Introduction to Aerodynamics
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Module No. # 01
Lecture No. # 42
Boundary-Layer Theory

We have seen that in an inviscid irrotational flow the lift produced by an airfoil or a body is proportional to the circulation around it and what we have found that drag force is zero. Of course, in an three dimensional inviscid irrotational flow the drag force is not zero as you will be seeing later that a drag force comes in, but as far as two dimensional geometries are concerned the drag force is invariably zero, if the flow is inviscid irrotational.

Now this is of course, unrealistic as is known from experiences that all bodies experiences drags. The lift may or may not be there, but the drag force is always present also in case of lift we see that lift is proportional to circulation and if we remember kelvin's circulation theorem which says that rate of change of circulation around a material closed curve in a fluid of uniform density in which the body force is derived from a scalar potential is zero in an inviscid irrotational flow. Now then a question comes that if the lift is proportional to the circulation where does this circulation comes from where does this circulation comes from.

We used an artifact of kutta condition to fix the value of circulation; we say the kutta condition will is such that the flow will leave the trailing edge smoothly. So, this is the artifact that fixes the value of circulation, but the question that where the circulation comes from still remain unanswered. So, in both the question of lift and drag that inviscid irrotational flow cannot answer what are required, it cannot answer where the circulation comes from it gives the value of lift correctly that it is proportional to circulation and it is rho q gamma got to correct, but how the circulation comes if we when we place the body in an irrotational uniform flow.

Uniform flow is irrotational as you can see clearly by taking the curl of that uniform flow is all gradients are zero so, curl is also zero that is rotation zero no vorticity. See if the vorticity is zero to start with then if we consider any material closed curve across that material closed curve the circulation is also zero and an inviscid irrotational flow the rate of charge circulation rate of change of circulation around any material closed curve is zero. So, what our zero initially that will remain as zero so, how that circulation comes in. Now to answer these questions or other to some of these questions to start with.

We need to look into say the viscous flow or irrotational flow to find the answer and since we are talking about vorticity and circulation. Let us see the vorticity and circulation within the framework of a viscous flow of course; we will still retain the assumption of incompressible flow. So, next couple of lectures we will discuss how a viscous rotational flow answer this questions. If we so what will vortex dynamics on the vortex dynamics on it shown is a subject you can call.

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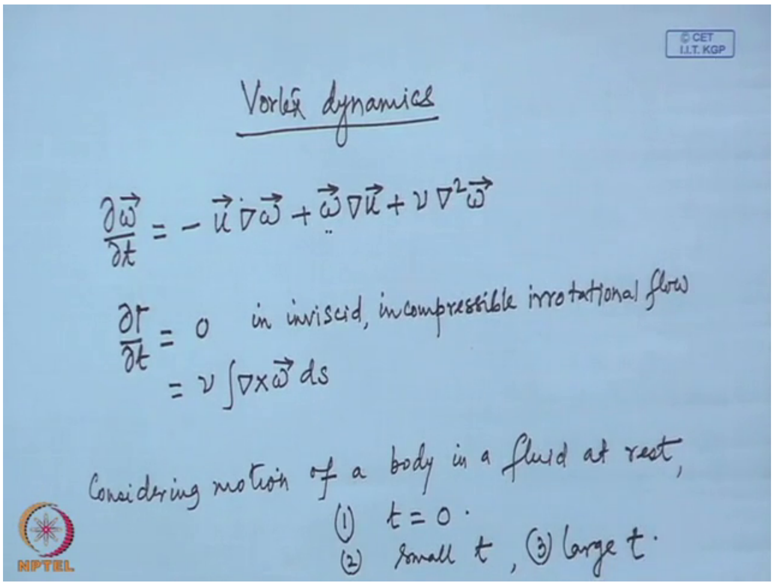

Of course, we will be discussing some preliminary aspects of vortex dynamics. I hope you remember that we earlier derived the navier stokes equations or the equations of motion in the form of vorticity. Which we named as vorticity transport equation to remind you once again the vorticity transport equation was which we remember that it was derived obtained simply by taking curl of the navier stokes equations.

But if you in deriving this equation if you remember that we considered the definition of circulation the line integral of velocity about a material closed curve and then we differentiated it. And for the term d u d t we replaced the incompressible inviscid Navier stokes equation or the Euler's equation. You can derive the same equation, but replacing d u d t not by the Euler's equation, but by the complete Navier stokes equation retaining the viscous term and then this will give a term something like this something like this that is once again it shows that the in a viscous flow the circulation around a material closed curve can be changed due to again the diffusive action of viscosity. But how the vorticity comes in or how the circulation comes in it has to it has to be there only then it can spread or diffuse.

Now let us see it from other way considering the flow I think we discussed it once earlier, but let us do it once again. Consider a motion starting from the rest to do that let us think about a situation where we have body emerged in a uniform fluid at rest body is emerged in a fluid at rest and let us for simplicity once again consider that the fluid is of uniform density the body force is derived from a scalar potential like gravitational force and then imagine that the body starts impulsively from a velocity zero two some velocity u and then moves steadily with that velocity it changes impulsively from zero to u and then moves with that velocity uniformly what is likely to happen.

The instant the motion starts the solution of this flow motion or flow will be invariably inviscid irrotational the instant just think about that instant when the motion is starting at t equal to zero. Let us say that is t equal to zero as long as the fluid is at rest obviously viscosity has nothing to do viscosity has no role to play even if the fluid is viscous the viscosity has no effect because the fluid is at rest viscosity acts only when the motion starts. So, just at the start this motion will start as irrotational and since at before t equal to zero there was no vorticity within the interior of the fluid then at t equal to zero also there will be no vorticity in the interior of the fluid in the interior of the fluid, but consider the next instant t equal to zero plus that time the motion has started.

So, viscosity has started playing its role and what is the role of viscosity it enforces a no slip boundary condition at t equal to zero when the motion starts this motion was inviscid irrotational and you know that inviscid irrotational flow is completely obtained by zero normal flow boundary condition which gives a slipping motion. So, at t equal to zero when the motion starts the motion is slipping motion the fluid over the solid body slips at t equal to zero, but then at t equal to zero plus the viscosity has started playing its part. What it wants that the no slip boundary condition must be satisfied there must not be any slip. Then see at t equal to zero what we had initially there is a jump in velocity on the solid wall there is jump in the velocity the fluid layer adjusting to the solid body was moving with the velocity u. If we consider the relative velocity the relative velocity is u, but at time t equal to zero plus this relative velocity must cease it will become zero. What then happen I finite seat of vortex, a seat of finite vortex because there was a jump in velocity at t equal to zero that has created a seat of vortex at t equal to zero? So, this is the mechanism of generation of vorticity in the first place the no slip boundary condition on the solid wall that at t equal to zero plus creates a seat vortex.

If we remember that we discussed earlier about seat vortex and we show that across a seat vortex there is a jump in tangential velocity, we consider a seat vortex on the lower surface we had a velocity of gamma by two in one direction on the upper half gamma by two in the other direction giving rise to a jump in velocity equal to gamma the strength of the vortex seat or the circulation about the vortex seat.

So, here also there is a tangential jump in tangential velocity which is equivalent to a seat vortex. So, as the body starts moving at t equal to zero a seat vortex has developed on the surface of the solid body not in the interior of the fluid not in the interior of the fluid it is generated on the surface of the body or at the boundary. The solid wall is one of the boundaries the other boundary let us consider at infinity where the flow fluid is at rest so, there is no question of any vorticity coming through that boundary. So, this is also another way this can be answered that according to this vorticity transport equation or circulation kelvin's theorem.

If there was no vorticity or no circulation in the interior of a fluid it cannot be created there so it must have to come from boundary itself if the vorticity cannot be created within the interior then it has to come from the boundary and we see that is how it is produced at the boundary. So, now considering the motion from rest we can explain this development of this motion in three stages one at time t equal to zero, one at a small time after t equal to 0 and then 1. Later at the final steady state when the time is large this will be the situation.

Now let us give it a little more definite meaning of that large time considering a problem which is reaching to a steady state what should we call is a large time or time infinity. Consider a body of length l in the direction of the flow the body length in the direction of the flow or the relative fluid motion in the direction of the relative fluid motion let it be l then how longer a fluid element is likely to reside the in the neighborhood of the solid wall definitely l by u what we have taken as the velocity it is the characteristic velocity. So, this l by u can be taken as a very large time. What we call large time? That is of the order of l by u remember.

We are talking about of the order of the characteristic velocities of the order of u it is not that characteristic velocity has to be u exactly know that we have seen earlier that if we consider flow over a circular cylinder the maximum velocity reaches two times the free stream velocity in case of airfoil also it behaves like that. So, that velocity can become two times or three times is immaterial but, it is of the order of that. So, by large time or we can consider that l by u is the large time.

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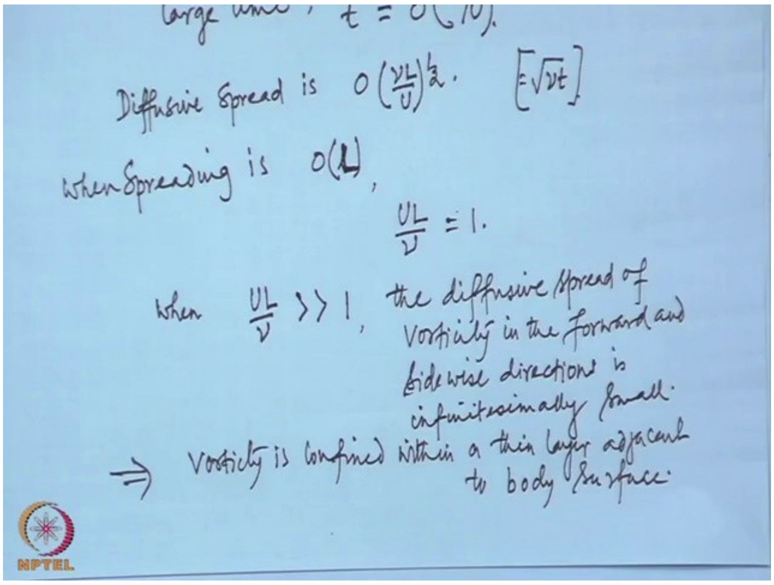

So, by large time we mean time t is of the order of we consider this time the velocity u or u infinity q. So, if you spread consider the spreading how much will be the spread at large time nu t is the square root of nu t is the distance to which the vorticity spreads after time t. So, when time approaches to infinity when t approaches infinity that is of this order then the spreading by viscous action is nu l by u.

Now whether this spreading is very small or of the order of the body length itself when will be this spreading will be of the order of the body length body size characteristic size of the body is l if we need that if we try to look for the condition at which the spreading is of also of the order of l. So, we can said that the spreading will be of the order of l what do we get from here?

Student: U l by nu.

U l by nu is of the order of one. So, this is of that say u l by nu this is the condition is of the order of one and if u l by nu is much larger than one.

Student: (( )).

Spreading is (( )). Spreading is very small infinitesimal small.. or we can say the vorticity is confined within a thin region near the body surface. Now let us see what is this u l by nu u l by nu think about the navier strokes equation the convective forces are or the convective force is given by the term u grad u if you look to the navier stokes equation the left hand side is rho into d u d t plus u grad u when you write in your standard form each term is u d u d x, u d u d y, v d u d x this sort of term that is what we called the convective acceleration what is order of this term now.

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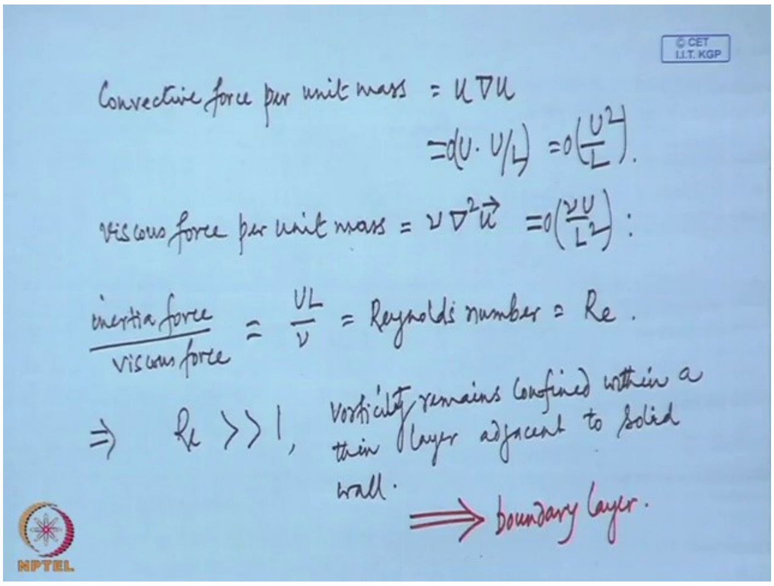

Say we will consider force per unit volume only or say force per unit mass there beta than rho will not come in picture only u grad u convective force per unit mass..

What is it is order?

Student: (( )).

No, I am not asking for the dimension say this is an acceleration you can say very clearly l by t square hence is the order of magnitude the characteristic velocity is u that capital u that we are using and grad u is u by l. So, this has should we write order of what is the viscous force per unit mass what is the viscous term is the navier stokes equation nu laplacian u when you divide by rho per unit mass is nu laplacian u and its order.

Student: (( )).

Again order of so, what is the ratio of this inertia force and viscous force see that that will give us the strength of relative strength of the convective force and viscous force earlier we said that the convective force is dominant if the convective force is dominant then this is what is going to be the situation. Now how do you measure dominant that one is much larger than the other so, this is what it that quantifies what it is now.

Student: U l by nu.

This u l by nu or rho u l by mu nu can be replaced by mu by rho. So, rho u l by mu this is what is called the reynolds number is usually designated by reynolds number. We have earlier come across this also and we will usually denote it by r e and then going back to that earlier condition, if reynolds number is much larger we earlier had this condition when u l by nu is much larger than one vorticity remains confined within a thin layer. So, u l by nu is reynolds number. So, when reynolds number is much larger than one.. and also we can see that as reynolds number goes on increasing the thickness of that layer becomes smaller and smaller and you see this is the assumption that was used to for that irrotational flow or potential flow.

That if the viscous actions if the viscous action is confined to very thin region near the wall say when you consider the complete flow field and a very thin region there where there is viscous action outside there is no viscous action.

So, it would be a good approximation to consider the entire flow field to be inviscid and irrotational and we saw that it is justified to some extent, but not fully. So, this is the region which is commonly called as boundary layer the very thin region near the body surface where viscosity plays its role where vorticity is non zero and velocity changing normal to the surface. What actually the spreading of vorticity means come back initially we had the vorticity was created only at the surface it remain at t equal to zero it was on the body surface then we said that which is equivalent to a vorticity seat vertex a jump in tangential velocity is equivalent to a seat vertex and then this vorticity spreads.

What actually spreads this spreads that velocity gradient when you said that the vorticity spreads actually that velocity gradient is spread velocity gradient which initially occurred over a distance of zero the velocity gradient that occurred at time t equal to zero over a distance zero that has spread to a distance of the order of the one way reynolds number or square of one of inverse of square inverse square root of reynolds number. That velocity gradient that is the change from zero to u that change in velocity initially occur over a distance of zero, but as time passes or as viscocity plays its role this change from zero to u is now occurring not over a distance of zero, but over a certain distance which is the distance is of the order of inverse square root of reynolds number that is what is the spreading of vorticity means that the velocity gradient or the velocity jump which was occurring basically as a jump is no longer is a jump, but it is gradually changing is smoothly changing over distance from zero u at the end of that layer the velocity is again u, but on the wall the velocity is zero.

So, from zero to u that change or that jump has been spread and now has become smooth on continuous variation over the distance, which is proportional to inverse square of reynolds number and within that layer in which this velocity variation takes place that is what is the boundary layer. So, for a high reynolds number flow a boundary layer develops over certain geometries and within this boundary layer the velocity changes smoothly from zero on the surface to the inviscid flow velocity at the edge of the boundary layer. The inviscid flow velocity at the edge of the boundary layer and within this layer the vorticity is non zero the flow is rotational also think that when a reynolds is really large considering practical aerodynamics where reynolds is of the order of 10 to the power 7, 10 to the power 8 then inverse square root of that is practically very small.

Over a wing chord of say two meter or so may the velocity boundary layer will be at a flying reynolds number of the order of few millimeters. The maximum thickness of this boundary layer will be of the order of few millimeter and you see that the assumption that the entire flow field is inviscid irrotational is in that sense justified when you think about say meters of area or volume. Just few millimeters are really negligible, but you see that that has this effect that gives the drag force zero or in a three dimensional case non zero, but still not correct far away from the correct value of drag.

So, this is the concept first brought in by frankled in more than a century ago in 1905. Where showed that you can use your inviscid flow analysis, but you must satisfy that no slip boundary condition that means you must bring in the boundary layer to bridge the gap.