Introduction to Aerodynamics

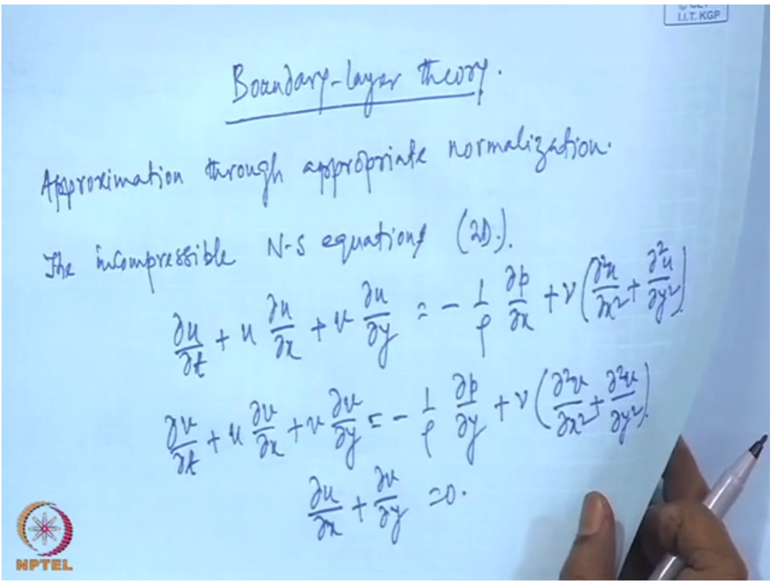
Prof. K. P. Sinhamahapatra
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 44

Boundary-Layer Theory (Contd.)

Last, we have discussed about the development of boundary layer, how a boundary layer flow develops from the start of the motion. And we have also seen that the distance to within, which the viscous effect. That is the diffusion of vorticity or velocity occurs; is of the order of square root of or inverse square root of Reynolds number. And on that basis, we have made some order of magnitude analysis and derived approximate form of the boundary layer equations or what is known as the boundary layer approximations; the equations that are valid for boundary layer. We would like to see the same equations again, through normalization; we would like to normalize the equations properly.

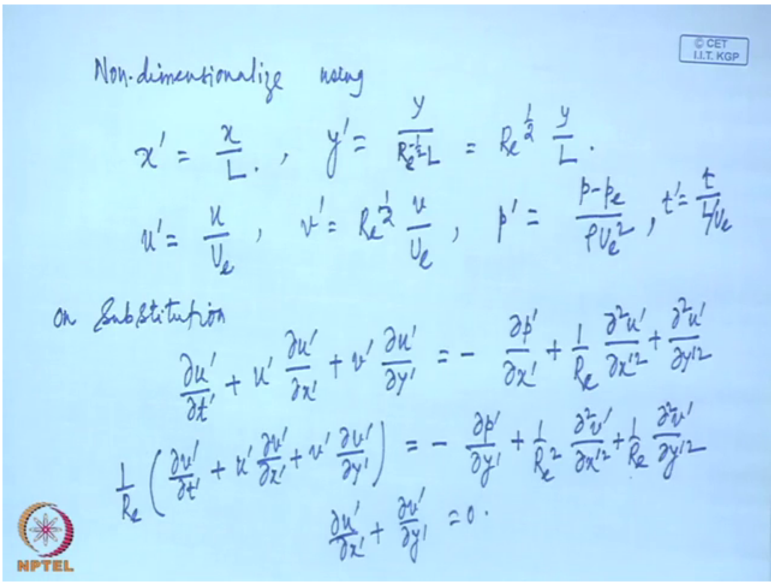
(Refer Slide Time: 01:45)



So, let us have the navies stokes equations, we will once again derive the equations so, this is actually not nothing new, whatever we have done yesterday, we will be doing the same thing. But, using a little different approach, we will make this approximation through normalization. We have the incompressible navies stokes equations which can be written as, the x component of momentum equation which is d u d t plus, the y component of momentum equation and the mass conservation or volume conservation equation or continuity equation.

Now we would like to normalize or non- dimensionalize these equations, now once again, we again earlier once earlier we non- dimensionalize these equations, where we used some characteristic length and characterise each of these term. If you remember for velocity, we used u infinity and for x y z coordinate length, we used l. But, in discussing boundary layer equations, we have seen that, even though the characteristic length in the stream wise direction is l; the characteristic length in the normal direction is not really l. It is of the order of inverse square root of Reynolds number. So, this time, we will use appropriate length scale and velocity scale to normalize these equations.

(Refer Slide Time: 06:04)



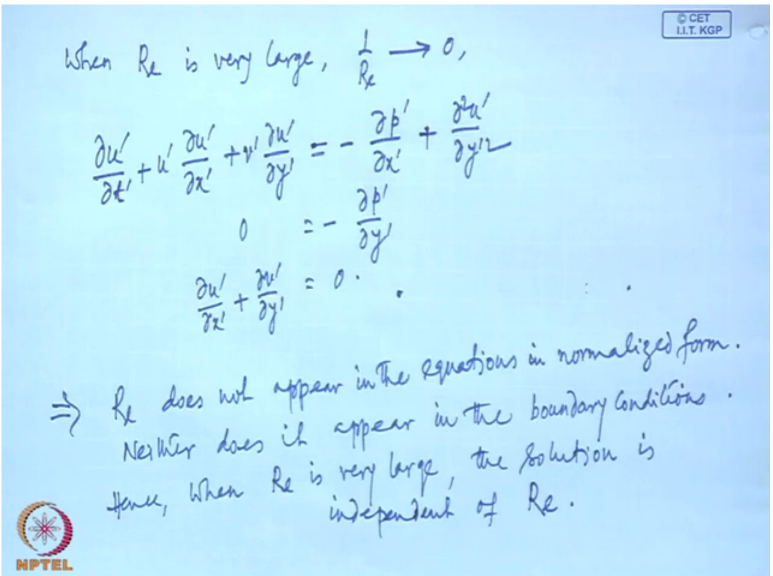
And we normalize or non- dimensionalize these equation as follows, say the non dimensional x prime is as usual x by L; where L is the reference length in the stream wise direction. y prime is not y by L but, i to the power minus half into L. Because the length in the normal direction or the thickness of that boundary layer is, of the order of inverse square root of Reynolds number, so that we that is y by delta, where the delta is the boundary layer thickness of the order of the boundary layer thickness. u bar will make it, U by U e where, as before that any parameter with a subscript e means, that it is the value at the edge of the boundary layer or it is the inviscid flow solution.

The inviscid flow solution just like y prime and for pressure, we normalize following this and t of course, you can prime of course, you can think that t by L by U e. Now if we substitute these parameters, what happens to the equations? If we substitute these in the equations, the navies stoke equations which we have written it here.

We substitute this non dimensional parameter here, what we will be getting is. The x component of momentum equation in this normalized form remains almost the same, only one by R e comes here and the other terms have no coefficient. The y component of momentum equation comes as and the continuity came, remain as it is.

Now see if we assume that Reynolds number is very large, so that one by R e approaches zero and in that situation, all the derivatives are finite then, we can see that from here. We can neglect all these terms in this equation, this term and in this equation all these terms; except this the pressure.

(Refer Slide Time: 14:15)



So, what do you get one thing is clear from here. That the equation for very large Reynolds number, do not contain the parameter Reynolds number. There is no Reynolds number in the equations. See it is obvious that the solution, at least in this form solution of these equations where your parameters are u prime, b prime, p prime will not contain Reynolds number. Because in the Reynolds number, do not appear in the equations, neither do they appear in the boundary conditions. So in the limit of very large Reynolds number, the Reynolds numbers do not appear in the equations in the non dimensional form, neither do they appear in the boundary condition.

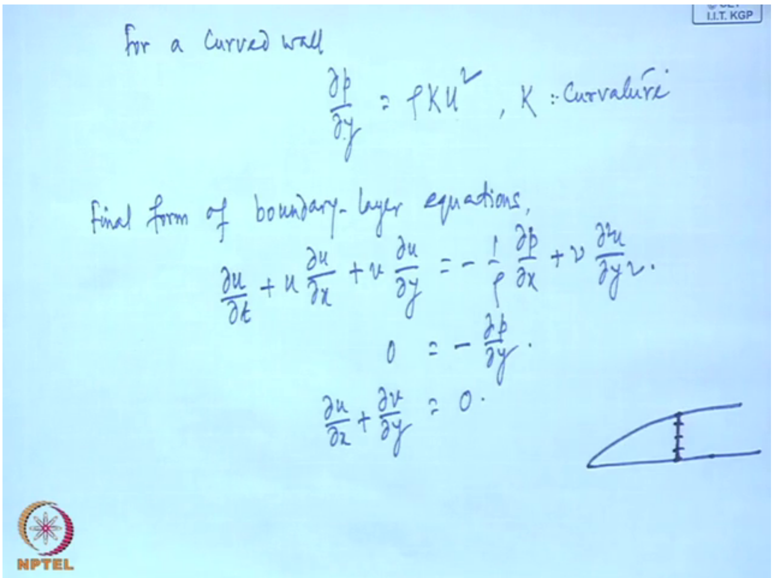
The boundary condition is simple that on the solid wall, there is no slip at the edge of the boundary layer. The two solutions will merge smoothly and one more thing, that is also required to solve these equations. That is at some x location, at some location- x location, you have to provide the solution. For a steady flow case, at some location that is at some x, you have to satisfy give the solution, then you can get the solution for x greater than that value. If your solution is given at say, x equal to x naught, then you can get the solution for x greater than x naught. That is because of the nature of the boundary layer equations, the boundary layer equations in this form or the form that we had let us say, they are parabolic in nature. So you have to give an initial solution.

So, if we consider steady flow where there is only x and y; some x location is an initial condition. At some x location, you have to give the solution only. Then you can find the solution not so at low Reynolds number, but at very high Reynolds number, the solutions do not contain any reference to Reynolds number. And even if the Reynolds number changes something, the solution will remain same meaning. Let say as an example, this may not be large but, usually what you find is quiet large enough. For airfoil type of problems, whether your Reynolds number is say ten to the power seven or ten to the power eight or ten to the power nine; the solution is more or less same.

But, if it is ten to the power four or ten to the power five or ten the power six in that range, it changes or even smaller than that, it changes. But, when the Reynolds numbers reach to that limit, the solutions do not change. These equations you have considered or derived considering that the wall is straight for three dimensional case, you can extend the equation very easily in the same form; you can do it yourself without any difficulty.

For curved wall case that normal momentum equation that is d p d y equal to zero that needs a little modification. When the wall curvature is considerable, not small curvature like the curvature that an air foil has, for that type of curvature is not required. But, if the wall has very large curvature, then that equation needs to be changed and the approximate form of that equation will become …

(Refer Slide Time: 21:51)



If the curvature is small enough then, this is not required. You can set straight, you make it zero; d p d y equal to zero. But if the curvature is large, then one need to use this equation, replace this d p d y equal to zero.

Now, so the final form of the boundary layer equations. You are again using that the dimensional form, not the non- dimensional form. So these are the boundary layer equation, if you look to the equations compare to the full navies stokes equations, they are definitely much simpler, compare to the original equations, the original navies stokes equations when you compare to them, these equations are definitely simpler. However, if you compare to the inviscid flow case, where the only equation is Laplace phi equal to zero. Obviously compare to that, these are difficult.

We now have three separate equations, remember we cannot use velocity potential here. This is a rotational flow; the boundary layer flow is rotational flow. So phi or the velocity potential is not defined.

Not only that, within the boundary layer, you cannot even use you are so called Bernoulli's equation; that p plus half rho u square equal to constant; that cannot be used within the boundary layer. You can use it at the edge of the boundary layer and outside the boundary layer, but not within the boundary layer. Within the boundary layer which is rotational flow, so if you want to use p plus half rho u square that constant will change from streamline to streamline. Can you say that what will happen to the total pressure within the boundary layer?

Within the boundary layer, what will happen to the total pressure? We already have d p d y equal to zero meaning the static pressure, p is remaining constant within the boundary layer, as you move from the edge of the boundary layer to the wall.

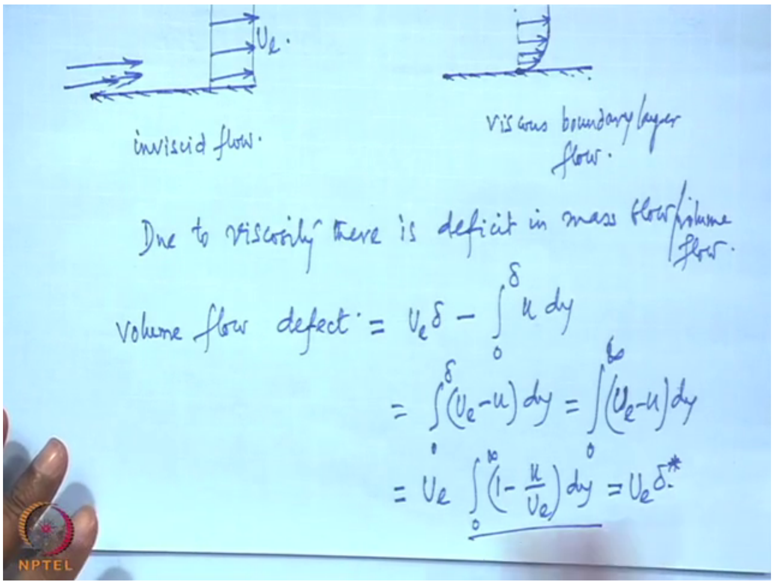
Considering a simple case say this, say this is the edge of the boundary layer and accelerative view. So the pressure is p is constant in this part, at any point here or if you have large number of points at each of these points p is same, the static pressure is same. What will happen to the total pressure? As you move to the wall from the edge of the boundary layer, as we move to the wall, what happens to the total pressure? This is very simple question it decreases total pressure is in an inviscid flow, the total pressure is p plus half rho u square; p is constant. And u we know is decreasing, as we move to the wall, u is decreasing. So half rho u square is decreasing, so the total pressure or stagnation pressure is decreasing as we move to the wall, stagnation pressure is decreasing.

In an inviscid flow, the stagnation pressure always remain constant, in an inviscid flow the stagnation pressure always remain constant, it does not change. But in the viscous flow, we see that the stagnation pressure changes and as you move from the edge to the wall, the stagnation pressure decreases.

Now before we start thinking of solving this equation, let us look to some other boundary layer effect. Due to the presence of boundary layer over a solid wall, consider again a flat plate case. Due to the presence of the boundary layer, what happen? The velocity from the on the wall is zero; it gradually increases and at the edge of the boundary layer, it reaches the inviscid flow solution after that of course, it remain same.

If there would have been no boundary layer, then for the from the plate surface to infinity, everywhere there were been inviscid flow solution. So however in this cases you see that, there is a small region where the velocity is less than what it would have been in an inviscid flow.

(Refer Slide Time: 29:39)

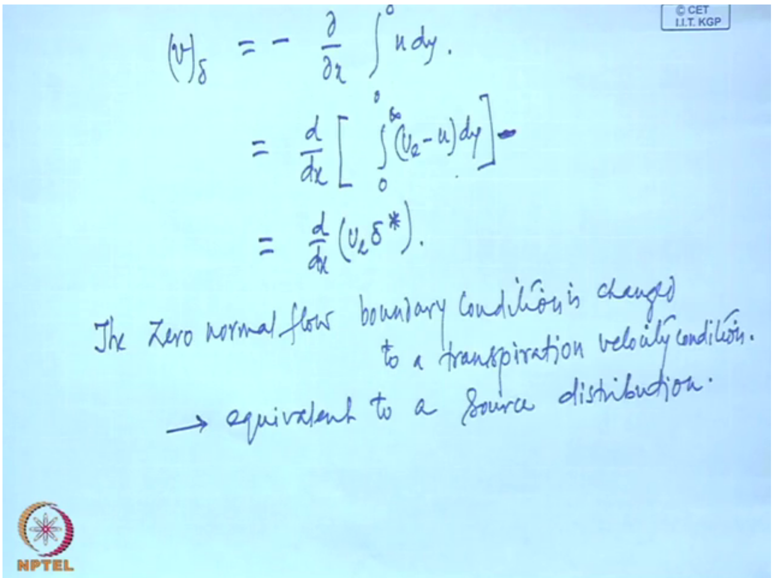


Agreed? If we had an inviscid flow, say if would have an inviscid flow, say this is the plate. Everywhere the flow velocity each, so the inviscid flow solution. For all this part, our flow velocity each, let us call it that inviscid flow solution u v. You know, our viscous flow, in our viscous flow the velocity is zero on the wall and then, increasing gradually and then. So, if you consider this up to certain distance, the velocity has decreased and then it has increased and remains constant. But, in this part the velocity is smaller. Can you see now that due to the presence of this boundary layer or due to this viscous effect, the total mass flow will be little less in this case, the total mass flow will be little less in this case. See that there is a deficit or defect in the total mass flow in an incompressible flow mass flow is equivalent to volume flow.

So you can say that, there is a deficit of volume flow or volume flux whatever it is. Actually it is mass flow but, since incompressible flow, mass flow is analogous to volume flow and how much is that? How much is the mass flow defect or volume flow defect?

Student: (( )) what is this left hand side? Left hand side can be integrated explicitly.

(Refer Slide Time: 44:13)



V at delta, V at zero is zero. V at zero is zero, so V at delta that equal to zero; the right hand side is zero, right hand side bring it in the form of delta star. and since we are integrating across y. Since we are integrating across y, then these quantities are no longer function of y. So these derivatives we can write as an ordinary derivative. Yes, what you have got?

Student: A into delta star i.

Yes.

Student: V into delta star minus.

V e. how V e is come here?

Student: U e.

U e.

Student: Into delta star minus delta.

U e into delta star minus delta. How you got the delta star minus delta? From the equation of volume flow that is. Since you have got the delta star, you must have brought in U e also here. And once it is U e minus U that integration you have got the limit can be straight way extended to infinity. A definitely you have done it, because you have got delta star make it U e minus e U, extend the integration to infinity and the constant term that we are using that is derivative is zero. You are reading a constant term here know to get it delta star form. You are reading a constant here and the derivative of that constant is zero minus what? U e into.

Student: Delta U into delta.

Why delta? Why delta is a constant, it is not delta; it is not an, it is not delta, where is delta coming? Let us say this is this integration U e- d y extended and that constant derivative of that constant is zero. So eventually, this is what this is or it becomes? Now see, this is can be thought of in another way. V e- is the normal component of velocity at the age of the boundary layer or where the inviscid flow solution starts. So this happens to be the boundary condition for the inviscid flow.

So, our original inviscid flow boundary condition was a zero normal flow, V is zero on the flow, U is anything V is zero, now what we have got? U is still anything but for V, we have got something which is related to the boundary layer. So in other way that the effect of the boundary layer is to modify the normal boundary condition and since a normal flow is added. So it is equivalent to a source distribution as, if a source is distributed on the wall.

So, this is also another effect so these are two alternative. This is the alternative effect to the inviscid flow, the inviscid flow will see either way, that either that the wall has now got a thickness or the wall has been displaced by a distance delta star or it will see that there is an additional source distribution placed on the plate whose strength is given by this .

So, if as we mentioned earlier, that in an interacting situation where one affects the other which is of course, the natural case that the inviscid flow affects the viscous flow and viscous flow in turn affects the inviscid flow. Either of these two approach is to be used for interaction that is say, you are solving viscous flow by this approach, first you solve the inviscid flow using a zero normal boundary condition at this stage. You do not have a boundary layer. You find the pressure distribution or velocity distribution of the surface which is to be treated as, the velocity or pressure on the age of the boundary layer using that velocity and pressure, you now solve the boundary layer equation.

Once the boundary layer is solved, you will find the delta star or this quantity, either of these two. You may use delta star that means then you change the thickness of your body at delta star and recalculate you are inviscid flow on that new body, again using zero normal boundary condition. Alternately, you use another source distribution at another source distribution which is given by this but, your body remains the same, body remain as it was earlier. But, your source distribution has changed and this is the way to proceed until convergence.

Now this is what you considered a defect in mass flow similarly, as you can understand that it will create defect in momentum flow as well as energy flow. Consequently there is a thickness associated with momentum defect as well as energy defect.

So there is various type of boundary layer thickness; this is displacement thickness. Then if you consider a momentum deficit, there will be a momentum thickness; then if you consider energy deficit its (( )) be taken as energy effect. If there is a temperature variation taken into account, then you will get a thermal boundary layer where temperature will change. Then that thermal boundary layer has its own length or own thickness. So we will consider a few more cases and then perhaps look for solution or some other approaches.