Satellite Attitude Dynamics and Control
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Lecture – 28
Gravity Gradient Satellite

Welcome to the number 28. So, today we are going to start with the Gravity Gradient Satellite. So, what is the basic principle involved here?

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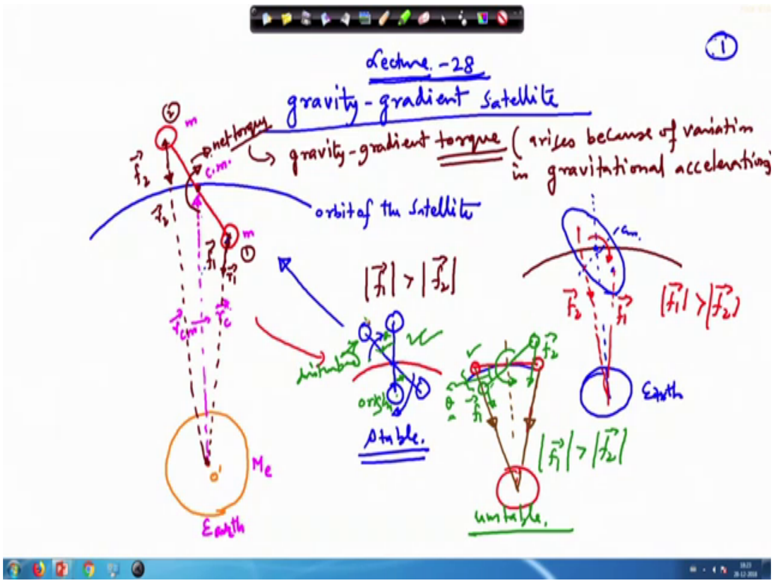

See if we are start with one very simple representation say, this is the orbit of the satellite and here say earth is present. Centre of the earth let us represent this by O prime and will take one very simple case. So, case of a dumbbell. Let us assume that this is r cm which we will write as r c for brief ok. This is the centre of mass of this dumbbell and both the masses are equal m and m and mass this is the earth.

So, this is earth, it can be earth or it can be any satellite heavenly body. So, what we observe from this place if we join this say, this is the first mass, this is the second mass and we write this as r 1 and this is the r 2. So obviously, you see that this distance from the centre of the earth is less and this distance is more. And therefore, the force acting on this particular mass here in this direction which is we can write as f 2 and here along this direction we can write as f 1. So, f 1 magnitude, this is greater than f 2 magnitude for this particular configuration.

So, what does it imply that at this point which is the centre of mass of this dumbbell so, there will be a torque acting like this because, sorry the torque will act in the opposite direction because this torque this force is more. So, this force is more. So, and here the force acting is less as compared to this one. So, there will be a net torque along this direction so as shown by this arrow. So, this the net torque and this is nothing, but your gravity gradient torque; net torque which we write as gravity gradient torque means the torque arises because of variation in the gravity; arises because of variation in gravitational acceleration.

So, this simple figure it says how your satellite can get affected because of the gravitational pull of the earth. Now let us say another case we take here in this place say, we have the case of a ellipsoid ok. This is the centre of mass of the ellipsoid and earth is somewhere here; this is your earth, this is the centre of the earth. So, this is here in this situation, you can see that on this half; on this half there will be more gravitational pull as compared to this half. So, here if we indicate by f 1, on this side the force by f 2. So; obviously, z f 1 is greater than f 2 and this implies that there will be a tendency for this satellite to turn like this ok.

So, means this implies that if your satellite is in a of a shape where there is a net torque due to the gravitational force about the centre of mass, then satellite will not remain in that position. And this kind of situation though here in this case what we see from this place that if my say here in this situation for this case, I am discussing at initially if your satellites these two satellites where sorry this two halves were like this means the dumbbell was initially in this side. And from this you have moved it to this configuration so; obviously, there will be a torque which will try to restore it back ok. So, it will try to pull it here in this direction.

So, once it disturbed from this situation if it is disturbed from this configuration to this configuration so, there will be a net torque here in this direction which will try to move it toward the original position. So, we call this kind of case; it is a stable is a gravity gradient stability means once we are discussing. So, this is simply a stable case, but it so happens that because the gravitational forces it is a conservative. So therefore, this will keep oscillating and you need to damp out this things until unless, if there are other factors which is trying to damp it.

On the other hand, if you take a case where your satellite is in the form of a dumbbell and it is looking like this and it is in this configuration here is your earth. So, you can observe that the forces acting on these two masses will be same because of this configuration. Now if you disturbed from this configuration, then what happens? So, once it is disturbed from this configuration and say it comes to this position after disturbance, then what will happen? You see that this force the force acting on this and force acting on this if this is a f 1 and this is f 2. So, f 1 magnitude this will be greater than f 2 magnitude. And what this will do?

This will give it a net torque here in this direction. So, if your dumbbell which was lying like this dumbbell satellite which is originally in this position and it is disturb to this position. So, it will not tend to return back to the original configuration rather it will go and it will keep moving, keep moving and it will come to this position means for the stability earlier we have discussed that there should be tendency to return back toward the original equilibrium position. So, here this is a statically not stable, it is a simply implies. If we look here in this static configuration and moreover if it is left over a period of time, it will go away from this position and it will keep start oscillating.

So, this kind of configuration is a unstable because, the disturbance it increases over a period of time. You have given initial disturbance by say this angle, you have disturbed this angle is here something like theta. So, this angle will increase over a period of time. It will not decrease and therefore, this configuration it is a statically unstable. And dynamically what we see the here in this case we will have to do a lot more work before we finally, come to whether it is a dynamically stable or not. But if once we can say that if it is statically unstable that is no restoring force means it is a dynamically unstable.

So, this thing we are going to analyze mathematically ok. This is a very simple representation, but for our actual satellite modeling, once we are doing it for say the ISRO is working so and if is designing a gravity gradient satellite. So, this whatever we have discussed till now, it is not enough. We need to discuss these things in terms of mathematics and make the things very precise. So, that proper fabrication of the satellite and it is a how much time it will take to say here in this case as we are discussing this particular case that if it is this was the original position and this is the disturbed position.

So, if it is disturbed from the original position how far by certain amount so, we have disturbed it from this position to this position ok. So, by theta angle let us say that we have disturbed it. So, how much time it takes to go from this position to this position; so all this things you can work out, it is all possible ok. You can do various kind of a studies what will be the period of oscillation because the gravitational force is conservative. So, if it is oscillating all the time so, what will be the period of oscillation, whether it will affect your mission and so on many things need to be discussed.

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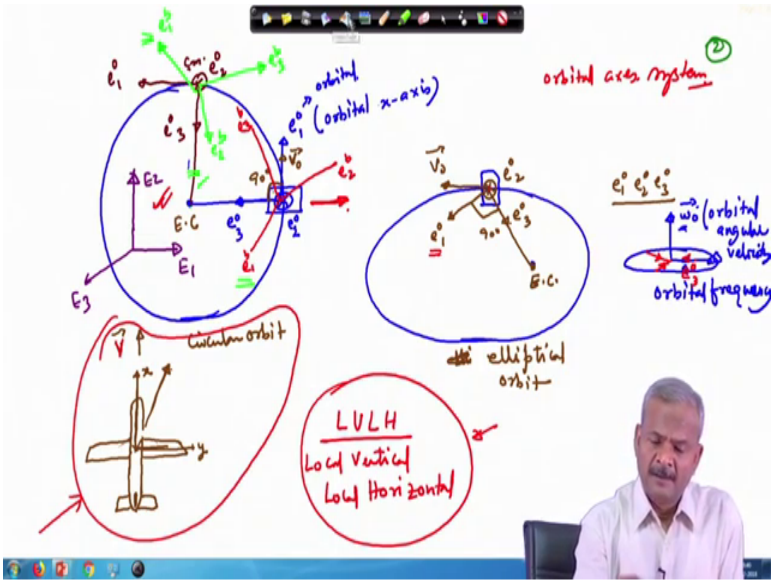

So, with this start within the main purpose say your satellite can be in an elliptical orbit or it can be in a circular orbit. Say, if this assume that this is a circular orbit and here this is the centre of the earth and this is your satellite. So, earlier in the very first class we have discussed about the reference system. So, let us assume that this is the velocity vector of the satellite. So, along this direction we write e 1 o means this is the first or the x axis of the orbital axis. So, this is the orbital x axis and how we have written this depicted the orbital axis e 1 along this direction and e 2 along e 2 taking the right hand rule.

So, it will be down and we will show it by cross this is e 2 and this o stands for the orbital and e 3 along this direction which is pointing towards the centre of the earth. So, this is your e 3 direction. So, this forms a (Refer Time: 13:51) ok. If the orbit is not circular in there are cases where your satellite may not be in a circular orbit rather an in an elliptical orbit. Let us say that I am showing a very highly elliptical orbit. So, it is a exaggerated figure. So, let us say that your satellite is lying somewhere here and this is the centre of mass of the satellite. So, we will choose e 0 1 along this direction, this angle it is a perpendicular, this is 90 degree, this will be 90 degree. Velocity vector will be tangent to the your V 0 will be tangent to this ellipse ok; velocity vector is always tangent to the trajectory.

So, V 0 is here in this direction. So, e 1 along this direction this is the centre of the earth, we can write this as EC earth centre circular orbit elliptical e 2 and this we are writing as e o 3 and; obviously, here e o 2 it goes inside. So, e 1 e 2 e 3 so, you can see that here is your velocity vector is also along the same direction. This is the velocity vector. So, both are coinciding. So, from where this notation is arising? We will follow this notation, this notation arises from the aircraft in the case of the aircraft ok; x axis is taken along this direction y axis like this and z axis vertically downward towards the value of the aircraft and you know that aircraft it flies along this directions. So, V is along the direction.

It may happen that V can be also along this direction, but the body axis it is a fixed along this direction. So, this is the body axis here in this case we are not talking here about the body axis this is simply the orbital axis and with respect to this orbital axis the e 1 e 1 0 e 2 0 e 3 0. This is the triad with respect to this your body is oriented. So, here I have shown in the body to be like this. So, let us say that your body is arbitrarily I can show it like this. This is e 1 e 2 e 3 and if we put b here so, these indicate this is for the body axis.

Similarly, here in this case I can show it, but I will not make it clumsy. So, remember that your e 1 axis will be always shown like this. Here this is the orbital system the orbital axis system it is a always shown like this. This direction I will show by e 3 this direction going vertically down into the page by e 2 and e 1 along this. So, e 1 has the same sense as V 0. Here in this case if you look here in this place so, you can have rotation about the velocity vector.

So, here also if you look this is the velocity vector so, you can have if the velocity vector is having the same direction as the x. So, you can have rotation about the velocity axis, but; obviously, for the aircraft thus things are much more complicated. It is a representation where there are the angle of attack side slip and so many things are there that needs to be discussed before anything can be mentioned about the aircraft. So, we confine ourselves to the satellite and discuss further on this. So, this is the basic description ok. There is one another notation which is called the LVLH. This is Local Vertical Local Horizontal. If I look into the journal papers so, some of the journal papers instead of considering e 0 here in this direction, they will take e 0 along this direction as shown by this arrow.

So, e 0 3 along this direction and e 0 2 then it will be coming out of the page. So, it will be coming out from this place. So, this particular notation we will not follow ok; we will follow this notation.

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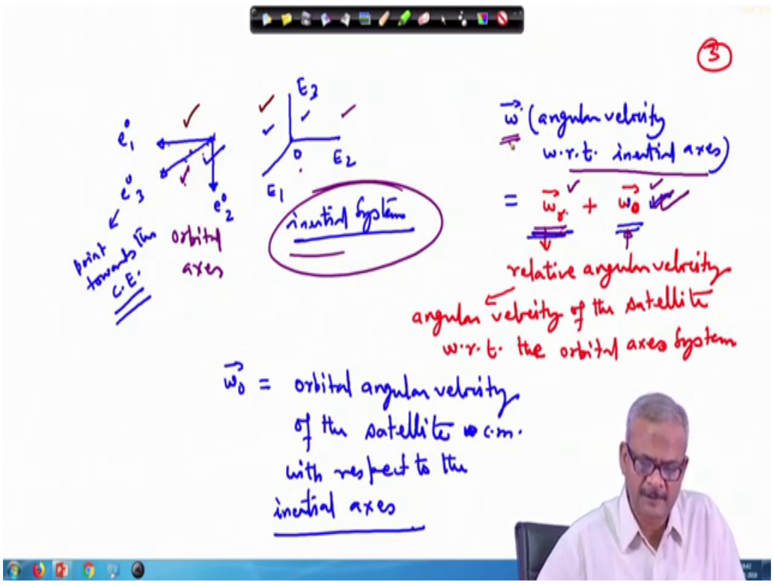

So let us start. So, let us assume that say if my e 0 1, it is here e 0 2 is vertically downward and e 0 3. So, e 1 e 2 e 3 e 1 e 2 e 3; this is the right hand rule we are applying and this points towards the centre of earth. Now, this is about what we have discussed this is your orbital axis system. Now what about the inertial axis system? Inertial axis system let us say that I can fix here in this place as you know that inertial axis system it is a non rotating system. So,, but for describing the inertial axis system itself it is a very complicated ok. If you go to the astronomies of the axis system used in the astronomy, it consists of 2 semester course.

So, we need not go into all those details because we have to focus on the satellite dynamics. So, what we assume that if this is the situation I can say I can take here the inertial axis system something like this or I can have any other orientation and with respect to this then this system is oriented e 1 e 2 e 3. This I can say that this is my inertial system already rotation we have discussed.

So, with respect to this rotation is indicated; now going back again here in this figure. If the satellite is having velocity along this direction so, where will the angular velocity of the satellite? So, I will make another figure here. Let us say this is the circular orbit being shown if we look from the top. So, your angular velocity of the satellite, it will be this is the, this is called the orbital angular velocity this which is different from orbital angular velocity. It is different from the angular velocity of the satellite with respect to this body axis.

Like I have this mobile and this mobile is rotating. So, I will say that this is the angular velocity of the satellite, it is having certain value along the 3 axis it is having certain values. But this as a whole this is also rotating in the orbit its going like this when besides it is rotating like this. It is rotating like this and going all around the orbit. So, in the orbit while it moves like this so that thing I have indicated here this omega 0. This is also called the orbital frequency. So, the same thing also we call as the orbital frequency.

So, if we want to show the angular velocity of the satellite so, this is the angular velocity with respect to the inertial axis with respect to the inertial, then how we can write? So, my satellite is rotating with respect to this axis and this axis is rotating with respect to the inertial axis. So, your satellite is here and this satellite is and in this direction your e here; it is your e o 3 ok. So, e o 3 is changing over a period of time. After sometime it will be along this direction after sometime it will be here right; now it is here ok. So, this means change its direction.

So, with respect to this we describe the motion of the satellite as omega r. So, omega r and plus omega 0 what this is this is the relative angular velocity which you call the angular velocity with respect to angular velocity of the satellite with respect to the orbital axis system. While omega 0 omega 0 this is the orbital frequency or orbital angular velocity, we will not call this as the frequency. We call this is a vector. So, this is orbital angular velocity that implies.

So, if we vectorially add we know that angular velocity can be added. So, if we vectorially means this frame is rotating with angular velocity omega 0 with respect to the inertial frame ok. So, orbital angular velocity of the satellite that is its centre of mass is moving of the satellite centre of mass with respect to the inertial axis. So, with respect to this e axis, this frame is rotating as shown here. So, as the satellite goes here. So, this frame right now it is here after sometime the same thing it will come in a position is of this will be e 1 0 e 2 0.

And so obviously, you can see that this is a centre of mass of the satellite. Satellite this figure the blue line the rectangular poor part I am not showing here. So, these lines this co-ordinate axis, this reference axis as rotated from this place to this place. So, there is a rotation involved for the orbital axis itself it is a very obvious its rotating about the this vertical axis which is passing through the page of the paper ok. So, this axis rotates with respect to this and the satellite then rotates with respect to this axis. So, in this axis, then we can as we have shown the orientation here in this place like this. So, your orientation will change and it will come to some other configuration.

Let us say, it comes to it goes like this e 3 comes along this direction e b and e b 2 is here e 1 e 2 e three. So, this is e 2 b and here e 1 b. So, e 1 e 2 e 3 right hand rule ok. We are using the right hand rule e 1 e 2 and e 3 along the thumb. So, this orientation can change e 1 was here. So, e 1 has gone here in this place. So, there is a rotation of the satellite with respect to the orbital axis which is shown here by this brown line and also there is a rotation of this orbital axis which is shown by this brown line. Brown lines with respect to the inertial axis which I have not shown here in this place, but inertial axis let us say that if I try to show it here, I can put it like something like this.

Instead of putting it here, I am just putting it here for convenience. So, I can put it here in this place and I can write that is this is e 1 this is e 2 and e 3 along this direction either e 1 e 2 e 3 whatever the way I choose to write. Therefore, from this consideration the angular velocity of the satellite, then becomes with respect to the inertial frame. It will be sum of these 2 first with the angular velocity of the orbital axis with respect to the inertial axis system and then the orbital then, the angular velocity of the satellite with respect to the orbital axis system.

So, you have get this very clearly that the first this rotation is referring to this coordinate; this reference axis the orbital reference axis rotation with respect to the inertial axis system ok. This is your orbital axis system orbital axis and this is the inertial axis system.

So, this is rotating with respect to this and with respect to this then the satellite is rotating. So, both are then vectorially added to get the net angular velocity of the satellite ok. So, this puts the background for discussing further rest of the things. Now, we go back we have to again discuss about the rotation and other things, but I will come to that later on. Let us start with the development basic formulation further gravity gradient satellite

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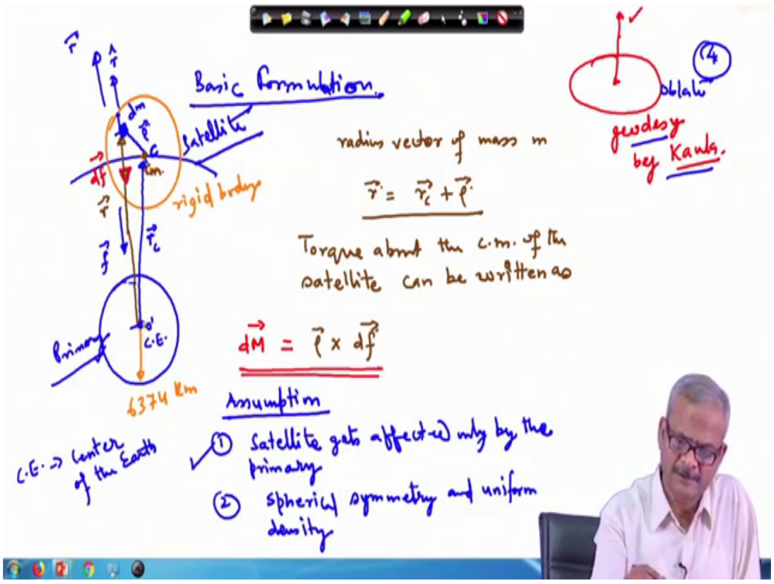

If we take this to be the earth, this is the orbit and I have a rigid body here. We are discussing with the respect to the rigid body, this is a rigid body here in this place. This figure is exaggerated. So, remember because this is earth is very large. This radius is around 6374 kilometers while your satellite maybe of few meter radius. So, do not take as it is ok.

If this is the situation, then the whole development whatever I am going to do here it will be totally different from that. So, this is the centre of mass of this satellite. We take a small mass inside this satellite and say this mass is d m this vector we write as a rho and the vector from the centre of the earth to this place. We write this as r c indicating that this is the centre of mass of the satellite this we can see set as c and then finally, this point we can join together and this is your r. So, the radius vector of mass m that becomes r equal to r c plus rho.

So, previously as we have discussed in the case of dumbbell, there will be force acting on this mass and we can take torque about this point about the centre of mass. So, already as per our earlier discussion in the previous lectures that if we write the torque equation about the centre of mass, then no extra term appears, but if we write about some any other point, then the extra term appears. So, we therefore, we are choosing here the centre of mass and not any other point. Therefore, the torque about the centre of mass of the satellite can be written as rho cross d f where d f is the force acting on the so, d f will be along this direction.

Here it will be acting along I will show it by some other color. This is your d f, the force acting on this and this we write as d m equal to d f. Now, this case can be very complicated. If we consider the earth to be oblate ok, then the and moreover the density of the earth it is varying as you know, it is a not constant throughout the sphere of the earth ok. So, if we are start discussing that way it will be very tough to develop this topic particularly in this class. If you are interested in looking that how the oblate earth the acceleration due to at any point due to an oblate earth is represented so, you can refer to the book on geodesy. It is book on geodesy by Kaula.

So, this is our basic equation and what are the assumptions we are going to make here? We need to make certain assumptions that, is satellite gets affected only by the primary. Primary means in this case it is a earth that moon also, but moon consideration it cannot be will not take it and the sun is also present. So, if we are trying to do very accurate propagation of the satellite orbit so in that case the perturbation due to the moon sun and various other planets is taken, but for this the attitude dynamics it is not required.

So, if the size of the satellite is very large like it is if say you are considering moon itself for which is a satellite of the earth ok, but the distance now it becomes large. But if that this distance is reduced and say the its moving just near the surface of the earth size of the satellite which is of the size of the moon, then the we need to consider various other factors. So, here we will simplify it as a case that this is my primary body and about this is satellite which is moving.

So, the first this is the first assumption, then the we will assume that as you know from the Newton's law that the gravity due to this place this whole earth because the inside the density is varying ok. And therefore, the acceleration due to gravity at this point it can be very complicated. But we will simplify this assuming that the whole mass of the earth is concentrated at this point o prime which is the centre of the earth, c is the centre of the earth. So, here spherical symmetry is present, this is not oblate case; for the oblate case, this is the oblate. For the oblate case you have to look into the book on geodesy by Kaula; so, as spherical symmetry and uniform density.

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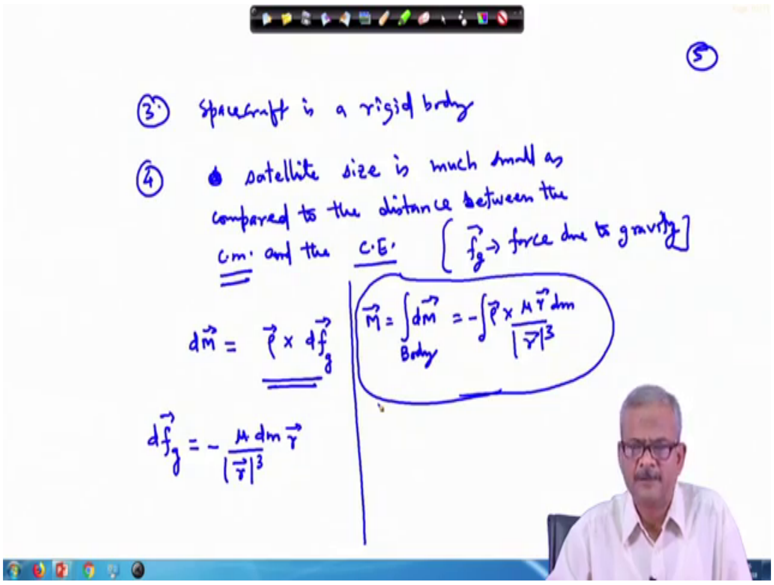

This spacecraft or the satellite is a rigid body, we are not discussing it for the flexible case and the fourth as we have discussed that this is a very small as compared to the distance from this place to this place. So, the fourth is satellite size is much small as compared to the distance between the centre of mass and the centre of earth. So, under these 4 assumptions, we can develop the satellite dynamics under gravity gradient.

So, already we have written that d m the torque acting on the satellite, it can be written as rho cross d f and. If we put a g here so, this just indicates that this is the force due to the gravity. So, f g the force due to due to gravity and as we know this d f g can be written as mu by this r magnitude whole cube going back here in this place. So, this is your r vector; r vector is along this direction. So, for magnitude of the distance from this point to this point this is r magnitude. So, r magnitude and then the unit vector along multiplied by here d m and the unit vector here in this direction.

So, because we have put here cube so, we will not write unit vector, but rather write this as r ok. If we have if we write here r square so, then will write in terms of a unit vector along the r direction. So, unit vector along this direction ok. This is r cap while r is a vector along this direction ok. Now, if we have to calculate the whole torque so, if we want m. So, we must integrate this over the whole body which simply I will write this as b or we can remove this b all together this is just integration over the whole body. And therefore, this becomes rho cross and here we need to put a minus sign because the gravitational force its acting along this direction.

It is a gravitational force is opposite to the r direction, r is here this is your r direction ok. While the gravitational force direction it is along here towards the centre of the earth and therefore, we must put a minus sign here in this place and if we put that minus sign so, we can write this as mu r d m and then ok. So, this is our basic equation and we need to develop it further to get the equation of motion of this rigid body which is in this case the spacecraft under gravity gradient.

Thank you very much.