Satellite Attitude Dynamics and Control

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Lecture – 27
Stability of Torque Free Rotation (Contd)

Welcome to the 26th, 27th lecture. We have been discussing about stability of torque free rotation of the satellite. So, we will continue with that of some last portion it is remaining. So, we will finish up that and then we will go to the gravity gradient satellite.

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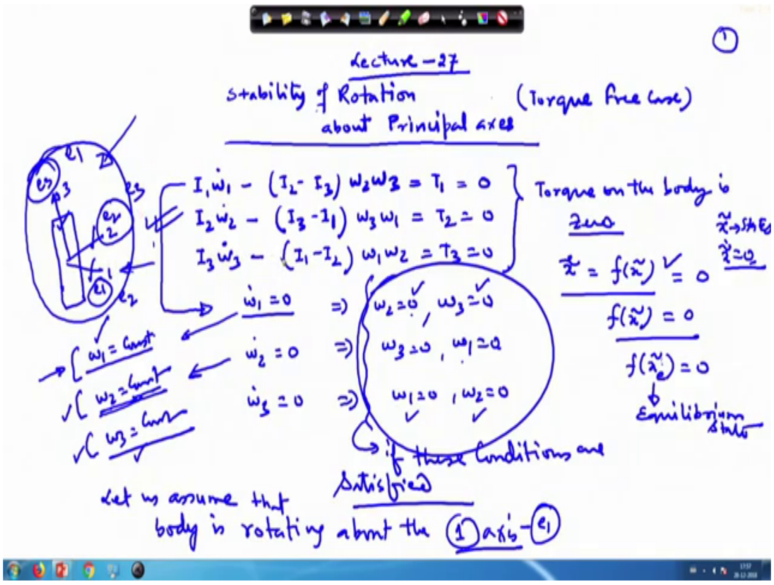

So, what we have observed that Euler equation we have I 2 minus I 3 times omega 2 omega 3, and for the torque free case we have written this quantity as 0 ok. So, as we have learned last time we got the equilibrium and other things. So, you can we can see that. So, first let us write like this x tilde dot this equal to f x tilde, say this is non-linear system.

So, if we are looking for the equilibrium point, so that at the equilibrium point we must solve this for 0. So, we have to solve it, because it is a stationary state of the system means the state of the system defined by x tilde, these are the states ok. So, and the stationary states implies x tilde will be equal to 0. So, x tilde setting it to 0, and then we have to solve for this, and the solution then we get here out of this say this is we will write this as x e.

So, once we solve this, so this gives you the equilibrium state. So, here in this case if we look into this equation. So, it is already torque free case ok. So, when the equilibrium solution will exist. So, for that obviously you need to set it to omega 1 equal to 0, omega 1 dot this equal to 0. Once you set it to 0, and then we need to solve it for this part. So, this implies either omega 2 equal to 0 and omega 3 equal to 0 ok.

So, if suppose we want to so continuously rotate what I mean here that is it possible to rotate about the first axis, the first principle axis this is your I 1. So, about the once we have taken the case of the cylinder and this we have written as 1, 2 and along this the 3. So, can we sustain rotation along this axis or along this axis or along this axis, this is the question.

So, similarly if we look for omega 2 dot ok, if we are looking for the equilibrium along this axis. So, omega 2 dot equal to 0 and this implies omega 3 equal to 0 and omega 1 equal to 0. Similarly, omega 3 dot this equal to 0, this implies from here. So, this itself tell that omega 1 dot is 0 means omega 1 is not varying. So, omega 2 we can keep it 2 0 and omega 1 is having certain value omega 1 equal to a constant ok. So, this condition is satisfied for the equilibrium.

Now, similarly we look for this one omega 2, we get the omega 2 equal to constant, omega 3 equal to a constant. So, we can maintain either a constant angular velocity along the 1 axis or the 2 axis or the 3 axis as shown here in this place. If it is a torque free case and for that what is the condition required that if these conditions are satisfied. Satisfied, this implies that if you have omega 2 equal to 0, omega 3 equal to 0 means it is not rotating along the second and the third axis, then it is a possible to maintain a constant rotation about the first axis.

Similarly, this implies that if omega 1 is 0 and omega 3 is 0. So, it is possible to continuously rotate this body about the second axis, their own thereby we are writing here omega 2 equal to a constant. In the same way, if omega 1 omega 2 they are 0 along this axis, then we can rotate the body along the third axis.

Now, suppose that the body is let us assume let us assume that body is rotating along rotating about the 1 axis ok, often we have written this as the e 1, e 2 and e 3. So, this is the body as fixed. So, if the 1 axis means, this is e 1 axis. So, if it is rotating about this e

1 axis about this axis. So, in this condition if we perturb about keep some deviation in the angular velocity ok.

And also displacement in the angle about the 1, 2 and the 3 axis, but those displacements are small ok, angular displacements are small. if we give this angular displacement, so what will happen to such kind of system; whether this rotation will way bounded or this will explored over a period of time. So, we can look from the equations we have written. So, this equation we are already having.

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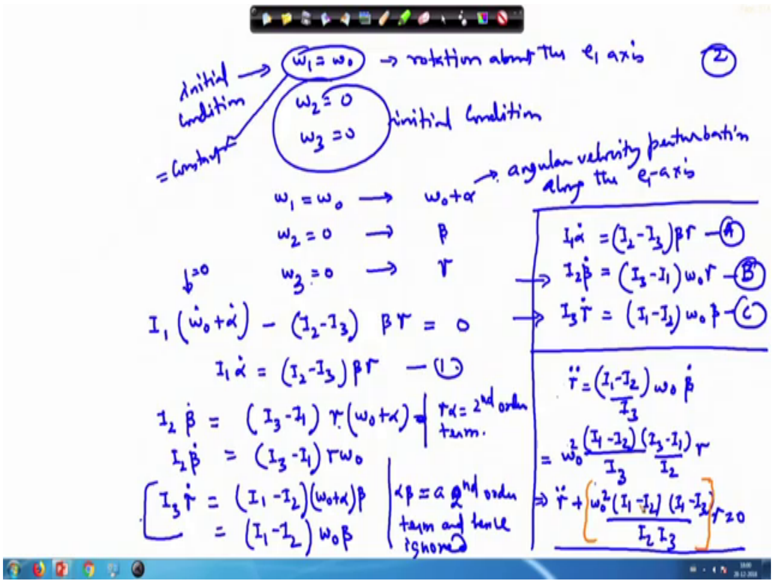

So, we will utilize this equations and write the necessary other necessary equations for this. So, we have I 1 times omega let us say that omega 1 this equal to we can write this as alpha, beta, gamma this is alpha. So, it is rotating about the e 1 axis, rotation about the about the e 1 axis. And the other one we will assume, omega 2 this is 0, and omega 3 this equal to 0. So, this is initial condition, this also initially.

Now, from this value the system is perturbed or maybe we can put it this as something like omega 0. Now, a perturbation is given, so omega 1 which is equal to omega 0, this is perturbed to omega 0 plus alpha, where alpha is the angular velocity perturbation along the e 1 axis. Similarly, omega 2 which equal to 0, this is perturbed to beta. And omega 3 which is also initially 0, this is perturbed to gamma.

Now, we can write our equation I 1 times omega 1 dot. So, omega 1 dot omega 0 dot plus alpha dot, this quantity is 0 because omega 0, this is a constant this is your constant. So, this quantity will be 0 minus I 2 minus I 3 omega 2 omega 3. So, omega 2 here omega 2 was initially 0, it has got perturbed to beta. So, we write here beta and this is omega 3 is gamma and there is no torque acting on the system, so we make it 0.

Now, we can write this as I 1 times alpha dot this equal to I 2 minus I 3 beta times gamma, this is my equation 1. Similarly, we will have I 2 times omega 2 dot. So, omega 2 omega 2 dot this part is 0. So, only what we get here this equal to beta dot this will be equal to I 3 minus I 1, and omega 2 is here, so 3 omega 3 omega 1. So, omega 3 is gamma and omega 1 is omega 0 plus alpha. And this we can approximate as gamma times omega 0, gamma times alpha this term we are ignoring.

Here, we can write gamma times alpha this is a second order term. So, we are ignoring it gamma times alpha and writing like this. So, I 2 times beta dot this equal to I 3 minus I 1 times gamma times omega 0. In the same way I 3 times gamma dot this can be written as I 1 minus I 2 times 1 and omega 1 and omega 2. So, omega 1 is omega 0 plus alpha and omega 2 is beta. So, this gets reduced to I 1 minus I 2 times omega 0 times beta. Here also, alpha beta this is a second order term. So, it is a ignore second order or second order term, hence ignore ok.

So, we have this three equations finally. We get I 1 times alpha dot equal to I 2 minus I 3 times beta gamma, this is a equation A; and I 2 times beta dot omega 0 gamma this is equation B; and this is equation C. Now, we can utilize these three equations to study the stability of the system. So, the second and the third equation we chose. So, second equation we have here, we go back this one and this one, these two equations we differentiate them ok. So, once we differentiate this quantity.

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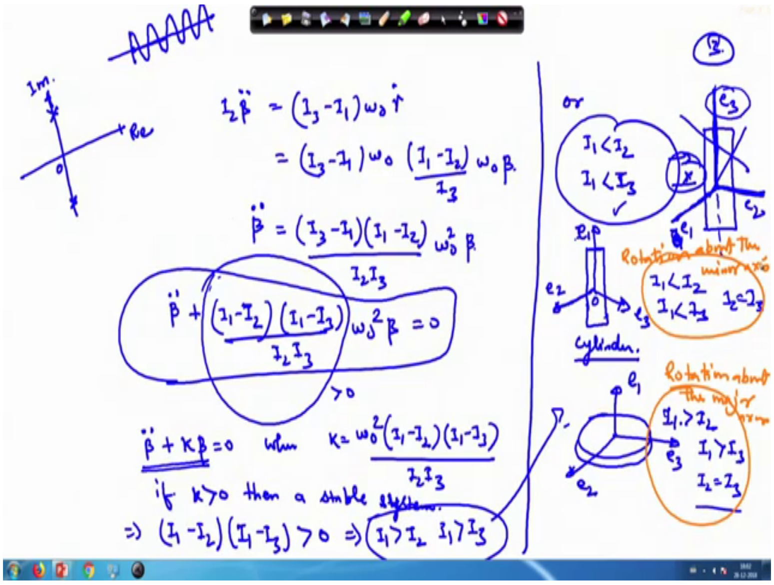

So, the b. So, I 2 times that will be I 2 times beta double dot this equal to I 3 minus I 1, I 3 minus I 1 omega 0 this is a constant, so this will not get differentiated and then we have gamma here. So, we write here gamma dot and this gamma dot we can insert from this place equation C ok. So, I 3 times gamma dot, this we can rewrite so I 1 minus I 2 divided by I 3 ok, I 1 minus I 2 divided by 3 omega 0 times beta, this is omega 0 times beta.

And this I 2 also we can remove from this place, and we can bring it on the right hand side. So, if you look here in this equation this is nothing but I 1 minus I 2 times I 1 minus I 3 divided by I 2, I 3 times omega 0 square beta this equal to 0. So, this is the equation of simple harmonic motion, simple harmonic motion equation format you are getting.

Provided this quantity is positive. If this quantity, if this greater than 0, then you will get a simple harmonic motion equation; if this quantity turns out to be less than 0, then what will happen this equation will indicate on unstable system ok. So, we have here beta double dot we can write this as K plus beta equal to 0, where K equal to omega 0 square I 1 minus I 2, I 1 minus I 3 divided by I 2 I 3.

So, if K is greater than 0, then a stable system. So, this implies the condition that must be satisfied is I 1 minus I 2 times I 1 minus I 3 this should be greater than 0; and this implies I 1 should be greater than I 2, and I 1 should be greater than I 3 or I 1 should be less than I 2 or I 1 should be less than I 3.

So, if we look here that I 1 is less than I 2, I 1 is less than I 3. So, this implies this is the particular case, this is pertaining to say p if you have a cylinder e, you are taking this e 1, e 2, e 3. So, if you look here in this case this is your I 1, this is I 2, and this is I 3. So, here this one is the axis of list moment of inertia while these two are equal. So, obviously this is not related to this.

So, I 1 will be list in which case, so what we can do that we can change the axis, say we define the axis e 1 along this direction and in the same way e 1, e 2 we can define e 2 along this direction, and then e 1, e 2, e 3 along this direction, so that makes a right handed system, and this is the body axis you were fixed. So, if we discard this, and we look here in this figure.

So, even here in this case you can see that the moment of inertia this is a cylinder. So, moment of inertia about this axis it is a minimum in e 2, e 3 they are equal. So, there many may be many cases, you can make many cases where this condition is satisfied, but what it says that this two together ok, and this one refers to the case where I 1 is greater than I 2 and I 1 also greater than I 3; so that you can indicate like this, this is a disk and here you have e 1 and this is e 2, e 1, e 2, and this is e 3.

So, here in this case your I 1 is greater than I 2 and also I 1 is greater than I 3. It is so happens in this case that I 2 and I 3 are equal, here in this case I 1 is less than I 2 and I 1 is less than I 3 also. It so happens that I 2 equal to I 3 here in this case I 2 equal to I 3. So, this equation we have just do not get confuse by the equation we have written here. We have just tagged here e 1, e 2, e 3 we could have equally tagged here; instead of writing here e 1, we could have written here the e 2, this we could have replaced by e 3 and this we could have replaced by e 1 ok. So, e 1, e 2, e 3 it does not matter the same equation applies. So, there is nothing to get confused about.

So, it is says that the beta will remain bounded means if this happens so you know that in your from your linear, this is the real axis, this is the imaginary axis, this is origin. So, linear control system you know that if the poles are lying over the imaginary axis. So, it is marginally stable system means the, if the oscillations are there. So, oscillation will be continuously be at the same level, it will not spread out, it will not grow over period of time ok.

In the same way, if you look for this equation this particular one. So, if you differentiate this may be I will take this space and write here itself. So, I 3 times gamma double dot I 3 will make move to I will like move to I 3 is moved to the right hand side. So, this is I 1 minus I 2 divided by I 3 omega 0 beta dot. And then beta dot will just insert from this place, so I 1 minus I 2 divided by I 3 and omega 0, we will put it here in the front and beta dot we need to replace from this place which is I 3 minus I 1 times omega 0.

So, we will make this as omega 0 square and then gamma. So, this implies gamma double dot plus omega 0 square I 1 minus I 2 times, I 1 minus I 3 divided by and here one more thing is missing, we have inserted beta dot. So, I 2 will also come here in this place. So, this is I 2, I 3 times gamma this equal to 0. And this is exactly of the same form you will see that if this quantity is if the quantity here if this turns out to be positive, then this will indicate is simple harmonic motion equation; if this quantity turns out to be negative, then this will be simply an unstable system. So, for the stability it is required that as we have discussed here in this place.

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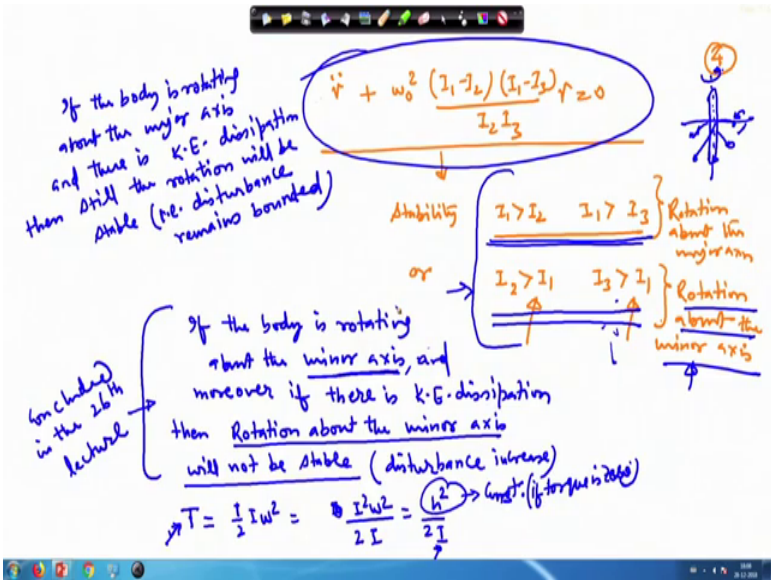

So, from this equation gamma double dot e 2 omega, here in the denominator I 2, I 3 are there. So, gamma double dot plus omega 0 square I 1 minus I 2 times I 1 minus I 3 divided by I 2 I 3 times gamma equal to 0. And this also exactly gives you the same condition that is for stability, we must have I 1 greater than I 2 and I 1 greater than I 3 or I 2 should be greater than I 1 and I 3 should be greater than I 1. So, this is rotation about the about the major axis. And this gives you rotation about the, because you are rotating about the first axis ok. So, rotation about the minor axis.

The same way here also, this set is rotation about the major axis about the major axis and this set refers to rotation about the minor axis about the minor axis. So, if you remember that already we have discussed that if the body is rotating, what we have earlier concluded that if the body is rotating about the minor axis, then and more over and more over, if body is rotating about the minor axis that means, this is the situation.

And more over if there is kinetic energy there is kinetic energy dissipation kinetic energy is dissipating, then rotation about the minor axis will not be stable; this we have concluded earlier in the last lecture, in the 26th lecture ok, so that implies though here what we are getting that even if the rotation is about the minor axis system will be stable, because this is of the simple harmonic motion format means the disturbance remains bounded.

But what if we look back into the system, where the there is dissipation of energy also, this is not giving you any information about dissipation of energy. So, if there is dissipation of energy then the rotation about the minor axis will not be a stable as concluded from this place, it is centenary to this ok. So, here because we have not considered the kinetic energy dissipation and therefore we got this condition.

On the other hand, if the body is rotating about the major axis as we have shown in this place ok. So, if the body is rotating about the major axis and there is kinetic energy dissipation, then still the rotation will be stable that is disturbance remains bounded ok. Here in this case, this is not the situation this disturbance increases ok.

The body will deviate from its original situation, as we have looked into this T this equal to we have written as 1 by 2 I times omega square, and then we wrote this as I square omega square divided by 2 I, which equal to h square divided by 2 I. So, if this dissipates is, if this decreases so obviously this quantity this is a constant in the torque free condition, this is constant; constant if torque is 0 ok, if torque is 0.

If torque is 0, this quantity remains constant. And if this is decreasing means, this must increase. So, if the rotation is taking about the minor axis, then it is bound to flip the axis of rotation means as you have seen in the case of the explorer, it was rotating about the minor axis. And there were four turn style antenna of during course of time because of this vibration in the turn style antenna, energy the kinetic energy got dissipated. and then this explorer started instead of rotating about this axis, it started rotating about this major axis ok, it has started rotating about this axis.

So, this was a big lesson learned. And so as we design certain thing and when find that after while it is put into the work. So, then we find that the certain thing it is not working properly, then we learn a lesson from that observation. And then we go back to the theory, and look into what we have done the mistake, where that lacking was there and then we try to correct it.

And this way the science and the engineering both of both these extreme; obviously, the engineering basis is the science and mathematics so but itself the science and engineering both of them they evolve from the observation ok. Sometimes in the science some assumptions are made and based on that, then you can do certain derivation, you can say that this is happening, but it may be the case that whatever the assumption you have made that may not be correct ok, so that has to be verified through the experiments. So, whatever the hypothesis you make, you if you verify through the experiment and then only it becomes a theory.

And similarly here in the case of the engineering also, we start with scratch, we do some simple modelling and we see that whether my system is working according to my theory or not, whatever the model we have developed. So, if it does not work properly, then if the model developed and the actual system. If they are not matching, then the correction is done the in the model the mathematical model of the system, and this way the things progresses. So, we will continue in the next lecture.

Thank you very much.