Satellite Attitude Dynamics and Control
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Lecture – 02
Kinematics of Rotation (Contd.)

In the last lecture we have discussed about the general requirement of the representation of rotations and how it certain lecture to the kinematics and ultimately to the dynamics and which is required for the modeling the Satellite Attitude Dynamics and thereafter doing the Control.

So, in this lecture we continue with the rotation which is the primary requirement for going into the kinematics of the rigid body and there after once I finish this we will go into the rigid body dynamics.

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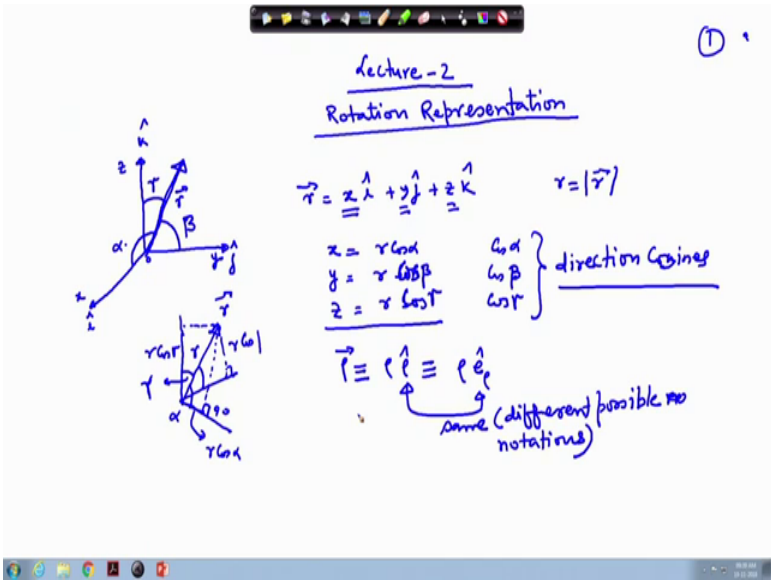

Let us say that we have one reference frame this is the orthogonal right handed reference frame. In which we have a vector we can represent it by a r rho whatever you like of. So, we can see that this vector is composed of three components x i cap y j cap and z k cap. Where i is the unit i cap in unit vector along the x direction and similarly the others one x y and these are the z direction.

So, how do we get these components x y and z? Basically, if you drop a perpendicular from this point to the x axis to the y axis and to the z axis so those components are related by certain angle which are called the direction cosines of this vector r. So, say this is alpha then this is beta and here this is gamma ok. So, if we take the projection of r vector on this axis so, this will be r which is r is the magnitude of the r vector.

So, r we can write as r magnitude. So, this is r cos alpha y is r sin beta and z is sorry this is r cos beta and this is r cos gamma. So, if it is a little difficult to look this way, but I will draw another figure in which it will be more clear so, if this is the r vector.

So, if I draw a projection from here to here this is a 90 degree ok. So, you can see that this magnitude is r and if this angle is alpha so, this quantity becomes r cos alpha. Similarly, you can drop a perpendicular from this to the z axis and this will this angle is gamma. So, this becomes r cos gamma and in the same way if you drop a perpendicular from this point to this axis so, that will be r cos beta, where beta is this angle.

So, any vector can be represented in terms of cos alpha cos beta and cos gamma which are called the direction cosines. If the same thing we want to write in some different way so, if they are multiple ways possible we can do this representation.

So, let us look into this representation I have any vector rho. So, rho I can write as a rho times rho cap or equivalently we can write this as rho times e rho cap. So, both are same these are same this is the different notations different possible notations. So, this is only one of the representation we have many other representation.

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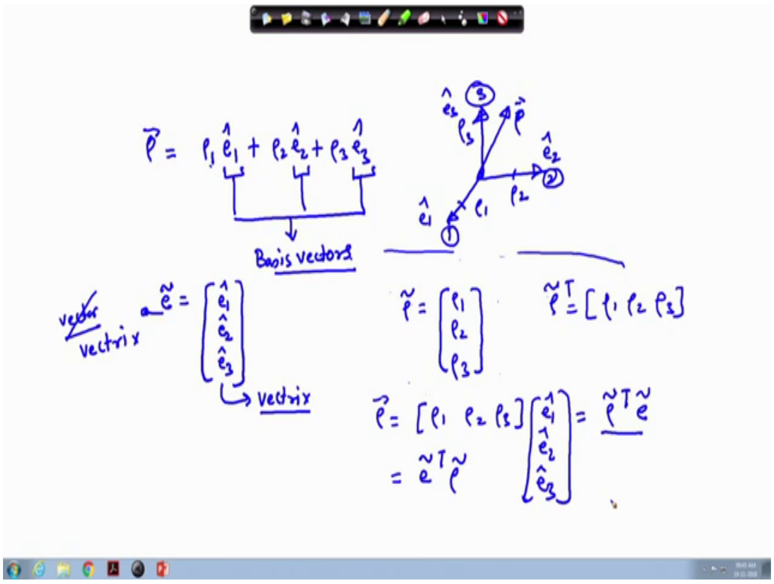

So, therefore, this rho vector we can write in the form of already we have written that this consist of three components. So, we can write this as rho 1 times e 1 cap rho 2 times e 2 cap and rho 3 times e 3 cap. So, where rho 1 rho 2 rho 3 they are the components so, this is the r rho vector.

So, along this direction you will have rho 1 along this direction rho 2 and along this direction you will have rho 3. And e 1 e 2 e 3 they are the vectors, unit vectors along the corresponding this you can say this is the 1 direction, direction 1 this is direction 2 and this is direction 3 and this 3 they are termed as basis vector these are vector basis vectors.

So, if I ever represent any shape if I use this notation and write this is the e 1 cap e 2 cap and e 3 cap. So, this is itself a vector in the matrix notation and this so, if this basically in the form, but each of them it is a vector at therefore, this is called vector x.

So, instead of using this vector we call this as a vector x. So, this is basically your vector x because, here all the three components itself vector ok. So, along this direction we have e 1 cap along this direction we have e 2 cap along this direction we have e 3 cap. So, this is one of the possible rotation representation.

Student: Sir.

So, we will represent rho tilde we will write as rho 1 rho 2 rho 3 and already e tilde we have written in this way. So, what we can say that we want to get this rho. So, can we write in terms of this two. So, you can see that if we write it as rho 1 rho 2 rho 3 and times here inner product basically e 2 cap e 3 cap. So, this we can write as rho tilde and a and transpose.

So, this is rho tilde transpose where rho tilde we have defined like this so, rho tilde transpose will be rho 1 rho 2 rho 3. So, rho tilde transpose times e tilde where e tilde we have defined like this. So, this is one of the representation and this is same thing can be written as e tilde transpose times rho tilde so, both ways we can write.

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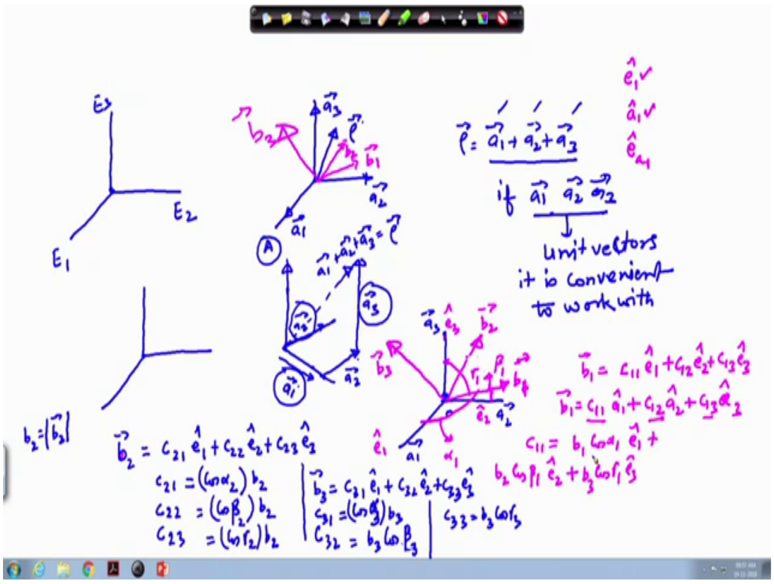

Now, let us consider that this is the frame E 1 E 2 E 3 or either we can call this also as the frame A where we have vector along this direction a 1 a 2 and a 3 and we will compose one vector here say if thing any vector rho which is written in terms of a 1 a 2 and a 3. So, rho is a 1 plus a 2 plus a 3 ok.

So, these are the vectors along this direction and the combination of these 2 these 3 this gives rise to the vector rho. So, if better way you can represent it like this, say along this direction you are measuring if a 1 that in along this direction you are a 2 is the your. So, if the parallel you can take from this place to this place and then go up.

So, this is your a 2 here what you have shown and this is a 3 which you are showing along this direction. And if you join from this place to this place so, this is your vector a 1 plus a 2 plus a 3 which you can write as rho. So, if the any vector because we have 3 dimensional space so, we using this 3 basis vector again they are acting like 3 basis vectors, using this we have written this ok.

So, we can describe in terms of a 1 a 2 a 3, but it is a convenient always to work in terms of the unit vectors here a 1 a 2 a 3 they are not the unit vector what will be defined a 1 a 2 a 3 to be the unit vector. So, of if a 1 a 2 a 3 they are unit vectors it is convenient to work with.

So, now let us say that this is the frame given here a 1 a 2 a 3 and we rotate this frame ok. And so, we are rotating such that the other vectors let us say it comes to this position a 1 comes to here. So, we rename it has b 1 and then a 2 goes here this is we rename as b 2 and a 3 comes here in this direction which we rename as b 3. So, what we have done that we have 3 vectors here, which are mutually orthogonal a 1 a 2 a 3.

And combination of this we have written in this way, now you say that if we instead of doing this if this is a unit vector and this is representing of frame; obviously, if whenever we discuss about the frame. So, we define the 3 mutually perpendicular direction taken together and joint at a point that constitutes your frame. But already we have discussed there is a difference between the coordinate frame and the reference frame. Coordinate frame it is a used for defining the coordinates of any particle, but in the reference frame we if define the equation of motion.

So, if this frame suppose this is the point 2 and we rotate about we rotate this frame about this point o. So, that in the new condition it looks something like this is the rotated frame and rotated frame we are showing by b 1 b 2 and b 3. So, as per the description here any vector rho can be represented in terms of the 3 components.

So, here it indicates that b 1 can be written in terms of the components a 1 a 2 and a 3 or either the vectors along this direction. So, if we have the corresponding vector let us say we write this as c 1 1 times e 1 cap where e 1 cap is the unit vector along this direction e 2 cap along this and e 3 cap along this direction.

So, c 1 2 e 2 cap c 1 3 e 3 cap equally instead of writing it like this we could have written as this is a 1 cap where a 1 cap or e e a 1 cap say the different notations we are using e 1 cap or either I can write as a 1 cap or either e a in the 1 direction so, a 1 cap.

Out of this is seems to be this two seen to be the simplest one. So, if as we have seen that any vector can be represented in terms of the 3 basis vectors. So, e 1 e 2 and e 3 these are the 3 basis vectors along the a 1 a 2 and a 3 direction. So, this constitutes either you write it like this or either the same thing you can write like in this, you should become frequent with this notations because in many places the notations will vary from one author to 1 other author.

And what these quantities are? Just we if you go on the first page. So, there we got this 1 has c 11 we had written as in terms of the direction cosines means this angle. Similarly, the c 1 2 we have written in terms of this angle. So, sorry for c 1 2 we have written in terms of this particular angle ok. So, this angle we have written earlier as alpha this angle has beta and the angle from this place to this place we have written as gamma.

So, you can say that if this is the b 1 vector and b 1 vector magnitude is b 1 ok. So, here you will have c 1 1 equal to b 1 cos alpha 1 now we will take this as alpha 1 beta 1 and this as gamma 1 because, now we have 3 vectors here ok. So, b 1 cos alpha 1 times e 1 cap plus b 2 cos beta 1 times e 2 cap plus b 3 cos gamma 1 times e 3 cap. So, if you can see that what this c 1 1 c 1 2 and 1 3 in this vector they represent.

So, in the same way here b 2 can be expressed in the same way. So, b 2 you will have c 2 1 e 1 cap c 2 2 e 2 cap plus c 2 3 e 3 cap. Where c 2 1 now this represents cos alpha 2 times the vector b 2. So, we are concerned with vector b 2 now. So, magnitude of vector b 2 so, b 2 is magnitude of vector b 2.

So, similarly c 2 2 this will represent cos beta 2 times b 2 and c 2 3 will represent cos gamma 2 times b 2 going along the same line you can also describe vector b 3. So, this will be c 3 1 e 1 cap plus c 3 2 e 2 cap plus c 3 3 e 3 cap where c 3 1 this becomes cos gamma 1 times b 3 c 3 2 this equal to b 3 times cos gamma 2 and c 3 3 b 3 times cos gamma 3. So, if and then the so, alpha beta gamma.

So, here this is we need one correction. So, we do this here the correction this is cos this is cos alpha 3 and this part will be c 3 2 similarly if that will be written in terms of beta angle. So, that will be cos beta 3 and here this is ok. So, we have the describe the whole thing in terms of the direction cosines.

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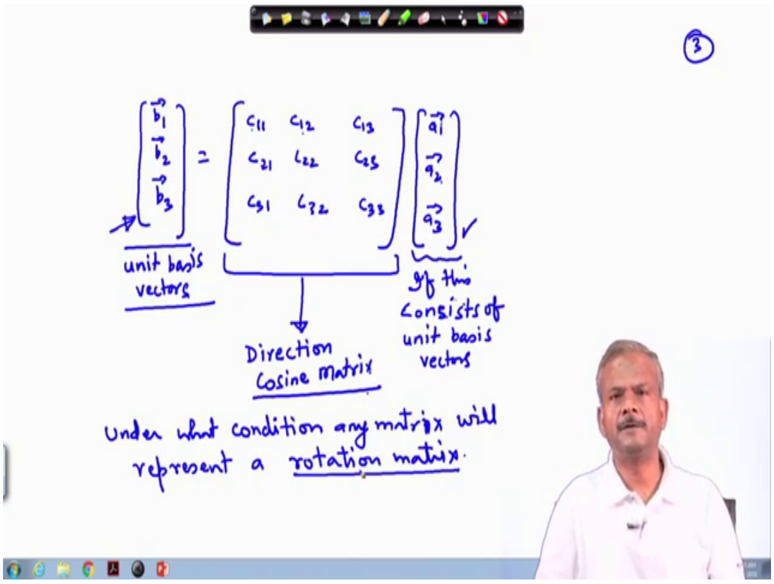

So, what we can write that b 1 b 2 b 3 these are the vectors which can be described in terms of the vectors.

Student: One.

A 1 a 2 and a 3 and can be written as c 1 1 c 1 2 c 1 3 you can look into. So, if we take the product of this so, c 2 1 c 2 2 c 2 3 c 3 2 and c 3 3. So, this you will multiply with this, this one with this, this one with this. So, we will get the first vector similarly for b 2 we will multiply this with this the second term with this one the third term with this one and so on so you get.

So, what we have done exactly that we have once we give rotation to any frame. So, that frame gets rotated to one another position, another angular position and that rotated frame can be represented in terms of this b 1 b 2 and b 3. So, if this is your unit vector if this consists of; if this consists of unit basis vectors you will also get here of the left hand side the unit basis vectors ok. So, this is basically a matrix and on the left hand side also you get a matrix and this of matrix that we have got it here this is called the direction cosine matrix because all the terms appearing here they are function of direction cosines and the cosine angles ok.

So, direction cosine matrix and obviously, we see that this is representing one rotation here ok, but the we have define the rotation in a more precise way so, we look into this topic further. So, what are the conditions under which any matrix will become a rotation matrix we will first we will have to look into that.

So, under what condition any matrix will represent rotation matrix. So, this is our next topic so, for going into this first we have to learn about the orthogonality and ortho normality of a matrix. So, we will continue with the orthogonality and orthonormality of a matrix in the next lecture which thereafter we can look into the how the rotation is perfectly represented and we will continue further thereafter going into the dynamics.

Thank you.