Introduction to Aerodynamics

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Lecture No. # 46

Boundary-Layer Theory (Contd.)

We have already derived, the boundary layer equation, both in differential as well as in integral form. Without going to a formal solution of these equations, either in the differential form or integral form, we consider a special case with the assumption that the solution is given to us. The solution of these equations that, as i mentioned earlier, that we would like to postpone it for the other courses. But what we would like to consider today, a situation where a solution is given, that may not be a realistic solution, but as an example solution.

For a simple case, where there is no pressure gradient, that is; there is no stream wise pressure gradient. In the inviscid flow, the pressure in the x direction is constant, pressure of course, within the boundary layer is constant; that is always, but here we are also assuming that, pressure in the x direction or the stream wise direction is also constant, this dp dx is zero. We have seen, what the form of the equation becomes in that situation, only the term dp dx become zero.

See, this is the usual situation for a flat plate type of flow, for a flat plate; as you know that if we put a flat plate in an uniform stream, then over the flat plate in an inviscid flow, there will be no change in anything and the pressure will remain constant. The velocity will not change, neither will the pressure. So this is a situation, where there is no acceleration in the inviscid stream; no deceleration, uniform velocity of the inviscid flow, so that is the situation we would like to consider. And in that, let us say the velocity profile is given, also it is stated that the boundary layer is similar, velocity profile is independent of x; that is at all x location, the boundary layer velocity profile can be expressed in such a manner, that it is same at all x location. So, this; that is called a similar boundary layer.

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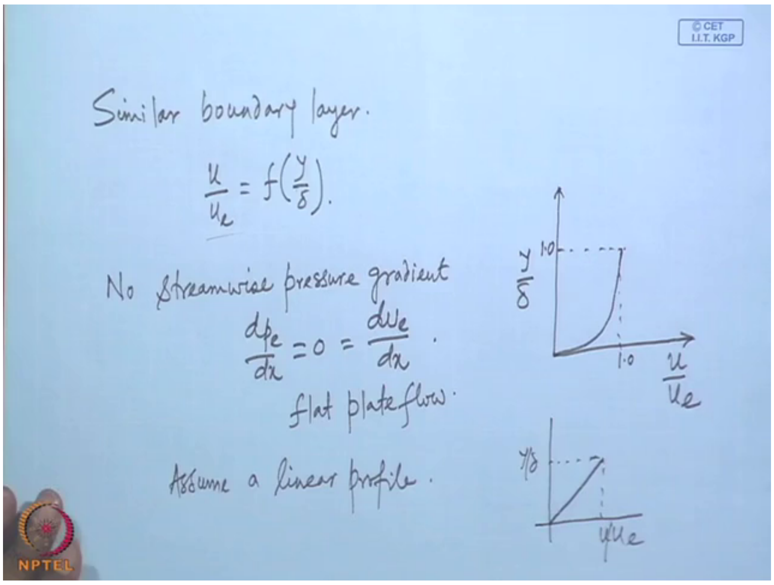

We can express in special form, where the velocity profile will behave as if it is independent of x. At all x station, it is the same velocity profile. You can have just only one velocity profile curve. As you can see here that; see this is, if we plot, u by u v along the x axis and say, y by delta along the y axis, then at any x location, we will have the same curve. That is; we can say that, u by u e is a function of y by delta alone.… So, now say that, we have given that; we have a flat plate type of flow, no pressure gradient and the boundary layer is similar. So, no stream wise pressure gradient.

Otherwise what it is 2; 2 x into under root 3 by R x 3 by R x. Or 2 root 3 x; 2 by 3 is complete then under root 3 by y,

No we will put it like this 2 root 3 x; root R e

Student: (( ))

R e,

This, let us see take for granted. Now once you have found delta, we can find the other parameters all, once you have found delta; we can find now, what is delta star? What is theta? And what is even C f?…

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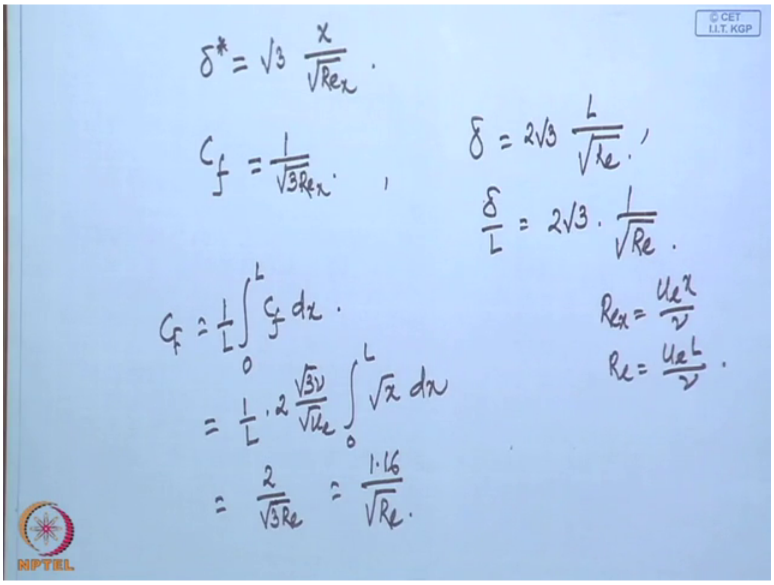

Delta star you can write, delta star is delta by 2. So if we take yours is correct, then this is… what is C f?... What is C f?

On the edge of the boundary layer

Student: (( ))

On the edge of the boundary layer du dy is zero, because in the inviscid flow, the u is no longer changing with y. So, all the derivatives are zero there, that is, is known. du dy; derivative of du dy is zero, when does it occur. If the function is not a constant, du dy is not a constant that you have seen. It might be a function of x; it might be a function of x. We are not considering any y x variation here, we are just simply differentiating with respect to y. At y equal to zero,

Student: (( ))

At y equal to zero, du dy equal to zero, will be zero. You want to say that, that du dy is zero everywhere or the velocity is constant in the boundary layer. That is of course, against the concept of boundary layer, it is not zero. It would have been a fantastic situation, if the du dy on the; at y equal to zero, would have been zero, then there would have been no skin friction drag. You know and your aircraft could have flow without any fuel. So, du dy on the wall is not zero, (( )) the derivative of a function zero, if the function is not constant.

Student: (( ))

Is maximum or minimum? So du dy will have highest value, at y equal to zero; largest value and definitely it is positive; and definitely it is positive. Because u is zero on the wall and then it is increasing. So definitely du dy is positive at y equal to zero in this case. And we can see that, du dy; starting from a positive value at y equal to zero, it decreases smoothly or continuously within the boundary layer.

(( )) the maximum double derivative is negative (( )). So, it is not exactly derivative, because the function is not defined after that. For y equal to negative, after that the function is not defined, so we cannot do that; is not maximum in that proper stance, that it will have the largest value.

So du dy, the variation of du dy will be something like this,… smoothly decreasing as y increasing… Now consider a pressure variation, where this dp dx is not zero. Let us first consider dp e dx is negative; dp e dx is negative. When dp e dx is negative, what we will call; the flow is accelerating or decelerating? If dp e dx is negative? Accelerating. Accelerating, because we have already seen dp e dx is minus u v dv dx.

But what about, d 2 u dy 2? What in this case, will there be any variation from the earlier case.… Remember simultaneously that, u is also increasing from zero to u e, that point also you must remember, while thinking about the gradient, that u will increase from zero to u e, from y to delta,

u is increasing from zero to delta. Since u is increasing from zero to something at the wall, so du dy, there is positive; du dy on the wall is always positive, not always say it is positive, when u is increasing. But, then d 2 u d y 2 is negative.

Student: (( ))

It will also be similar to the earlier one, similar to the zero pressure gradient case, almost as far as this trend is concerned. And in either cases, there will be no inflection point, in the velocity profile. A decreasing function in both cases; and in both cases, there is inflection point in the velocity profile.

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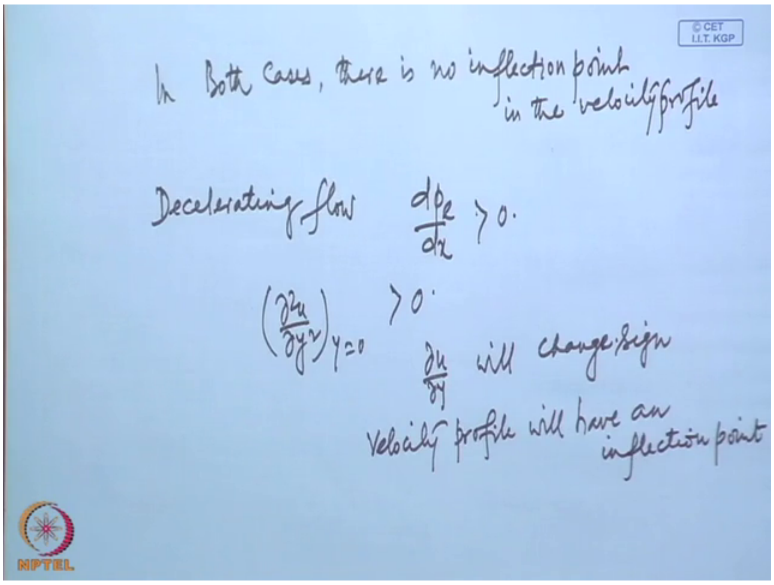

Consider now the other case, the decelerating flow. This time there will be an increasing, but then it has to decrease again.… So then, what else, du dy will; du dy will change sign or what does it mean, the velocity profile will have an inflection point; the velocity profile will have an inflection point.

Now then, what really happen in cases is that, a situation comes where du dy, as you can see that du dy on the wall in this case will be much smaller compared to the other cases, not only that as you move downstream, a situation will come, where du dy on the wall will become zero; a du dy on the wall will become zero. And subsequently, du dy near the wall will become negative. And what does it mean, that actually in that part, the flow is not from the direction that we considered earlier or that we were analyzing earlier rather it is from the other direction, that, that there is some backflow or the original main flow that has separated from the body; that has separated from the body, like this.

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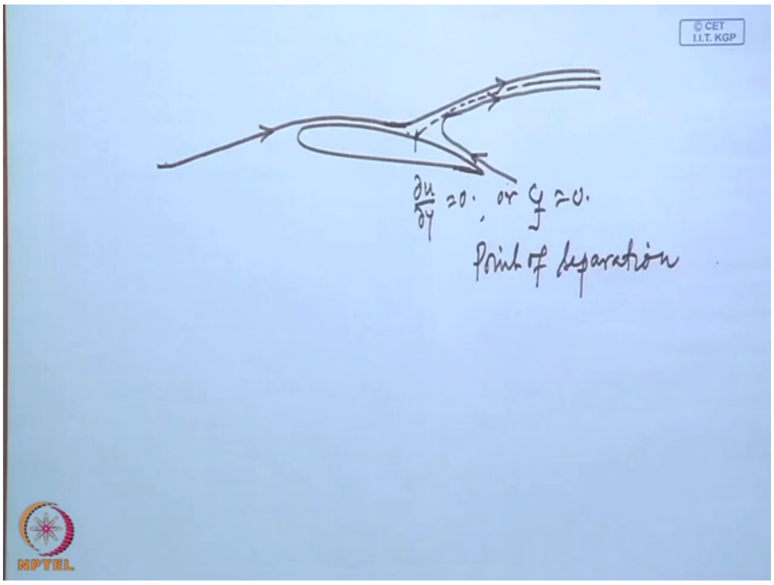

Let us say the flow is coming like this and after this, instead of going like this, it has separated and flow from this side has come like this…. This is the point, where that say du dy equal to zero or it means, the skin friction coefficient is zero.

Now in an airfoil type of geometry or a streamlined geometry, this separation occurs at certain angle of attack, not from; not at zero degree or one degree at that type of angle of attack, but rather of the order of 15 degree angle of attack, this separation starts. whenever the; when this separation occurs, the airfoil experiences a loss of lift; the lift coefficient started decreasing.

And if the separation is, say further here; somewhere ahead, then the lift will become so small, that aircraft may not be able to fly rather fall down like a stone, known as stall, which is because of this boundary layer separation.

However, as you will be knowing, that see immediately that, whenever there is a positive pressure gradient does not mean that the flow has to separate. As you said that, a positive pressure gradient, which is known as adverse pressure gradient, then the pressure is increasing downstream. That point of inflection is obvious, it it will be there, but later on you will see or you learn that, if the velocity profile has a point of inflection, then the flow is highly unstable. You might have heard of something called as stability, instability. The mathematical equations which is represent; which represents some physical system, they becomes unstable, anyway we will not discuss that.

But, if there is appoint of inflection in the velocity profile, that then that flow is likely to be unstable and what actually happens, that before this separation occurs, the flow changes its status. What essentially we are discussing are called laminar flow, where layer slides over layer, where fluid layer slides over fluid layer, just type of smooth regular flow, instead it becomes something called turbulent. And a turbulent flow can withstand this adverse pressure gradient much better than a laminar flow. So when the flow become turbulent, then the separation perhaps will not occur, where it was supposed to occur in a laminar flow, rather it will move downstream.

See, this process of separation can also be very easily understand. See, when the pressure is increasing downstream, the flow is moving against that raising pressure, it is like think of yourself. You are trying to climb a hill; you are trying to climb a hill, as you go up, upper you feel tired and if the hill is too high, you might not have the sufficient energy to go to the top. Similarly, as the flow moves against this raising pressure, it consumes its energy; the kinetic energy, it uses the kinetic energy to overcome that raising pressure. So, if the kinetic energy is not sufficient, it will not be able to move against that adverse pressure, after some distance it will lose all its kinetic energy and then it will be blown away, from, flow from the other direction is being blown away.