Satellite Attitude Dynamics and Control
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Lecture - 30
Gravity Gradient Satellite (Contd.)

Welcome to the 30th lecture.

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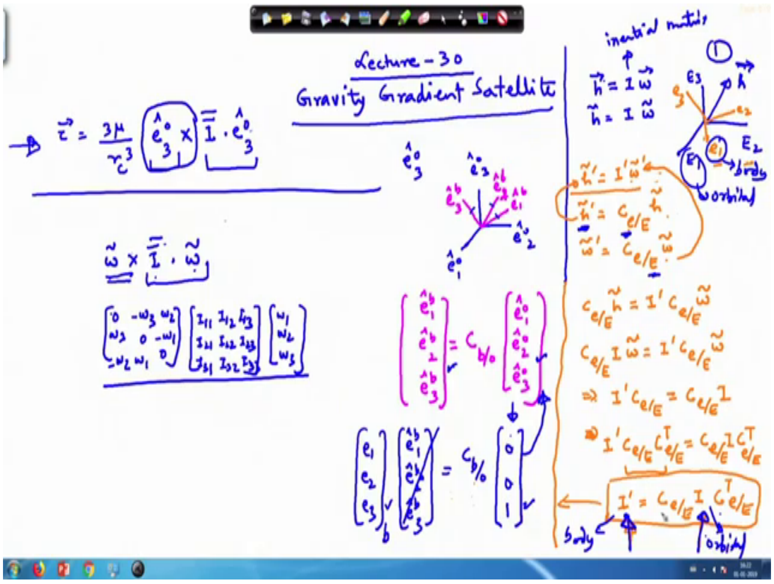

So here, we have been discussing about the Gravity Gradient Satellite and we derived this equation last time. So, we will continue with this. So, while working here, this dot is not there. So, while working for this, we will require certain conversion and we need to look into those conversion. So, let us write this h which is the angular velocity vector. This we have written as I times omega ok. This is in the vector notation and the same thing also in the matrix notation, we have written as I times omega tilde where I is the inertia matrix, inertia matrix.

Now, assume that first we have express this in one frame which is E 1, E 2 and E 3 and in this frame, this is my h vector. So, now, if I took a another frame say this is e 1, e 2, e 3. So, this vector is not changing, but it is component will be different in this e 1, small e 1, small e 2, e 3 frame. So, that there we write h tilde prime in this frame shown in the orange color ok. So, h tilde prime I can write this as I prime times omega tilde prime where I prime is the inertia matrix in this frame because as the frame will change, the inertia matrix will also change.

Also, we are aware of that h tilde prime this can be written as if c is the transformation matrix from capital E frame to a small e frame means we are going from capital E frame to e frame ok. So, this we can write it like this. So, h tilde, it can be described in this way. The same way omega tilde also can be written as c a small e slash E omega tilde ok. Now, we use this result here in this place. So, on the left hand side, we can insert this equation we can insert here in this and we get here from this place c E times h tilde this equal to I prime and omega prime we can insert from here. This omega prime can be taken here and this h prime is taken to this place, h prime we are inserting from this place to this place.

So, we have I prime times omega prime which is c e slash E omega tilde and then what we are interested in, that this h tilde this is nothing but I times omega tilde. So, this is I prime ok. So, this implies I prime c e slash E, this will be equal to because omega is a not a its a not an null vector and therefore, this quantities this matrix c times I this must be equal to each other. And if we so operate on this, so we can write as and what we can see that this quantity is identity matrix.

And therefore, this gets reduced to I prime equal to c e slash E I c transpose e slash E ok. So, here in this particular equation, the inertia matrix which is in one frame and we are able to convert it to another frame ok. So, for say this I was in the capital E frame and then we can go to here as per his one we can go to this the a small e frame and more. And as we know that this inertia matrix is always symmetric and therefore, this I prime if you take the transpose of this. So, transpose of this will be equal to I prime and here in that case, this gets reduced to.

If you take the transpose ok, we leave it because taking the transpose it does not matter here. So, we skip this process here we will take up this part later on. So, this way we can convert from one frame to another frame. So, we need to utilize all this information to work here in this place, yes. So, now we can use this results here and also as per our earlier discussion, this e cap o 3, this is referring to the orbital reference frame. These are the unit vectors along the orbital reference frame three directions. And with respect to this, our body frame is oriented say this is e 1 cap we can indicate it by a b to indicate this is along the body frame.

This is 2 ok. And you know that this conversion b 3 it is related to the this c matrix which is going from the going to the body frame from the orbital frame and on this side, we have e o cap 1. So, this is the way this gets converted. Now, we utilize this information to convert this from one frame to another frame. So, this we need to express in terms of the body frame system. So, here, let us say that this is a unit vector along the third direction of the body frame of the orbital frame.

So, if we write in a proper way, so this will be equal equivalent to this is 1 0 0 ok, this part. And then, we will have c b 0 and here on this side, then you will get all these quantities e cap b 1, e cap b here 2 and e cap b 3. So, this implies that this is your vector. This is nothing but this vector where because this quantity this quantity is only along the third axis. So, the other quantity, they will vanish. This as written in matrix notation and this is the vectrix notation. And here the same way like once we are dealing with this, so instead of writing it in a say the whole thing we do not want to mix up here. We have one unit vector here and this unit vector is converted to the another unit vector.

So, in that case I will not write it like this. Here, this cap will not be there, only thing that we will have here components. So, instead of writing this, then we should write in terms of some components say the e 1, e 2 and e 3. These are the components along the body reference frame 1, 2 and this 3 directions; 1, 2 and this 3. So, then this vector is converted into this vector. While we write in terms of vectrix, then we have to show it like this ok. So, this vectrix this whole vectrix is converted to this one, this particular one.

So, as you would remember that we can write this as, we can utilize this notation like this is a unit vector along this direction. So, we have converted this time this cross ok. So, this is e cap 3 cross because this is a unit vector and then, I double bar dot e cap 0 3. So, this part accordingly, we can change it. Let me express it in some other way. First I will give you a generalized form and there after we will come to this part. Say if I have something like this, omega tilde.

So, this is a vector there, this is a dyadic and this is a vector. So, this part obviously, you know that this can be written as 0 minus omega 3, omega 2, omega 3, 0, omega 1, minus omega 1, 0 and minus omega 2. And this part, if you remember to recall from the our earlier discussion, so this can be written as I 1 1, I 1 2, I 1 3, I 2 1, I 2 2, I 2 3 and I 3 3. And here, then omega 1, omega 2, omega 3 ok. Along the same line, we have to change it here in this place and then our job will be done and we will utilize these informations to put it in a proper shape ok.

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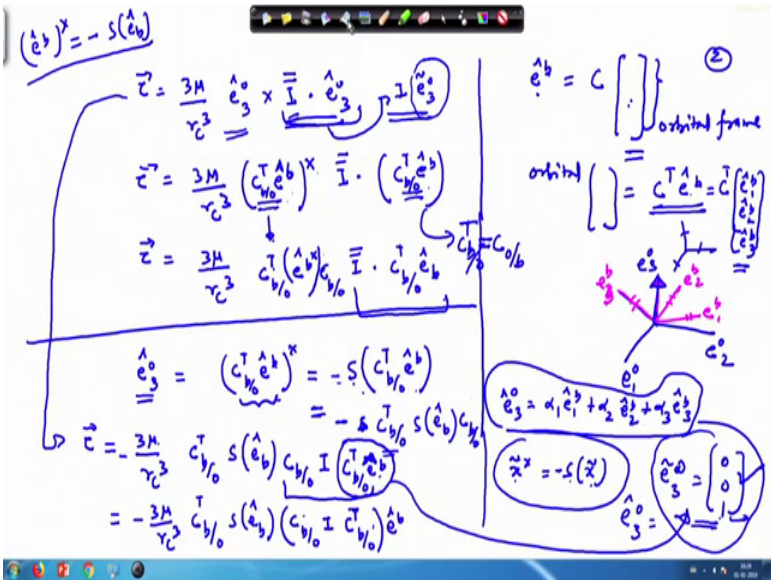

So, we have tau this equal to 3 mu by r c. Then, this part I double bar dot ok. So, this we write as say in terms of the C matrix. So, this will be c transpose times e cap b times 3 cross e cap sorry, this 3 is not there ok. So, this we are taking from the previous discussion where we have use this omega. So, this is a vector, which you get it from c times here, another vector which is in the orbital frame.

So obviously, if we are looking for the this orbital vector. So, this orbital vector can be written as in here in terms of C transpose e cap b. So, this is what exactly we have done. So, this e cap b, it does not indicate that this is just a vector along the any one of the body axis direction. It is a indicating all the 3; means I have just orbital frame e 1 0, e 2 0 and e 0 3. And then, we have the body frame here e b 3 ok. So, e o 3, this is the vector here. So, this vector will have components along all this three directions in the body frame ok. This vector this is a vector along the e o 3 direction ok.

So, that unit vector, so this is given by this quantity ok. So, this is what we have done. So, we have expressed in terms of e cap b. So, this means that this is C transpose e cap b

1, e cap b 2 and e cap b 3. So, obviously, you can see that in the as per our previous in the rotation case, we have looked into that e o 3 vector, this can be expressed as a combination of let us say this alpha one times e 1 cap b plus alpha 2 times e cap b 2 plus alpha 3 times e cap b 3.

So, instead of doing this, we are writing it in this format. So, this we go from the orbital frame to the body frame and from body frame to the orbital frame in which way we can approach. So, this is the thing which can be written ok. And because this is a cross here; this is a cross here in this place. So, we need to be very careful while working here. So, it is something like if you remember that this cross whenever we got a cross across this place, so sometimes we have written this as like this. So, this can be broken in a format where it can be written as C transpose and obviously, because we are c transpose you have written.

See, we are using like this e 2 e frame. So, we are going from orbital frame to body frame. So, we should write here b slash o e, b slash o means we this is from orbital to b. And then, we are taking the transpose of that to and this then this transpose this simply implies that C transpose b slash o, this equal to C o slash b. This is what we have learnt in our rotation ok. So, this can be written as b slash o e cap b where remember that e cap b is nothing but this part and then C t ok.

So, we need to do some simplification and then, we will get the final result ok. This we could have done purely in the form of a vectrix. And I will do that for you people because that will be beneficial to, because in terms of vectrix we have not done earlier much much of the things.

So, therefore, I would like to do this part in terms of vectrix ok. So, here we need to put a cross, this is this part ok. Let me explain this part and then this part will proof through the vectrix method ok. So, e cap o 3, if we write this as c transpose b slash o e cap b and then there is a cross. So, basically the quantity we are writing, we can write it in this format. We have changed to much there, there S is the skew symmetric matrix.

We are using the earlier definition that any vector x tilde cross this will be equal to minus; this is the notation we are using ok. So, if we use this notation here, so going into this part or either you just take this part ok. So, this is 3 mu by r c cube and this part can be broken like S times C transpose b slash o. See there is a minus sign here in this place ok. So, if use this notation and this part you write it as I times e tilde o 3. So, the whole thing will become very convenient to work with.

So, I take you to the next phase after this. So, now, pick up this part from this place, just minus sign is here. So, we will put the minus sign this place. So, the cross notation we are not using. Once we have written in this form, so this will be written like this ok. And this can be reduced to this format and this can be checked like, you can see that if you have c transpose b slash o, if this is an identity matrix, on the right hand side here this transpose is not there ok.

If you assume that C slash b transpose o C transpose b slash o this matrix if it is an identity matrix, so immediately what we can see that this part will be equal to minus S e cap b means here e cap b cross this quantity will be equal to minus S e cap b which is the definition we are using ok, x tilde cross this equal to minus a S times S tilde.

So, exactly the same thing is here. So, immediately for the identity matrix if your C happens to with the identity matrix, it is a verified. But here this is a rotation matrix case and for the verifying that, indeed this quantity is equal to this quantity we will have to expand it then it is a little longer expression. You can check it yourself ok. You just expand it and then check whether this quantity turns out to be the same thing or not ok. So, therefore, this part what we have been writing here, this particular equation here e cross, so this gets reduced to this part we have written from this place and then refreshing this as b slash o and thereafter, we have I times e tilde o 3.

So, e tilde o 3 if I write it in this way, so this implies, 0 0 1; while if we write it as like this, so this is just a unit vector in the third direction which is written by 1 here. So, this is the difference. So, these are some of the simple manipulation we can always do. So, if you are not working in terms of, the best way of doing this is to represent this in terms of vectrix and then carry out the whole operation otherwise convert it in here in this format and then work out. So, once we do this, then you check for some of the things here.

And here also, we need to convert it. So, this is I times this part we are writing equal to I times, this part we have written like this together then this also needs to be converted. So, if we convert this, so this will be c transpose o sorry, this is C transpose b slash o times e tilde o and this 3 we need to put it because until unless we put this 3, it will this meaning will not be apparent. Only through this meaning, this 3 it implies that this is 0 0 1. Otherwise, it will be something different.

Now, this part, what is this quantity? C b slash o I and c transpose b slash o. So, we need to go back here in this place and compare with this; C is you can check it. So, this was in your inertial frame and from inertial frame you have converted into the body frame. This I prime is the, I prime is referring to the e frame like here s prime is referring to the small e frame and capital E is referring to the inertial frame. The small e is to referring to the body frame. So, following the same notation, what we can observe from this place, this can be written as I or sorry, C b slash o I times this will put in one bracket and we will write it separately.

So, C b o I times c transpose b o, compare this. This is C transpose I and here C is less. So, in this format, what we can see that this is getting converted from if this refers to the orbital frame and this one refers to the body frame, so we can see that this is converting from body frame to the orbital frame to the body frame. So, this is referring to the body frame and this is referring to the orbital frame. So, using this notation, then this can be written accordingly.

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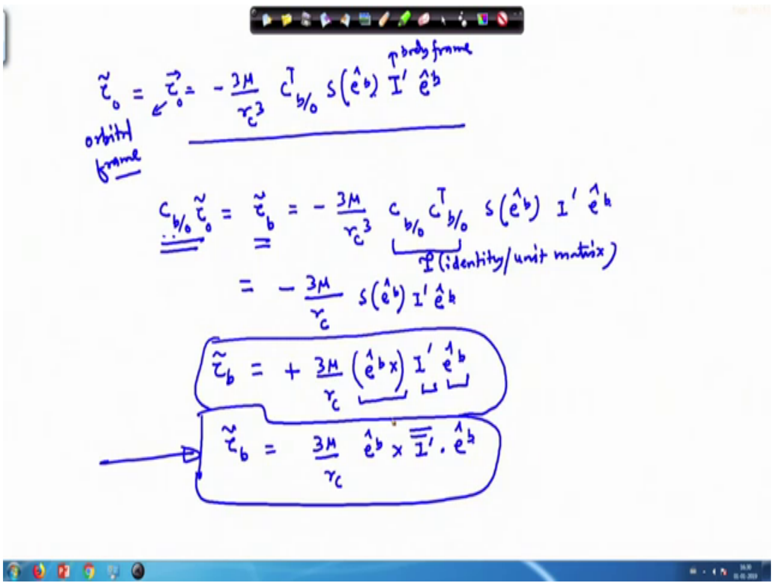

So, let us move to the next page. So, this tau then becomes gets reduced to minus 3 mu divided by r c cube and then, C transpose b slash o and then, we have this skew symmetric matrix S times e cap b and then, this quantity which is nothing but inertia matrix expressed in the body frame.

So, maybe I will put a prime here to indicate this is in the body frame; body frame and thereafter we have e tilde o 3, sorry this is we have already converted this. So, some where we have missed this part. This part we have written it. So, this part we need to correct here in this place. If e tilde b or e cap b whatever the notation we use, so we can use that. The best way is to write in terms of tilde but here we will continue with this part only.

So, this mark it, this is the change. So, these are some of the errors that keep in while writing because we have changed this ok. This part we are writing for e tilde o 3. So, this must be present here in this place where e tilde o 3 is representing this quantity which is shown here in this part ok. So, therefore, this can be reduced to e cap b. So, right hand side we have. Now, one thing you remember that still this is in the orbital frame. And we need to convert this into the body frame. So, instead of, because we have right hand side we have express the whole thing in terms of the matrix notation, therefore, I will put here tilde instead of putting a vector here in this place.

Then, multiply this both sides of this by we have c transpose b slash o. So, this is in the orbital frame and then, we want to take it into the body frame ok, so we will multiply it by C b slash o, perhaps over C we have not put the tilde. So, we will remove this tilde ok. So, if you operate on this tilde, so this will get reduced to and then we are expressing this in the body frame ok. This is in the orbital frame. I have not put the tag but this is in the orbital frame; this in the orbital frame or orbital frame to body frame conversion.

So, this is converted into the body frame on the right hand side. We have 3 mu by r c cube and then c b slash o times c transpose b slash o S e cap b and then, I prime e cap b. So, this quantity is nothing but identity because it is a rotation matrix and to rotation matrix this C transpose C times C transpose that will be equal to identity. And therefore, this gets reduced to r c. So, this is your identity matrix which we have represented like this. This is identity or unit matrix slash unit matrix.

So, therefore, this quantity can be written as S e b cap times I prime. And if you do not want to write it in this way, so this can be written as this minus sign, can be observed here and this can be written as e cap b cross ok. So, still this remains a matrix, this remains a matrix, this remains a vector. If you are looking to write this in terms of the inertia dyadic, so write it like this, I and prime I will put here prime for we have used b. Let me check here what the notation we have put here; I have not put anything.

So, this I prime is in the body frame. So, I will continue with I prime. So, this is I prime. So, e cap I prime dot. Now we have written in terms of inertia dyadic. Let it be and this part, this gets converted into tau b. So, here we write it tau b. One thing I will draw your attention that for this has been converted to the b frame by multiplying it by c b slash o, this is the transformation matrix which goes from orbital frame to the body frame, but the same thing, we have not done for the same thing. We have not done for the while converting from this.

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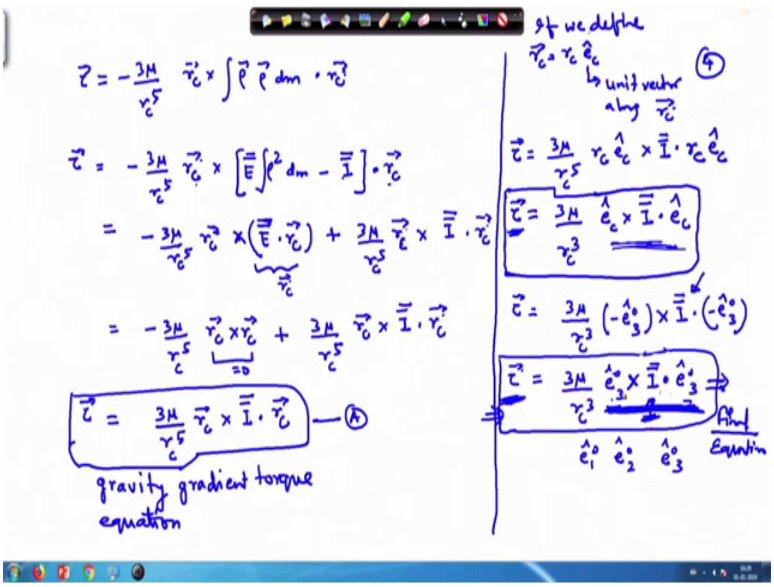

Here it is; so, c here in this place, that this is tau and here this is e c e c and from there, once we have converted in terms of the orbital frame here in this place. So, we have not multiplied on both side by the transformation matrix. The reason is very simple. This e c and is e 3 vector, they are in the same direction ok. Otherwise, I can also take the transformation matrix. I can multiply by the transformation matrix and I can do the whole thing but is, so only think that it is a going to consume time, you can see that this a small thing it has taken so much of time. So, this is for only one way of doing. The other way of doing it is also there.

So, but you need to understand it how we are converting and you need to be very careful. So, these are our two final equations that we have been attending it. And for you people, I will do this also in the vectrix notation so that, you become convenient with the whole thing ok.

So, we stop here and we will continue in the next lecture.