Introduction to Aerodynamics
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Lecture No. # 08
Kinematics of Fluid Motion (Contd.)

So, this since this derivative is computed while following a particular material.

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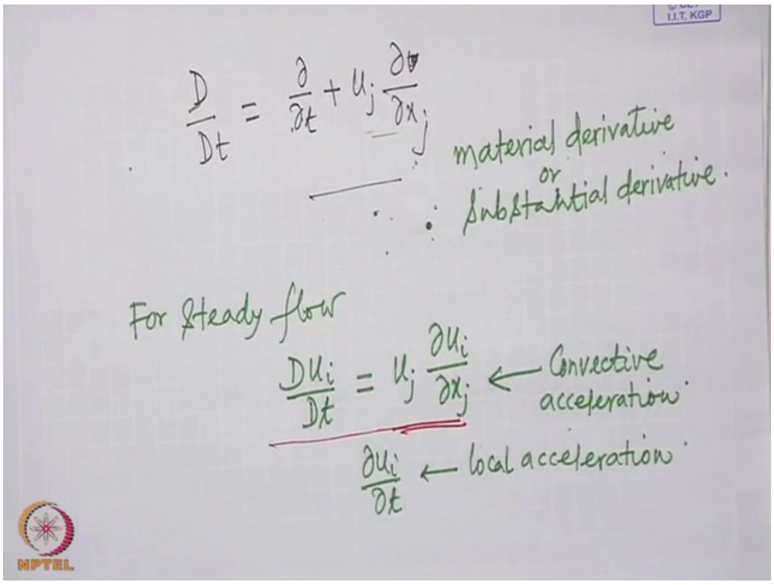

This material derivative D Dt or which is d d x j is called material derivative or substantial derivative, and as you will see that this will appear are almost in all equations in fluid mechanics this substantial derivative. That is of course, because of our consideration of the Eulerian concept. Because, most often we will be we are interested in properties of the material flow properties of the material element and this will come. And remember this derivative is not applicable to velocity alone it can be applied to any field variable. So, it can be applied to density, pressure, temperature with all flow variables in Eulerian description is a field variable. So, for all flow properties are all variables this substantial or material derivative may come. Now looks something special think the flow is steady we have already defined the flow is steady when u is independent of t.

Now what will happen? Then the first term is zero but this it is not zero, this is not zero. So, steady flow a steady fluid flow does not mean that it has no acceleration. You see but that is that would be little difficult to explain in lagrangian description. So, we can write for steady flow for steady flow acceleration is not zero, acceleration of fluid element is non-zero. This first term which refers to change in velocity at time t at position at the same position, the first term represents the change in velocity with time at a fixed position. This part is usually called the local acceleration; it is called the local acceleration. And the second part which is a change in velocity due to the change in particle position from one special point to another special position.

So, this due to a real transfer of material from one position to another and called convective acceleration. And in general these are called local change this is called convective change, when it is change in velocity of course, it is local acceleration and convective acceleration, but as I mentioned that this change may be for any other variable. Let us see if it is a change in density we are talking about this way then of course, this will be local change in density and the other will be convective change in density. So, this is local change and this part is we will call it convective acceleration and this part rho delta (( ))

Student:Rho.

Delta

Student: (( ))

Little louder

Student: Rho delta x delta y delta z

Rho delta x

Student: Delta y delta z

Is that the mass flow that is entering?

Student: Rho delta y delta y delta z

Rho delta y delta z

Student: U delta t u t (( ))

No per unit time let us forget about so rho U

In this case the component which is parallel to delta x rho u delta y delta z that is the mass flow entering through this face p q r s. Now what is the mass flow? let us say that is going out through this opposite face, the opposite face, this face.

Student: (( ))

Delta y delta z is alright the area of that face is still remains delta y delta z. But what about this quantity rho u? Again it is rho u see this rho u is changing over this length delta x so it should be rho u plus delta rho u, some change in rho u and to the first order of accuracy we can write that to be rho u plus d d x of rho u into delta x.

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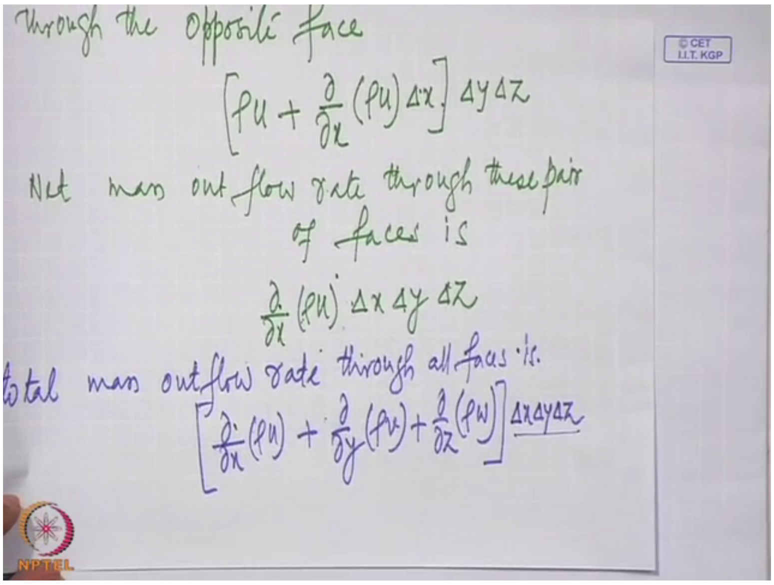

So, through the opposite face. Now for this pair of face the net mass flow, there is a difference between the two, the net mass, what should you call? Outflow So, net mass outflow. Now what is the mass of the fluid contained within the controlled volume?

Student: (( ))

Rho

D v integrated over v if you for the entire control volume. And see because of this mass outflow rate this mass will also change, depending upon if whether the if the inflow is more than the outflow then there will be increase in mass within the control volume. Similarly, if there is outflow is more that the inflow then within the controlled volume mass will decrease. So, what is the rate of change of mass within the control volume so mass fluid mass contained in the…Will be later on seeing that this is one of the important governing equation of our fluid dynamics, this is the mass conservation equation. However in fluid mechanics see it has a very popular name and people often call it by that name which is continuity equation. So, this is we will call it mass conservation equation which is really the correct name.

Now can you say what does this diversion of u mean? What is a diversion of u? Let us say to look to this diversions of u consider a material element and think about a small element only a small material element. And to differentiate the volume with v because v we are fixing the volume as a just a volume in space. So, we will differentiate it with respect with material volume material volume is just a volume found by a certain material this may occupy different position in space. Let us call this material volume element so consider a material volume element v, let us call this tau material volume element and it is bounded by a material surface and again instead of calling it a we will call it another name bounded by the material.

Now let us say that the material surface is moving either inward or outward or is deforming. We can sat the material surface is being deformed, what will happen as a consequence? If the material surface think about just a material, this is not no longer just a volume it is a material volume made up of only certain fluid elements, its volume will change if its surface moves outward or inward or if the surface is deforms then its volume will change. So, you can then write that d tau d t is (( )) How much the volume is move? How much the total surface is moving? And that is again which again by divergence theorem, why rho there is no question of rho? we are considering the movement of the surface so there is the surface velocity is the only thing that is required. So, the rate of volume change is not d v d tau.

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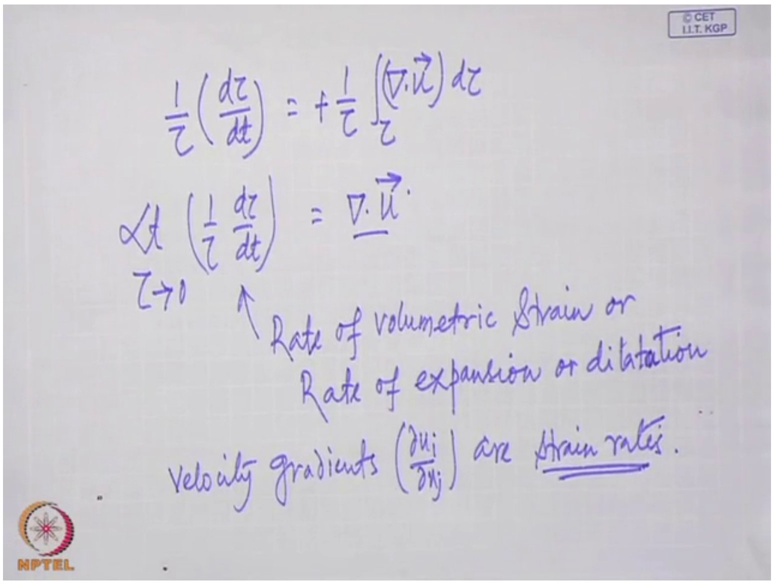

Now take this rate of change of volume per unit volume and will this minus be here it will be plus because

Student: (( ))

If the surface moves outward its volume will increase so this is no minus plus, this is rate of change of material volume divided by the volume or rate of change of volume per unit volume. If we consider this element to be very small infinitesimal element that is approaching this to be zero then this becomes this. Now what is the left hand side? This is very familiar to you or let us says, what is rate of change of volume per unit volume or change in volume per unit volume? That is not compressibility change in volume per its value change in length per unit length.

Student: (( ))

Strain change in length per unit length is strain so change in volume per unit volume is

Student: (( ))

Volumetric strain and this of course, not strain only it is rate also there is rate involved here it is not simply delta t delta tau by tau it is d, d t so this is the left hand side is rate of volumetric strain which is usually called as a rate of expansion or rate of dilatation. So, this side is rate of volumetric strain or expansion or rate of dilatation so that is what is the meaning of diversions u.

Student: (( ))

That is rate that is what a instead of calling time rate time rate it is call simply rate. Because at whenever you say strain that the special rate is already included in strain is always increase in length per unit length or increase in volume per unit volume. So, the special rate is always included in it so when you in addition say rate it is the time rate that is what you need and you can see also that this velocity has velocity can also be called as rate of strain or rather velocity gradient, velocity gradient can be called up strain rate or rate of strain.

This we can very easily see if you consider a small say as an example a small line element line element extending from point p two q again, where at p u the velocity is u at q the velocity is u plus delta u or u plus d u d x into delta x delta u is d u d x into delta x. And then you can see that over a small time interval, this element will elongate and the rate of strain will be d u d x. You can consider a small time element delta t so that the point p is moving from u to another location given by u into delta t the second point q is moving to an distance u plus delta u u plus d u d x delta x into delta t.

So, you see that the two points of the same element is moving by different distance that means the element has extended or stretched. So, how much the two points have moved if you take the difference of those that you will get how much is the total strain and from there you can again find that the d u d x is strain rate.

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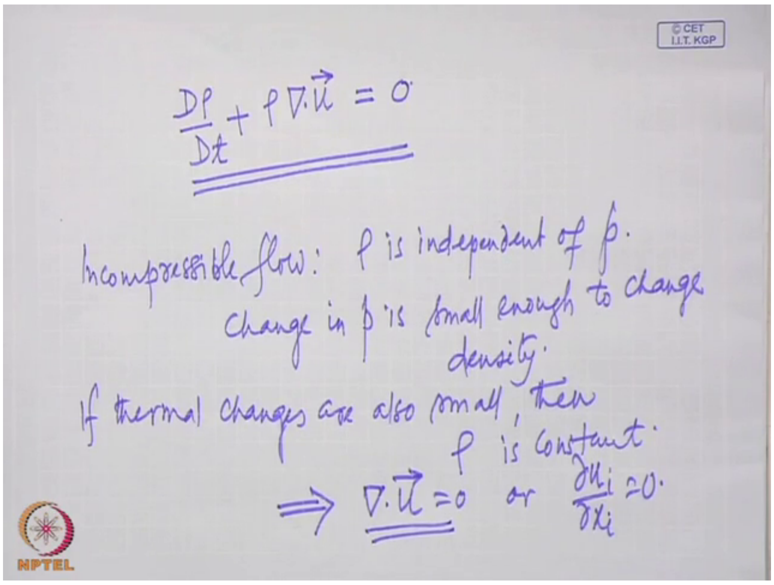

So, now let us think about the equation in this form. The continuity equation or the mass conservation equation in this form, we will define some flows to be incompressible where density is independent of pressure. Remember this very clearly density is not constant it says the density is independent of pressure, an incompressible flow is defined when density is independent of pressure not the density is constant. A constant density is not an essential requirement of incompressible flow; density may change in incompressible flow.

So, the two most common mechanism of a region of change in density one is of course, by pressure, the other is by thermal region thermal conduction. That also can cause change in density even when there is no change in pressure, but there is considerable thermal conduction then also their density can change which is quiet well known to you which is almost every time. You are experiencing that change in density due to thermal effect. This is not excluded in the definition of incompressible flow, change in density by thermal region is allowed in incompressible flow. I know whatever i say still many of you will continue to keep on saying that incompressible flow means density constant, but do you precise it is not show incompressible flow needs that density is independent of pressure.

Now you know that at least if you are a concerned with gas that for gas at least it is not true the gas density is not independent of pressure. But we will see that under certain circumstances which are not that very uncommon. The change in pressure is so small that the change in density caused by that pressure change is really negligible, that if the density changes are really very small compare to the actual absolute pressure. The pressure changes are so small compare to that absolute pressure. Then the density change that will occur due to that small change in pressure is practically negligible.

That situation comes quiet often, and in such a situation even for gas we will see it later that density is all most independent of pressure. So, we can further elaborate our definition of incompressible flow that is the incompressible flows are those flows where pressure changes are so small that the consequent change in density is negligible. And that is even in case of flow in flow of gas is not really uncommon, there are many situations where changes in pressures are really small.

Now in addition of course, if the thermal conductions are also very small there is not much of temperature difference even then the density change due to conduction thermal conduction is also negligible. If that happens that pressure changes are also very small and the thermal changes are also very small then of course, density can be assumed to be constant. And what will happen in this that case? Let us say that we have an incompressible flow in which even thermal changes are also negligible. Look to this continuity equation what will happen to it then?

And what will happen then look to the continuity equation the continuity equation then gives what?

Student: (( ))

Divergence of u is zero, or and what does it mean? Diversions of u already you have seen is rate of expansion rate of expansion or rate of volume change volumetric strain rate. So, an incompressible flow means that rate of expansion or rate of volumetric strain is zero. And you see that is the meaning of the word incompressible something which cannot be compressed rate of volume changes moles zero. You know this any vector which satisfies this relation, that is divergence of that vector is zero, those vectors are called solenoidal any vector whose divergence is zero is called a solenoidal vector.

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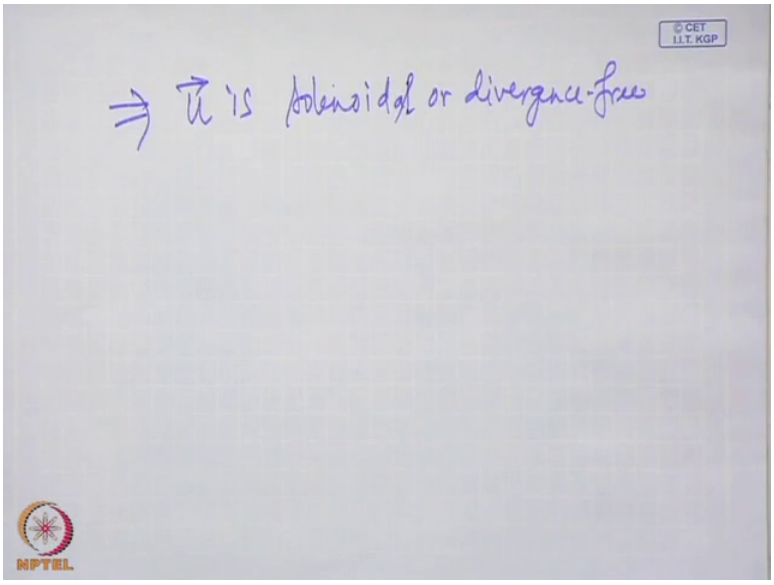

So, for an incompressible flow the velocity vector is solenoidal or divergence free solenoidal means diversions free. So, you see that incompressible flow imposes a certain restriction on the velocity field that any field, if field means basically you can say of that is the mathematical function, function of special position and time. But any function that you can choose cannot be a possible candidate for the velocity field, only those which are solenoidal which satisfies this relation divergence of u equal to zero can be an possible candidate for description of the velocity field.

So, not any field can qualify as velocity field for a constant density flow. For a constant density flow, the velocity field has to be solenoidal or divergence free. So, it is basically a restriction imposed on the possible solution that will come. We forgot to define something called a stream tube when we define a streamlines, it is another concept. A stream tube thinks about a closed curve. A closed curve is made up of infinite number of point through each point we can have a streamline. Now if we form a closed curve, if we think about a closed curve and then through each point we have the streamlines, then it founds a tube, all those lines together they forms a tube this is what is called stream tube.

A tube made up of streamlines that means if we draw the streamlines passing through a all the points of a closed curve, then the resulting tube that is our this stream tube. This divergence free velocity field needs that the stream tube cannot end anywhere either at the boundary it can end only at the boundary or it can make a closed curve or it can extend up to infinity.

There is the boundary if there is no other boundary then infinity. That is also a meaning of this divergence free condition divergence of u equal to zero. That a stream tube cannot end within the fluid it has to end either at the boundary of the fluid which may be infinity or it can end in itself.