**Satellite Attitude Dynamics and Control**
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**Lecture – 03**
**Kinematics of Rotation (Contd.)**

Welcome to the third lecture of in the last lecture we discussed about the rotation how we get one matrix how we get the rotation matrix in terms of the direction cosines. Today we are going to discuss about the orthoganility of a matrix because this is very basic to the attitude dynamics of the satellite.

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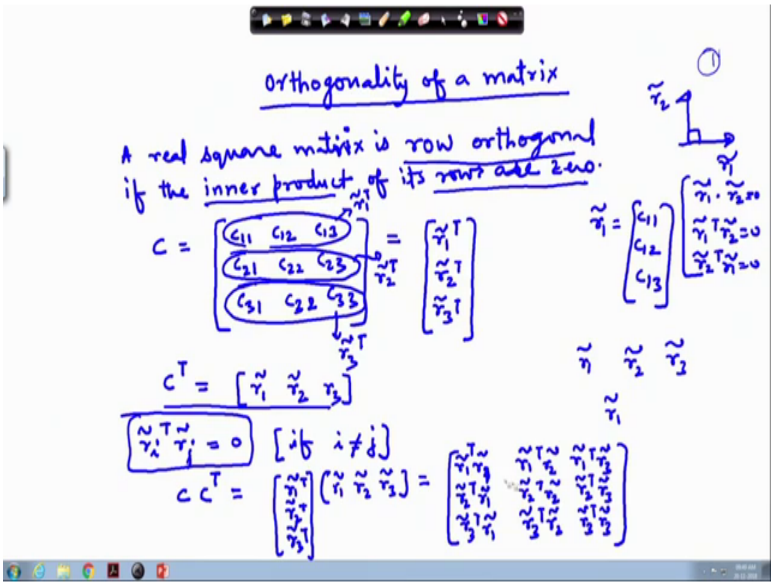


A real square matrix; a real square matrix a real square matrix is row orthogonal if the inner product of its rows are zero. So, let us consider this C matrix which we are working with last time. So, C 11, C 12 and C 13 this constitutes one row; similarly this one is another row and similarly this one this is another row. So, total we have three rows in this and each of the row of this we can write as if we give the indicate this as r 1 tilde which is a vector in matrix notation, this we indicate as r 2 tilde and this we indicate as r 3 tilde; so, this will write as r 1 tilde transpose. So, here r 1 tilde this is C 11 C 12 and C 13 see the difference, this is a row vector which we have written in a column form ok. So, this is r 1 tilde and transpose of this then it will come in this format.

So, better we should write here as r 1 transpose; so, this becomes r 2 transpose and this becomes r 3 transpose; so, we will have here r 2 tilde transpose; r 3 tilde transpose. And if we take the transpose of this three matrix; so this will result in, so taking the transpose of this; this is the here in the column format. So, this will go into the row format and r 1 tilde transpose. So, we will have to take the transpose of that and if you take the transpose of that it will get converted into r 1 transport; similarly, the other one will be r 2 transpose r 3 transpose. Now as we have written here a real square matrix is row orthogonal if the inner product of this; rows are zero.

So, here you have one vector r 1 tilde another vector r 2 tilde another vector r 3 tilde. So, if the inner product of them let say 0 on the say if orthogonality means they are perpendicular to each other. So, if you have two vectors here say this is r 1 and this is r 2 and orthogonality implies; they are perpendicular to each other and therefore, if you take the dot product of them. So, you will have r 1 tilde dot r 2 tilde; this will be equal to 0. So, you do not have any component of r 2 along the r 1 direction or r 1 either r 1 along the r 2 direction or the same thing in the matrix notation, you often write as r 1 transpose r 2 tilde this equal to 0 or way r 2 transpose two transpose r 1 tilde; this equal to 0.

So, this way each of the rows here they are perpendicular to each other this is what the row orthogonality implies and their inner product of their these rows are zero. And therefore, in general notation we can write as r i tilde transpose times r j tilde this equal to 0 if i not equal to j. So, i equal to y means j equal to y means you are taking the dot product of the same vector means r 1 dot r 1 dot. So, it will turn out to be unity or whatever depending on the magnitude of the vector r 1 ok.

So, from here; so what we can write that C times C transpose; C times C transpose, where C is the matrix r 1 this here just look into this. C is the matrix r 1 tilde times r 2 tilde times r 3 tilde and C transpose we have written as in this place r 1 tilde r 2 tilde r 3 tilde. So, if we write in this format you will get this as r 1 tilde times r 1 tilde; this is r 1, r 1 tilde here transpose is there this is transpose transpose transpose. So, transpose put here r 1 transpose r 2 tilde; r 1 transpose r 3 tilde and so on.

So, we need to fill this r 2 tilde transpose r 3 tilde r 3 tilde; now you can see that from this definition because if the rows are orthogonal to each other means only the diagonal terms will remain and off diagonal terms will vanish ok.

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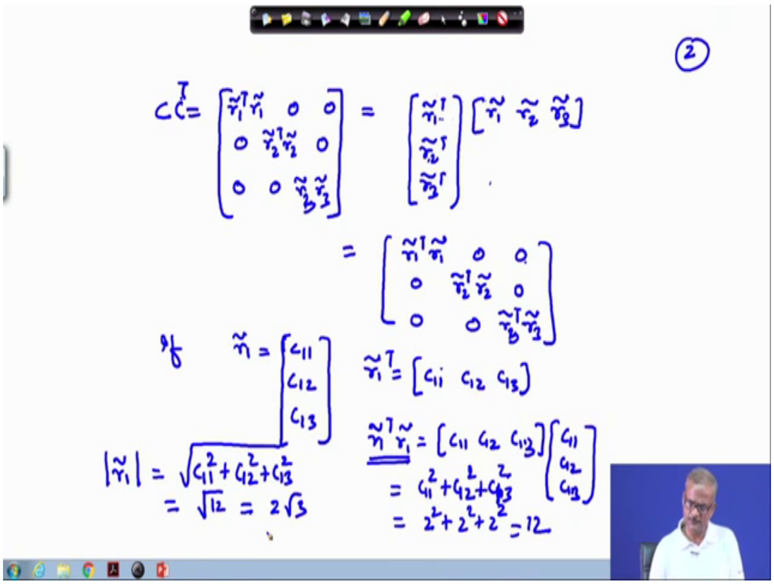

So, we have the C matrix and therefore, C times C transpose we are writing as r 1 tilde transpose r 1 tilde and off diagonal terms they will be 0; r 2 tilde transpose times r 2 tilde 0; r 2 tilde r 3 tilde transpose times r 3 tilde ok, which we are writing from r 1 tilde transpose, r 2 tilde transpose, r 3 tilde transpose and here on (Refer Time: 08:49) r 2 tilde transpose, r 3 tilde transpose. So, this gives us r 1 tilde transpose r 1 tilde and this inner product; this will be 0, similarly this inner product this will be 0 ok.

Because by our assumption they are perpendicular to each other; if they are perpendicular to each other then this quantity will this quantities will not be 0. Similarly this and this they are inner product it is a 0 and here we will have r 2 tilde transpose r 2 tilde; this will be 0 0; r 2 tilde transpose r 3 tilde transpose r 3 tilde. If say we have r 1 tilde transpose; so r 1 tilde transpose we have taken as (Refer Time: 10:12) better we write in terms of r 1 tilde. So, we have written as C 11, C 12, C 13 ok; so r 1 tilde transpose; this is C 11 transpose, C 12 C 13.

So, these are just scalar and until unless you have a vector where the they are different components this transpose you should not put say just scalar; so will put this way ok. Therefore, we will have r 1 tilde transpose; r 1 tilde equal to C 11, C 12, C 13; C 11, C 12, C 13. So, this makes it C 11 a square C 12 a square C 13 to a square ok. So, if C 11 is say all these are 2 2; so this will be 2 a square, 2 a square, 2 a square; so that makes total 12 and what will be the magnitude of this vector?

So, magnitude of vector r 1; this will be written as C 11 a square, C 12 a square plus C 13 a square under root ok. So, that becomes then 12 under root which is 2 times root 3.

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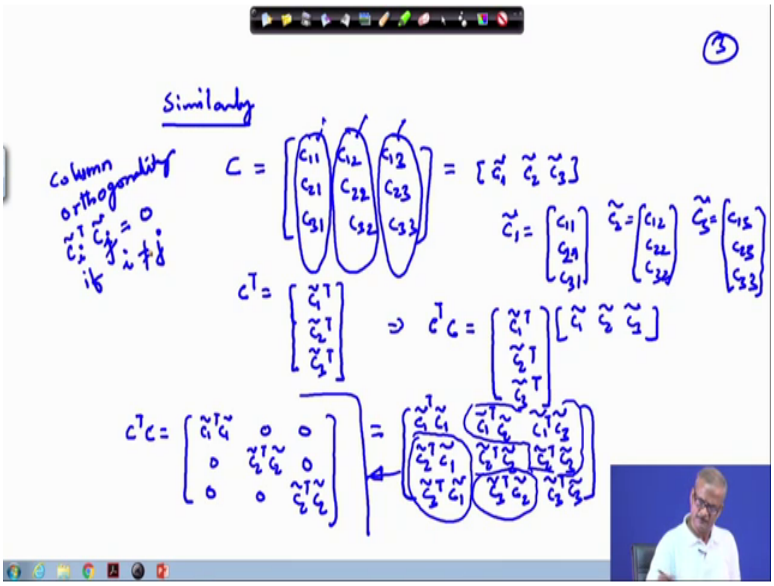

Similarly, we can also defined in terms of column vector. So, here C 11 the same way the C matrix we are choosing 2 2; now this is the column vector. So, C can be written as say if we indicate the column as C 1 tilde, C 2 tilde and C 3 tilde. So, here your C 1 tilde; this is C 11, C 21 and 31; similarly C 2 tilde will be C 12, C 22 and C 3 tilde C 13, C 23, C 33 ok.

So, C transpose then this will be written as C 1 tilde transpose C 2 tilde transpose, C 3 tilde transpose ok. And then C transpose C; this will comes C 1 tilde transpose, C 1 tilde and if we take this product here C 1 tilde transpose, C 2 tilde, C 1 tilde transpose and C 3 tilde; this is transpose (Refer Time: 14:20) ok. If this vectors; they are mutually orthogonal this is constitute in one vector, this constitute in another vector, this is constitute in another vector.

So, if this vectors are mutually orthogonal; so that implies that this off diagonal term means these two this one and these terms; they will vanish, these are the off diagonal terms. So, this implies that C transpose C will get as C 1 tilde transpose C 1 tilde 0 0; so this is called the column orthogonality, if i not equal to j.

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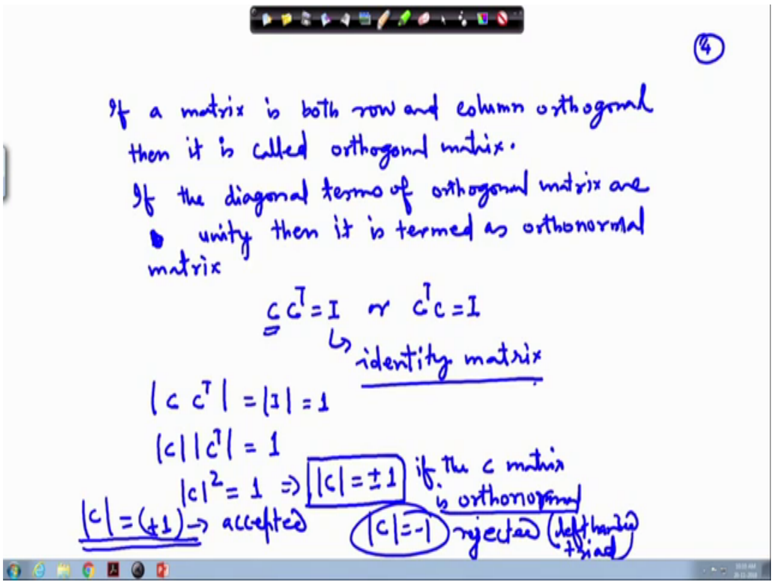


So, if a matrix is both row and column orthogonal; then it is called orthogonal matrix or if the diagonal terms of orthogonal matrix are unity; then it is termed as orthonormal matrix. So, this implies that we will have C C transpose equal to I and or C transpose C equal I; so there I is the identity matrix ok.

So, now if you take the determinant of the C transpose C; so on the right hand side you see that this quantity will be equal to 1. So, the determinant of product two matrix as it can be written like this and this is nothing, but C a square because transpose if you take the transpose of the matrix and take the determinant its determinant does not change. So, this is equal to 1, this implies C determinant this is equal to plus minus 1 ok. So, this happens if the C matrix is orthonormal and to qualify as a rotation matrix it is a required that this C matrix should be an orthonormal matrix; so here but the plus 1 value this will be accepted.

So, this will be accepted and C equal to minus 1; this is rejected because this belongs to this is the left handed triad; it belongs to left handed triad or the different system. So, we are not taking this we will take this one and it can be shown by a very simple example.

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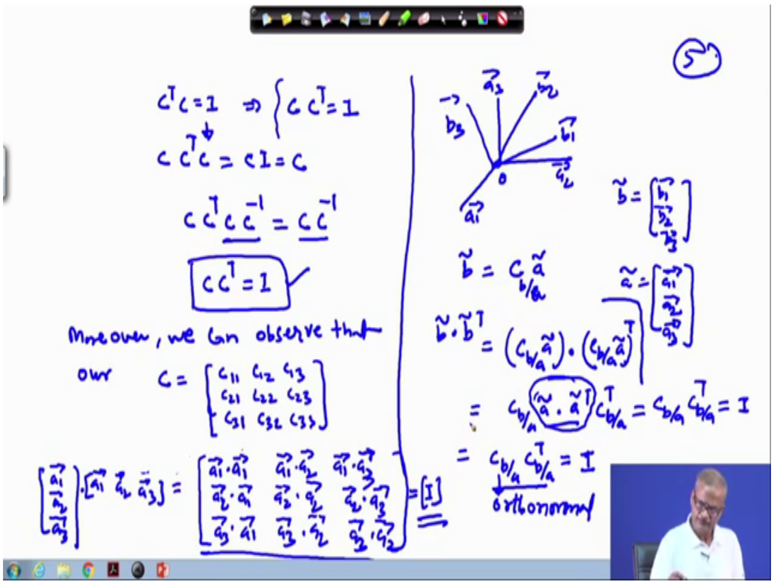


Also you can see that C transpose C this equal to I. So, this also implies C times C transpose equal to I which we can look by multiplying both side by C in this one. So, this is C times I which will be equal to C; now take multiplied by the universe on both the side. So, if we do so so C times C transpose C C times C inverse, this will be equal to C times C inverse and you can see this will be unity matrix; so this implies this.

So, if we have the C matrix which is denoting or rotation matrix must be orthonormal and we will later on also we will show this through writing (Refer Time: 21:19). More over we can observe that our C matrix which was defined as C 11 C 12 ok. So, if you remember we took one reference frame and then went to the other reference frame. So, this was a 1, a 2, a 3 and here b 1, b 2 this is a 3 and this is b 3. So, once we rotated it this is orthogonal matrix and once we rotate about this point; so this steps up this position and this we have written has b tilde equal to C times a tilde; where b tilde obviously, b tilde it consist of b 1, b 2, b 3; this a vectrix.

Similarly a tilde it consist of a 1, a 2, a 3; instead of this we can just write in terms of the unit matrix just not matter. And because this is a conversion from a to b; so we will write this as b slash a, to indicate that this is taking it from a frame to b frame ok. So, if you now work it like this b tilde b tilde transpose; so we can see that b tilde is C b a times a tilde; this will transpose and this is the dot product here ok. So, these are the extension of what you have learnt in your 11 12. So, C b a and a tilde this is transpose.

Now, if we this is the matrix transformation we are doing; so this will look like this. So, these two are vectors and this can be written as b a and already we know that this, if this is orthogonal matrix. So, this can be written as if this is the orthonormal; C is orthonormal then this can be written as I as the identity matrix; where a tilde a tilde this will be the quantity because already we have written here a 1 like this.

So, you have here a 1, a 2, a 3; a 1, a 2, a 3 and dot product in mid between. So, this can be written as a 1 dot a 1; a 1 dot a 2; a 1 dot a 3 and so on; a 2 dot a 1; a 2, a 2 dot a 3.

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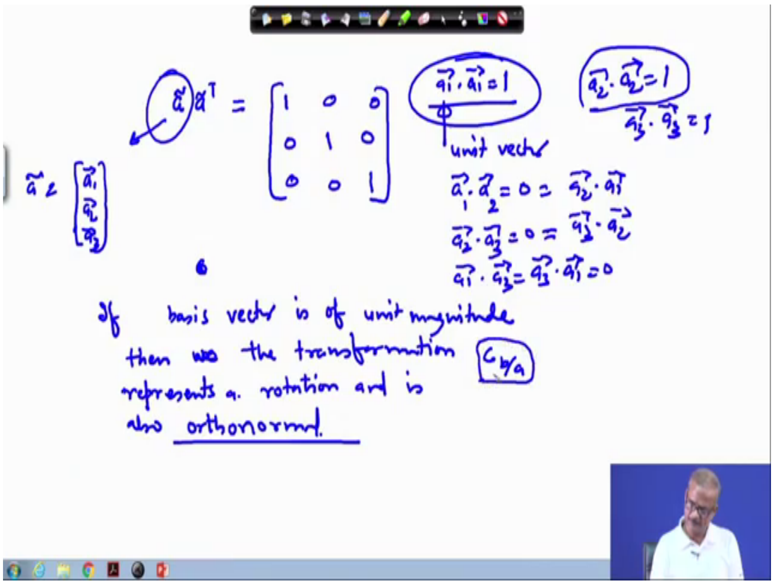


So, this a tilde times a tilde transpose; so if you have something like this a 1 dot a 1 dot, if this is unity means this is unit vector; if this is a unit vector ok.

And as we know that we have taken the three here; so a 1, a 2 all they are perpendicular to each other. So, this will vanish a 2 dot a 3 and similarly this quantities will be 0 ok. So, the off diagonal terms they will turn out to be 0 and if this is 1 and similarly you have a 2 dot a 2; this equal to 1 and a 3 dot a 3, this equal to 1. So, you can put here 1, 1, 1 means if this is a vectrix which are which is of unit magnitude ok.

This is simply telling that this vectrix which is a 1, a 2, a 3 is magnitude is unity then we can replace this by 1 and off diagonal terms will be 0. And therefore, what we can see that if we go to the previous page; this matrix, this turns out to be identity matrix ok. This is identity matrix as shown here; so this one gets reduced to an identity matrix and therefore, this matrix product will get reduce to C b a transpose is equal to I.

So, what we have shown here? That if your basis vector is of basis vectrix is of unit magnitude; basis vector is of unit magnitude unit magnitude, then the transformation C b a represents rotation and is also orthonormal ok. So to qualify as a rotation matrix any matrix should be orthonormal ok; if it is orthonormal, so it will qualify as the rotation matrix.