Satellite Attitude Dynamics and Control
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Lecture – 01
Kinematics of Rotation

Welcome to the course on Satellite Attitude Dynamics and Controls. We will go first to the representation of the satellite in the orbit, how it is oriented, how its orientation is represented.

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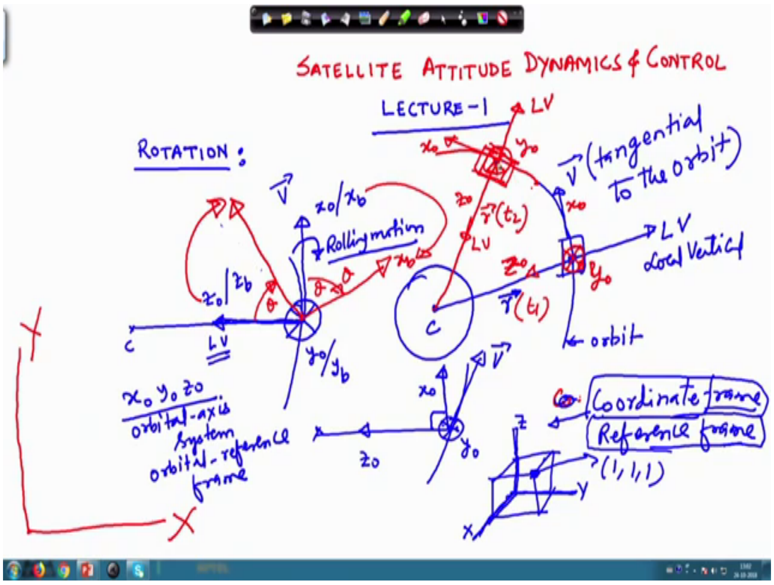

So, let us start here. So, let us consider that we have the earth here; it can be any planet or natural satellite of that planet. And there is a satellite here in the orbit; this is orbit. And satellite velocity vector can be represented as v which is tangential to the orbit ok.

And radius vector of the satellite which is from the centre of the earth, let us say this point is o, this point o here. Radius vector is up to this point to the point o from here to here. And this is the radius vector of the satellite from the centre of the earth to the point o, which we can say centre of mass of the satellite.

So, we are not sure right now we are not discussing about the dynamics of the system rather we are just looking into the orientation of the satellite system, how it is oriented in the orbit. So, it can be a circular orbit, it can be and it can be an elliptical orbit. So, this direction taking it to outward. This generally just called the Local Vertical – L V.

Some of the authors or some of the books may tend to represent the opposite direction that is towards the centre of the earth as a local vertical. So, respective of whether you indicate the local vertical either in this direction or either in this direction, it is. But you should continue with a particular convention that is if you are representing in this direction, you should keep continue with that you cannot keep exchanging in with between that whole then the calculation will be wrong.

So, separately representing here this is a centre of the earth and let us suppose that for simplicity, we have the circular orbit here. And in this direction we will assume that this direction is the local vertical ok. So, this is our local vertical. And we will indicate this as z 0; z 0 means this is about the orbital frame ok.

I will explain you in details about all these things. This direction will indicate as x 0. And y 0 then we can take the right hand rule, because will be working with the right hand side, so taking x to y, this is downward ok. So, downward we indicate it by a cross and at this point we will have y 0. So, we have the coordinates frame here which is x 0, y 0 is downward into the page, and z 0 is going towards the centre of the earth which we are calling as a local vertical.

Now, just as in the case of the aircraft those who are aware of the aircraft notation, so that all motion it is a always taken along the velocity vectors. So, here in the case of the circular orbit, your x 0 and the velocity vector v of the satellite, they will coincide, because in the circular orbit the velocity vector is tangential to the orbit and the also in the elliptical orbit.

But here in that if you take the elliptical orbit, suppose I have the centre of the earth here, and satellite is going inside elliptical orbit ok. So, my velocity vector will be here in this place, but if I take this to be the z 0 direction, then my x 0 direction will be along this direction ok; this is normal to the z 0. And as usual our y 0 will be pointing out pointing in downward. So, here v direction is not along the x 0 direction, but if at this place this as the same sense ok.

And in the aircraft as you know that the role access is taken along the velocity access. So, in the case of the circular orbit it gets simplified and the role access, we can take the right hand rule that if this is the velocity vector, so we can take the right hand rule like this. So, its a rolling like this ok, then we consider this is a positive role ok.

And so here the velocity vector is along this direction and this is a positive role. So, I will indicate on the board, it like this; this is our the rolling motion. So, what I have indicated here, this is called the x 0, y 0, z 0; this is termed as the orbital x, orbital x system or the orbital frame orbital reference frame.

And we assume the body axis, the body axis system to be coinciding with the orbital x system. So, here say in the beginning, I am fixing I have say in that case I have this mobile ok. So, along this particular axis, I fix this one axis along this direction. So, this I call as the x-axis, along this direction I fix the y-axis, as in the case of aircraft; and z-axis downward which is the normal (Refer Time: 07:59) axis in case of the aircraft.

So, following the same notation, so write in the beginning just assume that x 0 and x body axis they are coinciding. Similarly the z orbit and z body axis, they are coinciding; and similarly y orbit and y body axis both are coinciding. Now, once the satellite is rotated. So, right in the beginning, they are all coinciding with the orbital axis. All the three body axis, they are consulting with the orbital axis. Now, if I rotate like this, so obviously the z body x axis, it will no longer remain in the same direction as the z orbital x.

So, in the case of the aircraft as you are taking z to be vertically downward. Now, if I rotate, suppose I fix in the space once line like this, and this we assumed at this was the previous orientation of the z body axis. Now, if I rotate it, the set this aircraft. So, the perpendicular to this or the z-axis of this will rotate from this place to this place. So, this was the original position, now it has gone into this. So, in the case of the if this is satellite, if I assume z to be here and x 0 is already here; and obviously the x and y and z body axis axis they are considering with the orbital axis.

Now, I rotate it by certain angle. So, if I rotate about the y 0 x which is indicated here, which is the y direction of the orbital x system. Now, it will the x-axis will come to this place, which the body axis will get this place from this place to this place. And this is the orbital axis will get, this z body axis will get this place from the z orbital x to this place.

And let us say that this angle we have rotated this is theta and this angle is also theta, this is theta ok. So, the body axis rotation we are representing with respect to this orbital x.

Now, what we will see that as the satellite moves further ok. It goes from this place to this place. So, the centre of mass is located here, radius vector will draw from this place to this place. Now, obviously can see that the satellite orientation has changed if I assume this page, so if I write this as the x and y; and write this as the x in your inertial, this is the y inertial. You will see that the in this reference frame the satellite orientation is changing; right now it is like this, and now it has gone here into this position.

So, the orbit if we connect the centre of the earth and centre of the satellite. So, this is the new radius vector. So, this is a time t 1. So, this will at time t 2. So, in this new position, this will be the local vertical or either inside this will be the local vertical as per our assumption, we are continuing with the local vertical directed inward, velocity vector will change its direction it will be here in this place.

So, by going from one place to another place our orbital x orientation which is here, in this place let us say this is z 0 we have indicated and in this direction, this is the x 0 and vertically downward this is our y 0. So, with respect to this here you can see in this place that the orbital x system itself, it has rotated. So, this is now x 0 and this is z 0, and vertically downward this is our y 0. So, the orbital x system that is also rotating. And with respect to the orbital x system body will also be rotating.

So, there can be issue like either we want to so the satellite orientation in the inertial frame or either with respect to the orbital reference frame. And one thing you remember, there is a difference between the orbital this coordinate frame, there is difference between the coordinate frame and reference frame.

In the coordinate frame usually we describe the coordinates of a system, like I have a body; so if I have a body like this ok, and I want to describe this coordinates of this corner. So, let us say this is x, y and z vertically up and co if this is a unit cube, so this will have coordinate 1, 1, 1. So, this is coordinate frame while the reference frame it from differs from the coordinate frame, in the reference frame we describe the motion of the particle or it may be the motion of a body ok.

So, basically the reference frame is stands for the description with respect of the reference frame, basically we are describing the motion of the body or motion of a particle while with respect to the coordinate frame; we describe the position of a particle. If not necessary, the equation of motion will not enter into that. So, we will meet this distinction that coordinate frame especially, this is stands for only for the description of location of the particle.

While the reference frame, it will stand for the describing the motion of the particle. So, if orientation of the particle, it will here in this case. Orientation of the satellite, it changes from this place to this place; both in the this x y initial frame, and also in the orbital reference frame which is x 0 y 0 and along this direction z 0; similarly here x 0, y 0, z 0. And orbital x system that itself it is a changing its orientation with respect to the senior cell reference frame. So, which reference frame, we are trying to refer to the orientation of the body, it all depends on the our particular interest; what we are trying to do. So, let us go to the next page.

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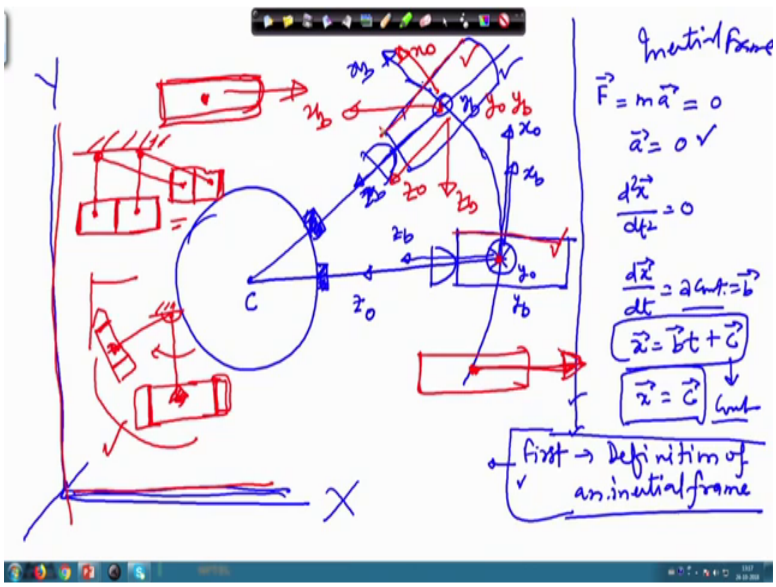

So, if I have worth here in this place. And I have a satellite here, on the satellite there is a transponder or it may be a camera ok. And this direction as usual we have assume this to be z 0, this will take as the x 0 direction ok. And going into the page, this will be described as a y 0. And with respect to this we are describing the body x system.

So, basically in the case of the aircraft, as you may be aware of and those who are not aware of you still, they can get to this notation that if my the body x, what we have represented on the previous page. It rotates with respect to the orbital reference frame. So, the role axis of the body frame that will be always indicated by x that is this x-axis of the body frame that will act as a role axis. While the z-axis will act as the your axis, and y-axis which is act as the pitch axis, this is the most common notation which everybody follow, because if you look in some other book, if you go to certain reference, then you will find that the same notation is followed.

And to maintain a synchronicity between my presentation, and whatever will be given in the book or any other reference material, I will follow this notation throughout the text. So, if you are looking for that your camera of the satellite should always points towards the center of the earth, then what is required that as your satellite moves in the orbit. So, right now it is here, after sometime satellite will go here in this position.

So, to orient the camera along, the new direction of the satellite if the camera is to point always towards the center of the earth, it may be required from the point of view that I have underground station; some object or something kept that I want to scan or send certain signal. So, I need to point my camera or the transponder, all the time here in this direction. So, you can see that my satellite has in the initial reference frame, it has rotated from this place to this place.

So, initial frame as usual we are indicating here in x y direction or whatever the way you choose. So, in the plane this plane you can say, this is my x direction, this is my y direction; and from in this reference frame, this I changed it orientation. And the body axis as we have indicated this is its body axis, and z body axis we have indicated here, and y body axis here.

So, if the camera is maintained along the centre of the earth all the time. So, again your x body axis should be here, z body axis should be here, and x body axis should go here in this direction, and y body axis will be downward. And obviously, your x 0 will also be along the x body x and so on. So, what we observe from this place that depending on our need ok. Orientation with respect to the orbital reference frame or initial reference frame can be given.

So, here in this case with respect to the orbital difference frame, if you see there is no rotation, but if you look into the inertial reference frame, so obviously with respect to the initial reference frame, the satellite orientation is changing. So, for describing the motion of the satellite it is very much required that we properly assign notation of the orbital reference frame, the body reference frame, and the inertial reference frame.

Inertial reference frame as you are aware of this the frame in which Newton's second law is defined, many of the students they are not aware of the this fact, as over a period of time I have learn that, but previously I thought that many people may be aware of. Let us discuss about that also little bit; so that you are aware that what the inertial reference frame is.

Suppose, F equal to m a, and we said this quantity to 0. So, this implies a equal to 0 means d a square x say or d a square r, whatever you are writing by dt a square this equal to 0. So, this implies dx by dt, this is a constant which is nothing but your terms out to be a velocity. And it is a position then let us say this constant you are writing as b. So, this is a vector. So, bt and plus another constant c.

So, the position of the particle it is a changing, if the acceleration is 0. And if suppose b is 0, so then this become c, x equal to c and c is a constant ok. So, this simply implies that particle is remaining in the same place, if there is no initial velocity. And another way this tells that these states about the Newton's first law which tells that until unless a particle is applied or on a particle until unless you exert and external force, it will not change its position; it will try to remain in the same position.

Then the question arises why do we need the first law then if the second law itself (Refer Time: 22:23). So, this is not the thing; the thing is that the second law is defined in the inertial reference frame, so this is defined in the inertial frame. So, second law we are defining in the inertial frame and first law this is the definition of inertial frame; so this defines an inertial frame. Now, what we can see that not only this information, this information is vital, because we always define workout with the Newton's second law in an inertial frame itself and whose definitions given by the first law which defines the initial reference frame.

Now, beside this in the first law also, there is inherent that because of the mass of the system. In the mass gives initial to the system because of that it tries to remain in the same position. The greater the mass, the greater the force will be required to change its position, but books that do not defined it and moreover you should be aware that the Newton's contribution was only the third law. The first two, the first law and the second law, they were already existing in the literature before Newton.

So, in this view we see that in a inertial frame we can describe the orientation of the body or with respect to the this orbital reference frame with we say with respect to the orbital reference frame itself; so we can describe the orientation of the body. So, both coming to this place, here in this place your it may happen that instead of you have the x 0 direction here, z 0 in this direction and y 0 here in this direction ok.

So, instead of this direction your of x b and y b, your x b may be looking something like this, this the x b, and this is the z b, and y b remains along the same direction; means your with respect to the orbital reference frame, your satellite has changed its orientation. This is a very simple representation here that we have rotated about the y orbital axis here in this case. So, rotating about the y orbital axis, the body orientation has changed to x b, y b and z b.

And y b is directly going inside. So, the rotation about the y b axis. In general this rotation may be much more complicated, and rotation can take place also about the x bx, z bx and y bx all the three axis rotation can take place. So, I have a body here. And this is the x body axis, this is the y body axis and downward this is the z body axis.

So, I can rotate a body about the z-axis, I can rotate the body about the x-axis ok, I can rotate the body about its y-axis. So, as I am rotating, so then body is also, this z body you see here in this case if I am rotating about the y body x; so this body axis also going up. So, we have to become convergent with this notation of the difference different reference frame with respect to which we are going to describe the equation of motion.

So, if coming to our conclusion, in this why do we require different reference frame, the first one is that we need to orient the satellite in the space itself. It may be required at safe; here it may be case that in this initial reference frame, your satellite should always point along this direction, it should be always like this ok. It is coming from this place to this place in the orbit, so it should go like this. And this may be case here, this may be case related to say the Hubble telescope, you are pointing towards particular star. So, star at many light years away from us.

So, almost the light coming from the star towards it will assume to be parallel. Basically in that case you have your satellite here, and you have your satellite here, it is a pointing towards a star all the time. So, in this case the your satellite is undergoing purely a rotational motion in the inertial reference, purely translation motion in inertial reference frame that you can check from this place that if I have say a pin here, pin here and two roads are hanging, and if I attach a body here, and there is also a pin ok.

In if I push it, so this will get post only in this way ok. So, it will come from this place to this place like this. If this is purely at translation motion, on the other hand if I have a body like this. It is attached here, here it can rotate, but here it is a fixed ok, there is no rotation involved here; so here I am fixing. So, as you rotate about this, suppose along this direction. So, it goes from this place to this place and it looks like this. So, you can see that here in this case, this line this two lines which we are like this now, they have changed here its orientation.

While in this case, if I have one line here or this line this three lines; so they are not changing the orientation they are remaining in the towards the same direction ok. So, this is purely translational motion while this is purely rotational motion. So, there can be a combination of rotation and translation which happens in the case of the satellite in the case of the aircraft.

So, here if you are pointing it initially, then it is a purely translational motion while you are trying to point about the center of the earth; then there is a rotation involved as you can see that this is rotating from this place to this place, it is a orientation has changed. And also the translation motion is involved, because this coordinates and this coordinate in the inertial frame they are different. So, the translation has taken place from this place to this place and also the rotation has taken place where this line, it has changed its orientation to this line.

So, we conclude this part introduction, introductory chapter which is related to the representation of orientation of the satellite in the orbit. And next time, in the next class, in the next lecture, we will go with how to present the orientation of the satellite in the orbit that we will learn. And slowly, slowly we will build up and once we are aware of the orientation of the aware of orientation, how to orient represent the orientation of any object in a space or with respect to the any coordinate frame or with respect to the any reference frame, then of course, it is a rotational motion also we can describe.

So, without considering the torque acting on the system or the force acting system, once we describe the motion of the particle of a rigid body, it is a call the kinematics. And beyond that we go, so where the forcing involved so that part we call as the dynamics, where the force or the torque is involve. So, first we will look into the rotation of the body, then we will go to the kinematics of rotation. And thereafter will start with the dynamics which is our final objective learning, how to manage the satellite in the space, how to orient it, how to control it, so that it can serve my purpose by say pointing towards the certain star, the Howell telescope, so that is one objective, it should do in a prescribed manner.

So, milli of arcsecond or the arcsecond accuracy if you require, so it is very precise control let say you have to do. And for those kind of precise control unit, the precise controller also. Precise controller means the controller suppose on the earth you put a small jet, on the x surface, so tangentially to the surface, so it can rotate the earth. But earth is such heavy that it will, its mass is so big that even if you have apply substantial force through that jet, it will hardly it affect the earth. So, for depends on the size of the body, how much accuracy you require and accordingly you have to choose the propulsive system or the torquing system which will meet your objective that you are planning for certain mission.

Thank you.