 square theta 3 and this equal to 0. Now looking back this term is 0 (Refer Time: 04:07) of second order and this term is also of second order. So, this is also 0. So, we are left with this term and this term. So, I 3 times theta 3 double dot theta 1 dot minus I 2 minus omega 0 theta 1 dot plus omega 0 square theta 3 is equal to 0.

So, whatever the terms we can combine we will combine here ok, if we take the, write it this way. So, what we want to do? That this term we can take it outside. So, this minus this minus that makes it plus. So, we can write this as I 1 minus I 2 plus I 3 times omega 0 theta 1 dot ok, this term from here to here and there after we are left with this term. So, that gives you I 1 minus I 2 times omega 0 square theta 3 is equal to 0. So, this is our equation C.

So, these are the 3 Euler's dynamical equation, but now it is a written in terms of the Euler rates and the Euler angles. So, benefit is that if as I have told you earlier that quite often we look for how the body is rotating. So, if we are looking in terms of the body components that is omega 1 omega 2 omega 3 we can visualise nothing. But if we look the same thing in terms of Euler angles then we can visualise how the satellite is moving, and for that reason for we have expressed it in this form and there we are assuming that the Euler angles are small and Euler angular rates are also small and therefore, the approximation can be done.

So, under these conditions how your Euler's dynamical equation it looks like then whether using this, the question is that using this what we are going to get how we are going to get benefited. The thing is that we can look into the system a stability; how do we look into the system stability? We always linearise a system and then we look for whether the system is a stable or not a stable, this is this is what we do most of the time we do the analysis in terms of eigenvalues ok. If the eigenvalues are lying in absolutely in the left of complex plane the system is stable, dynamically stable.

If it is lying on the imaginary axis so still we say this is stable, but it is called the actually marginally a stable, this kind of system its of in reality in the physical reality this kind of system also considered as unstable because once the noises the system absorbs the noise. So, that noise is basically the absorption of energy so and because of that the system may get become unstable. So, the system which are marginally stable it still its considered as stable, but in from the physical reality point of view this kind of system will ultimately it make create problem.

While if the eigenvalues are lying in the right of complex plane though then the system is completely unstable, there is nothing to do. So, like here I can say that x dot equal to f x ok; so, I can linearize the system and I can write it like this dou f by dou x times delta x ok. So, this is a linearized system, let look into this place and if you write it in this format that this is a vector and x tilde it is a vector. So, therefore, we can put a tilde here to indicate this is a vector, this becomes a matrix.

And in this format this is nothing, but delta x dot tilde this equal to a times delta x tilde and the you know that this is of the form x dot equal to a x which is a linear differential equation, and this equation. And how do we analyse it? As I have told you we get the eigenvalues of this, get the eigenvalues of A if you give command eig A. So, it will list you the eigenvalues of matrix A and from there we will be able to know whether your system is locally stable or not, while we do like this we just look around the equilibrium point ok. And how do we define the equilibrium point? We set x tilde dot equal to 0 and look into the find out the equilibrium point.

So, about the equilibrium point we are linearizing the system and then looking into its stability ok. Now once we have got this 3 equations so then it remains to study the stability of the system. So, we will have to accumulate all the 3 equations in a single place. So, we have the first equation.

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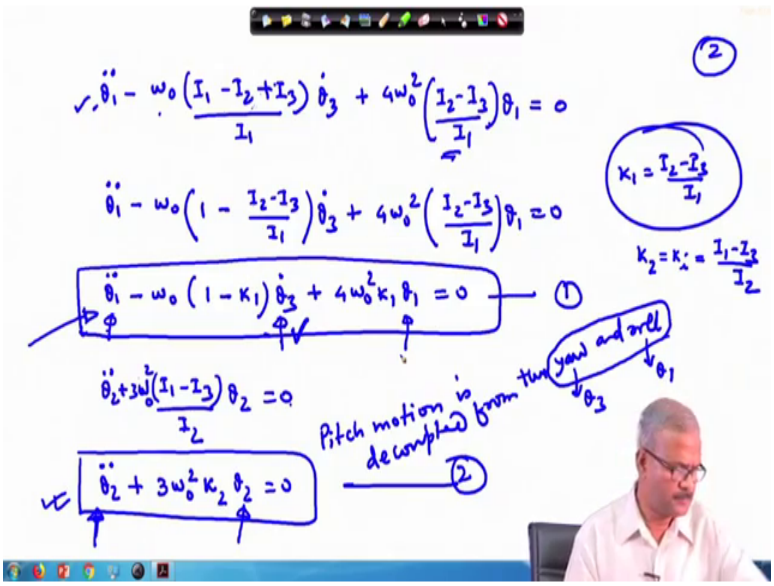

We have written as theta 1 double dot minus this is 2. So, the first question we have written as theta 1 double dot minus omega 0 plus I 3, and here there was I 1.

So, this I 1 I am eliminating from this place and this I 1 I will put here in this place and then we have the term of course, theta 3 dot. So, go back verify this we are writing it correctly, I 1 times theta one double dot I 1 minus I 2 plus I 3 omega 0 times theta 3 dot; omega 0 times theta 3 dot. Omega 0 I have write written here in this place and then the next term this is 4 omega 0 square 4 omega 0 square theta 1 4.

So, I am writing it in the beginning. Then I 2 minus I 3 I 2 minus I 3 and this also will get divided by I 1 because I 1 I have pulled up from this place to this place and then this is multiplied by theta 1 and this is set to 0 ok. So, this we can brief as 1 minus I 2 minus I 3 divided by I 1 times theta 3 dot plus omega 0 square I 2 minus I 3 divided by I 1 times theta 1 this equal to 0.

Let us say this is equation number 1, here K 1 equal to I 2 minus I 3 divided by I 1. Similarly the pitch equation, this is your pitch equation this we can write as theta 2, here itself let us first write and then we will write on the next page. I 1 minus I 3 divided by I 2 times 3 omega 0 square theta 2, this equal to 0. So, theta 2 double dot I 1 minus I 3 divided by I 2.

Theta 2 double dot plus I 1 minus I 3 plus I 1 minus I 3 divided by I 2 then this 3 factor is there 3 omega 0 square theta 2. So, this 3 factor we will put 3 omega 0 square theta 2. So, we will put it like this, 3 omega 0 square theta two this equal to 0 and this we will write as K two and quite often it is also written as K I which is I 1 minus I 3 divided by I 2. So, this can be written as theta 2 double dot plus 3 omega 0 square K 2 times theta 2 this equal to 0, this is your equation number 2.

What we can look that theta 2 is the pitch angle ok, I will explain you why I am calling this as pitch. Earlier also I have told you here you can see that in this first equation the theta 1 and theta 3 both are involved, while here in this only theta two is there theta 2 dot double dot and theta 2. So, this is not coupled with the other displacement or other Euler rates. So, this implies that the pitch motion is decoupled from the yaw and roll. Yaw and roll it is a related to is it related to theta 3 and this is related to theta 1, we will return back to this part first let us wind it up.

And then theta 3 dot finally, we have to write. So, theta 3 dot we go back into the equation theta 3 dot is here. So, this theta 3 dot let us first write here in this place theta 3 double dot this equal to I 1 minus I 2 plus I 3 divided by I 3 theta 1 dot I 1 minus I 2 times omega 0 square and of course, we have to divide it here, slash I 3 omega 0 square and this is in bracket times theta 3 this equal to 0.

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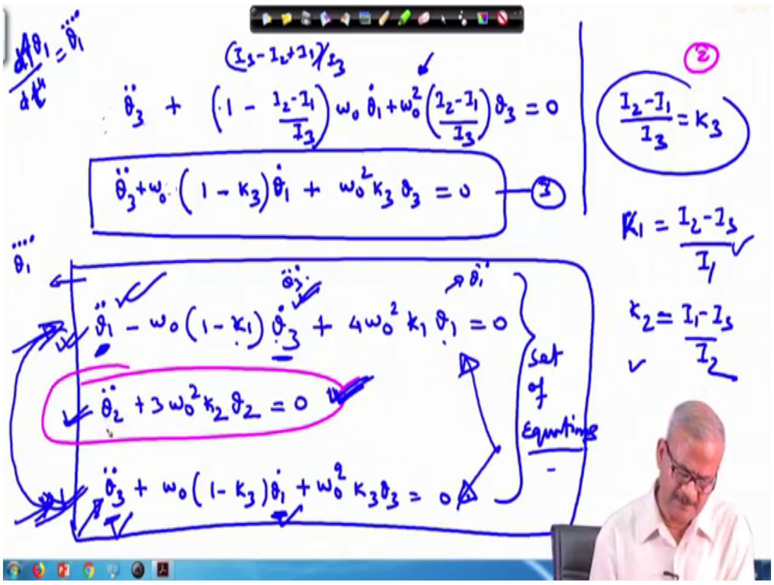

So, we have theta 3 double dot plus I 1 minus I 2 and all these quantities are there. So, we can re write this as I 3 divided by I 3 this will be equal to 1 ok, and then this I 1 minus I 2 we can write this I 2 minus I 1 ok, we can write it like this ok. It is the same thing, if we look here if we multiply it like this will get reduced to I 3 minus I 2 plus I 1 divided by I 3.

So, we can write in this way, it is a little advantage that we will see how it this is the thing omega 0 times theta 1 dot omega 0 times theta 1 dot and then is the last term I 1 minus I 2 slash I 3. So, this we will write as I 2 minus I 1 plus I 2 minus I 1 divided by I 3 times omega 0 theta 3. So, omega 0 we can put in the beginning here itself, theta 3 this equal to 0.

Now, going back here I 2 minus I 3 divided by I 1 we have defined as K 1 ok. So, similarly I 2 minus I 1 divided by I 3 we will define this as K 3 and if it we find this quantity, this quantity then we can write this as 1 minus K 3 times and omega 0 again I will put it here. This is plus omega 0 times here theta 1 dot, here this is there is omega 0 square this particular term one phase extra blank page has appeared.

So, here this quantity is omega 0 square, two extra blank pages have turned up. So, minus omega 0 this we have written as I 2 minus I 1. So, this gets deduced to plus and this omega 0 square is there. So, here omega 0 square, this square we have to put and this term is K 3 times theta 3 this equal to 0. So, this is the equation number 3. So, combined together now we have the, we can summarize the whole thing 1 minus K 3 times theta 1 dot plus omega 0 square K 3 times theta 3 this equal to 0, then theta 2 double dot plus 3 omega 0 square K 2 times theta 2 plus 3 omega 0 square K 2 times theta 2 this equal to 0.

And theta 2 and this one last one; so, theta 1 double dot minus omega 0 theta 1 double dot minus omega 0 times 1 minus K 1 times theta 3 dot 3 dot plus 4 omega 0 square K 1 times theta 1; 4 omega 0 square K 1 times theta 1 this equal to 0. So, is the set of equations we get, no this equations from here we can study the stability of the motion. So, once I will verify all this equations that somewhere some sign problem or other things has not taken place.

Just quickly going through this, this part is and 3 omega 0 square K 2 we have written as, K 2 we have written I 1 minus I 3 by I 2 this is fine and then theta 2 this equation is fine. And this equation just now we have derived and theta 1 double dot this minus omega 0 1 minus K 1 theta 3 dot plus 4 omega 0 square K 1 times theta 1 where, here itself we will write all those things. Pick up this terms K 1 equal to I 2 minus I 3 by I 1; K 1 equal to I 2 minus I 3 divided by I 1 and K 2 equal to I 1 minus I 3 divided by I 2 I 1 minus I 3 divided by I 2.

1 minus K 1; so, K on is I 2 minus I 3 divided by I 2 this is fine plus 4 omega 0 square K theta 1 ok, and theta 3 double dot plus omega 0 theta 1 dot 1 minus K 3 omega 0 square K 3 theta 3. So, this is the final equations we are getting, this equation is very easy to analyse the pitch equation. So, why we are calling this as the pitch equation and its a decoupled from the other the two equations.

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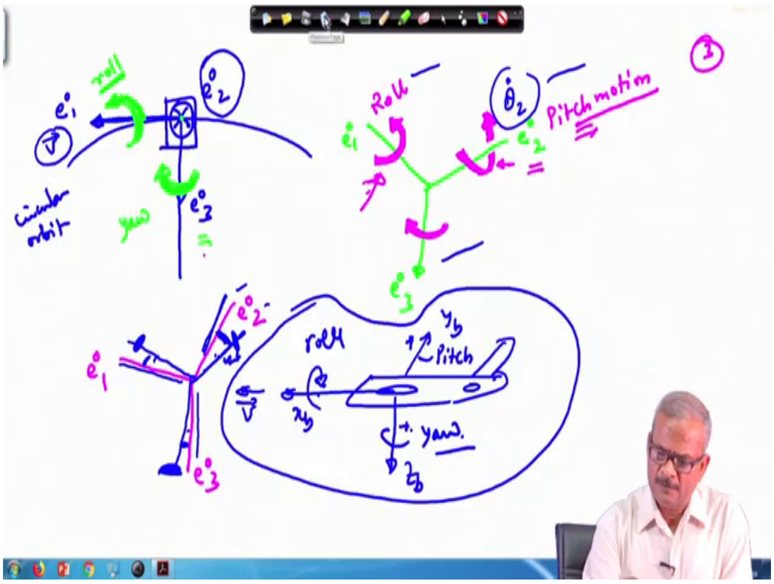

The reason is that once the satellite is going into the orbit this is your e 0 1 direction here you have e 0 3 and into the page this is e 0 2.

So, as you know the velocity vector is along this direction, velocity vector the satellite and this is the circular orbit circular orbit ok. So, using the right hand rule, if your velocity vector is along this direction; so, using the right hand rule I can show it like this see it is passing over this blue line. So, this is the anticlockwise motion and this is the roll motion; similarly the motion about this axis the third axis. So, it will be going something like this ok, it is passing over the blue line means I am doing like this ok. So, this is also a positive motion and this is called the yaw motion, this is the yaw ok; then what is the pitch?

So, about the 3rd axis we are taking the yaw about this we are taking this as the roll and about the two axis. So, two axis how the two axis is going into the page it is a going directly inside the page. So, this is something like it is a nose up motion ok, if you if I draw this e 2 direction say if I write here as e 1 this is e 2 and vertically down this is e 3. So, this is e 0 1 e 0 3 and this is e 0 2. So, motion about this axis so, this is passing from the below and here it is a like this, wherever it is the green line is visible means the your red line this pink line is going below that.

So, in this direction this is the pitch motion; generally the pitch motion is about the roll motion is this notation is coming from the aircraft and as I have told you the along the velocity direction we take this as the roll and then along the y direction we take this as the pitch. So, this is acting as a y direction. So, here roll will be along this direction and how the roll will be? Its a going like this. So, here again you see the green line is below the red line here in this place and z 1, z 1 is also like this as shown here in this place.

So, this pitching motion equation, if the body is slightly perturbed from the this orbital axis means this is your orbital axis system e 0 3 e 0 2 e 0 1 and body is slightly perturbed from this position little bit perturbation ok, not large here it is a looking large, but this angle is this angles are small so, that the linearization remains valid. So, little bit of perturbation. So, in that case you can see that this line is still it is a near this line, this line is a still near this line, this line is a still near this line ok. So, the motion about this blue line I can consider this as the pitch motion, about this line I can consider this as the roll motion and about this line I can considered as the yaw motion ok.

While actually we have define the roll along this roll along this pitch along this and yaw along this. But it so happens that whenever you take the case of the aircraft. So, in the aircraft we fix up the 3 body axis, this is the x b axis this is the y b axis and z b axis its a pointing down ward. So, the motion about this it is a consider the roll about this body axis.

So in fact, this roll pitch yaw whatever I we say this is the this motions are about the body axis system. But here it so, happens that the body axis and the orbital axis they are quite near to each other and therefore, I am telling that this is a roll, this is the pitching motion and then the yawing motion ok.

So, similarly here in this place this will be your pitch, this is roll and the yaw using the right hand rule so yaw is like this. So, this is your yaw. So, this is a case of the air craft and your velocity vector direction let us assume that this is lying along the same direction ok. So, from there I have described this notion that how the roll pitch and yaw they are related, but consider that the body axis are coinciding with the orbital axis itself and body is just rotating about the e 2 axis e o 2, that is its a just rotating about this axis. So, the that is just the pitching motion involved; isn't it?

And if I disturb the satellite by a small angles along the all the 3 direction so, how the behaviour of the satellite its a motion can be described? So, that is about the spinner stabilization we will come to this is a separate topic which we will take off later on. So, right now we are concerned about the gravity gradient problem.

So, here what we have observed that this is my pitching motion. Now you can understand that why I am telling this is a pitching motion because, your theta 2 you are writing along this direction theta 2 dot, this is your theta 2 dot. So, approximately this is valid ok.

Now, this part and this part, this is the roll part and this is the yaw part this is the yaw part and this is the roll part. So, what we see that these two equations are coupled together because theta one appears here. So, theta one also appears here in this place theta 3 appears here. So, it also appears here in this place. So, this two equations are not independent of each other, they are related to each other and in fact, I can take double derivative of this equation ok. So, theta 1, so here if I take the double derivative; so, theta to the power r there will be 4 dots over theta 1. So, this means we can write this as like this.

This is theta 1 and then 4 dot, this is 4 here d to the power 4 here like this. So, this is the fourth order differentiation. Now if we do this kind of things so here we will see that theta 3 double dot will appear in this place, here theta 3 triple dot it will become and here this will become theta 1 double dot ok. So, we can utilise you can differentiate this particular one ones. So, theta 3 triple dot it will become I can insert here. So, this way I can eliminate this, but it is a much easier to do in terms of Laplace transform. So, these two equations I will do it in terms of Laplace transform rather than a stretching in terms of the differential equation and this part we can take up separately. So, first of all let us take this pitching motion equation.

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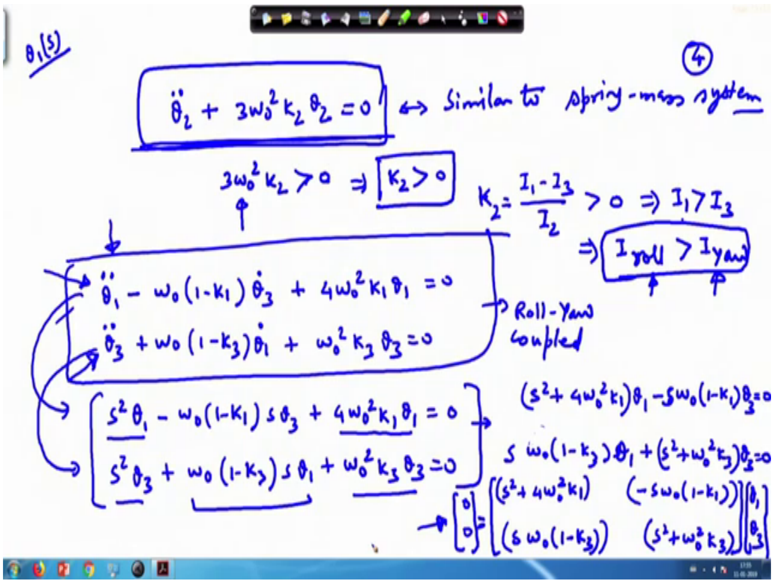

So, our pitching motion equation is theta 2 double dot plus 3 omega 0 square K 2 times theta 2 plus 3 omega 0 square K 2 times theta 2 this equal to 0, and if you remember then this just the similar to is spring mass system. So, this implies that this will continuously. If you are looking for stability of this kind of system so, we must have 3 omega 0 square times K 2 greater than 0 and this quantities always positive. So, therefore, this implies that K 2 should be greater than 0; and what does it mean? The K 2 the quantity we have written on the previous page K 2 is I 1 minus I 3 divided by I 2.

I 1 minus I 3 divided by I 2 this is K 2. So, this quantity should be greater than 0. So, this says that I 1 should be greater than I 3, in another way we can write this as I this is 1 which is related to the roll. So, I roll is greater than I yaw ok, this quantity must be satisfied; that means, the moment of inertia about the roll axis should be greater than moment of inertia about the yaw axis ok. Next we pick up the other equations; these two equations then we have to copy on the next page.

So, the first equation we have see here on the computer is a quite a difficult copying from one place to another place, on the page paper its very easy I whatever I write here on one page I can directly show you, that here I have to go back and forth again and again. So, for not creating any confusion so I am wiping out all this things ok, this is fine. So, theta 1 double dot theta 1 double dot minus omega 0 1 minus K 1 minus K 1 times theta 3 dot times theta 3 dot plus 4 omega 0 square K 1 theta 1; 4 omega 0 square K 1 times theta 1.

So, this equation I have written here and the other equation also I will copy, theta 3 double dot theta 3 double dot and then plus omega 0 1 minus K 3 times theta 1 dot plus omega 0 1 minus K 3 times theta 1 dot then plus omega 0 square K 3 times theta 3, this equal to 0 this equal to 0. So, these are the two equations where roll, yaw, coupled.

Now I do the Laplace transform of this; so, Laplace transform of this will be S square instead of writing theta S I will just write it as theta 1. So, understand that this is the Laplace transform omega 0 times 1 minus K 1 then S times theta 3 plus 4 omega 0 square K 1 times theta 1. So, these are all theta 1 here I am writing. So, this is nothing, but theta 1 S, it is a function of S. So, this is Laplace transform similarly this we have to do the transform S square theta 3 plus omega 0 times 1 minus K 3 times S theta 1 plus omega 0 square.

So, these are the two equations which is Laplace transform and obtained from these two equations. So, as I have told you that I could have double differentiated this equation and then inserted from these equation to get a single equation and I could have eliminated this theta 3 dot from this place ok. And we will get quartic equation, means of the differential equation of order 4 in theta 1, but instead of doing that what we have doing that we have Laplace transform and you can see that theta 1 theta 3 all these are appearing.

So, this in the combined equation we can write this in the matrix notation. So, one step I will have to go more and combine the terms; so, if we combine I have written write here on this side itself. So, you look here in this place this is theta 1 is here, theta 1 is here.

So, we can write this as S square plus 4 omega 0 square K 1 times theta 1 minus omega 0 1 minus K 1 times, S I will put here in the beginning this is S here theta 3 this equal to 0 and again picking up this one. So, here theta 3 is here theta 3 is here. So, this becomes S square that term is related to theta 3. So, first I will write the term for the theta 1. So, this is the theta 1 term, this is omega 0 1 minus K 3 S times theta 1. So, theta one will put here S I will put here and then the other terms which is plus S square plus omega 0 square K 3 times theta 3 this equal to 0.

And this can be written in a matrix format as S square plus 4 omega 0 square K 1 and then S times omega 0 1 minus K 3 and here similarly we have S times omega 0 1 minus K 1 and this particular term S square plus omega 0 square K 3. So, these are the terms and this is operated on theta 1 theta 3 and this equal to 0, right hand side we do not have place so I have written it here.

Now for the non trivial solution its a determinant must be 0 ok. So, on the 5th page we have so far its determinant to be 0. So, S square plus 4 omega 0 square K 1 minus omega 0 S times 1 minus K 1 its fine and then S times omega 0 1 minus K 3 S square omega 0 S square K 3.

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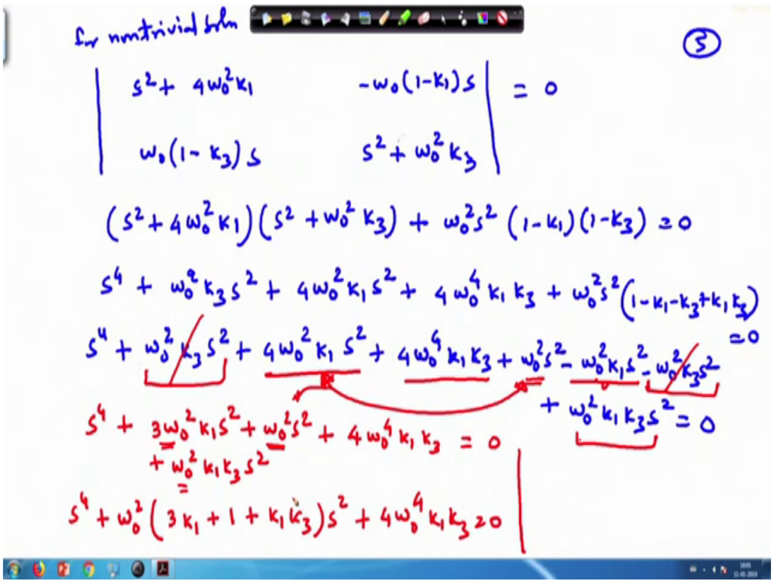

So, we must have for non trivial solution S square plus 4 omega 0 square K 1 and here omega 0 times 1 minus K 3, S 1 minus K 1 times S plus S square plus omega 0 square K 3 ok.

So, this determinant this has to be said to 0 for the non trivial solution, I hope you are aware of all these properties of the matrix this is the preliminary requirement for this case. In this course the quantity S square plus omega 0 square K 3 and then this becomes plus S square 1 minus K 1 times 1 minus K 3 this equal to 0.

And we need to expand it and work it out. So, this quantity gives us S to the power 4 and plus omega 0 square K 3 times S square plus 4 omega 0 square K 1 times S square and this one gives us 4 omega 0 to the power 4 K 1 times K 3 and from here we have omega 0 a square S square and this also we can expand it. So, this will be 1 minus K 1 minus K 3 plus K 1 times K 3 and this equal to 0.

So, few more steps are required to get hold of all these things, omega 0 square still we keep it like this S square plus K 1 times K 3 and then expand it, omega 0 square S square minus square K 1 times S square minus omega 0 square K 3 times S square plus omega 0 square K 1 K 3 times S square this equal to 0.

Now, what we can observe that can we simplify it? This what we are looking for. So, we have here term like the 4 omega 0 square K 1 S square and here this is omega 0 square K 1 S square, also if you look here in this place omega 0 square K 3 S square here with minus sign this is present.

So, we can write this as S to the power 4 and omega 0 K 3 S square. So, this term will get cancelled and this term and this term we can combine them together for this becomes 3 omega 0 square K 1 times S square this is a constant term then we have this term present here which is omega 0 square S square.

And the last term which remains the these two we are combining here in this place and the last one then remains is 4 omega 0 to the power 4 K 1 K 3 and we have one more term here, this term is remaining. So, this term also we put here in this place omega 0 square K 1 K 3 times S square and this equal to 0. So, S to the power 4 plus omega 0 square we can take out from this place this omega 0 square omega 0 square omega 0 square.

And the S square is also common. So, 3 this becomes omega 0 square then 3 K 1 is S square will be taking it outside this becomes 1 and here omega 0 S square goes outside. So, this is K 1 K 3 and here this S square in the last term we have to put here K 1 K 3 this equal to 0 ok. So, to summarise we can go on the maybe in the next page and summarise this equation.

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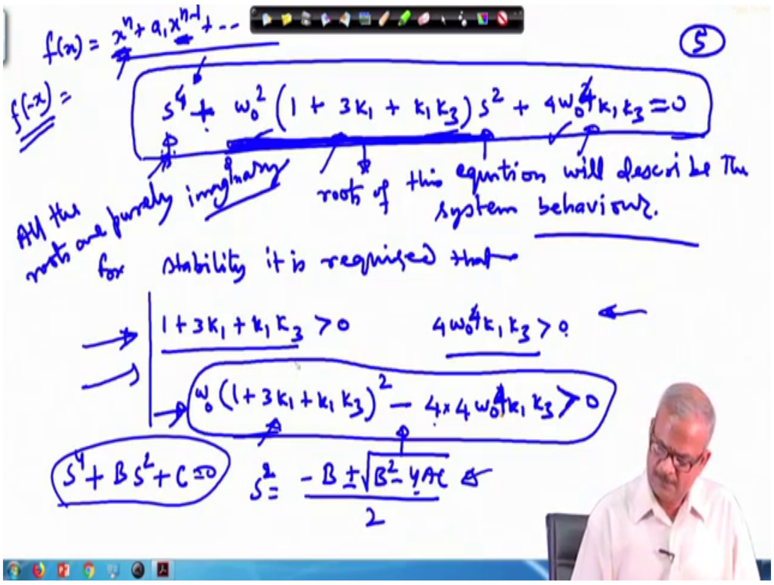

So, what we are getting the combined equation it looks like this omega 0 square 1 plus 3 K 1 plus K 1 K 3 times S square plus 4 omega 0 S square times K 1 K 3 this equal to 0. And the roots of this equation will describe the system behaviour this is I am trying to wind it up. Now if you look here in this equation. So, for stability it is required that 1 plus 3 K 1 plus K 3 this must be greater than 0 4 omega 0 square K 1 times K 3 this should also be greater than 0.

In addition 1 plus 3 K 1 plus K 1 K 3 square minus 4 times 4 this term omega 0 square K 1 K 3 this should also be greater than 0. This you know very well that the coefficients of this polynomial in S this must be positive this quantity is positive. So, this must be positive, if we do not want to have any roots in the right half complex plane ok.

So, if so happens that the roots are in the right half complex plane means the system becomes unstable from the Descartes rule you know it know that, in any polynomial the if we are writing it like this say the polynomial is f x and you are writing here as x to the power n times say a 1 times x to the power n minus 1 and so on ok. So, the nth order polynomial.

. So, as many sign changes. So, number of roots in the right half complex plane it is bounded by the number of sign changes that takes place in the coefficient of this polynomial this is the Descartes rule, but what we are looking for that there should not be any root in the right half complex plane for the stability it to be maintained. So, therefore, I do not want any sign change means all this coefficient this and this they must be positive.

Similarly, if we are looking for the roots in the left half complex plane ok. So, what we will do? We will change this to the f minus x and. So, x here will be replaced by this minus x and then we will look for how many sign changes are there, this also part of the Descartes rule. So, that will tell you the what is the upper bound on the number of roots in the left half complex plane or the on the on the negative side.

So, here it so happens that if you put here S equal to minus S. So, only in these two places there is no nothing changes here in this equation. So, this simply implies that if these are positive so there is no roots in the left half complex plane. So, it is a purely imaginary roots exist in this case, and moreover this condition must be satisfied. You can check yourself by taking some simple example of the this quadratic form and then trying to find out the rules. So, if this quantity is not greater than 0. So, your condition will not be satisfied for the stability, means we know here in this case that all the roots are purely imaginary, all the roots are purely imaginary ok.

We cannot take what I mean that if here what we have written that S to the power 4 if I write the equation like this, B S square plus C. So, you can write S square equal to minus B plus minus B square minus 4 AC and this is under root divided by 2. So, this is the part we are talking about. So, if this condition must be satisfied if we do not want any roots in the right half complex plane and you can check using a very simple for this.

So, this 4 into 4 this is nothing, but 16 here, in this place so, on the next phase maybe we can write.

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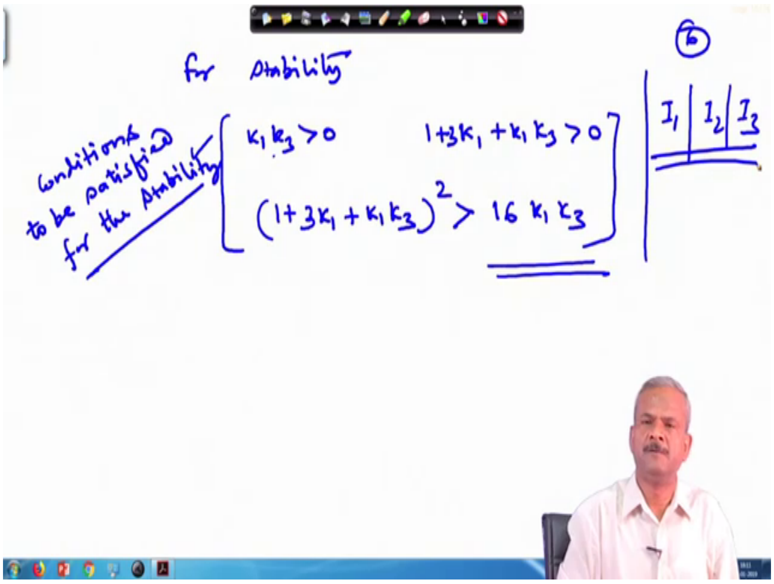

So, the in summary for stability this K 1 times K 3 this must be greater than 0 then 1 plus 3 K 1 plus K 1 times K 3 this must be greater than 0. And moreover this condition should also be satisfied 1 plus 3 K 1 3 K 1 plus K 1 times K 3, its whole square must be greater than 16 omega 0 square ok, here we are missing this omega 0 part; so, omega 0 square. So, this is omega 0 we have picked up this part from here to here, omega 0 square is there. So, omega 0 we need to put here in this place somewhere we have missed some particular this square part here we have here this is the 4 ok.

4 omega 0 to the power 4 K 1 K 3. So, this 4 we have missed instead we have written here S 2 so, this is your 4 ok. So, here this should be 4 and here also this should appear as 4. So, omega 0 this quantity is always a positive quantity. So, the we can summarize like this K 1 times K 3 should be greater than 0 1 plus 3 K 1 times K 3 this should be greater than 0. And here if you would look for this part so this is omega this also becomes omega to the power 4 and here there is omega to the power 4. So, this can get eliminated because its a positive sign; so, positive quantities. So, this we get that this quantity a square should be greater than 16 K 1 K 3, this is sixteen K 1 K 3.

. So, these are the conditions for conditions to be satisfied for the stability. So, these are the condition, now we have to analyse it we have got the condition that under this condition we get the purely imaginary roots and based on this how this conditions will help you in designing the spacecraft. And what is about the design? We want to choose the values of capital I 1 this is we want to choose the inertia matrix basically I 1 I 2 I 3 these are the things we want to choose. So, will it guide me in doing so? So we will take up this problem in the next lecture and for the time being.

Thank you very much for listening.