Introduction to Aerodynamics
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Lecture No. # 40
Transformation (Contd.)

So, as said we will consider few examples to further illustrate this concepts of transformation. And the first example, that we will take is a very simple one in which we will be transforming certain streamlines from one complex frame to the other. How the streamlines look like, and for that we will consider the simplest possible streamlines the uniform flow. You know the uniform flow is given by straight streamlines all streamlines are parallel. We will apply a particular transformation and see then what happens to these uniform flow streamlines. So, let us consider that in the physical plane that is in the z plane, we have a free stream which is uniform and consider the stream flow is parallel to the real axis or parallel to the x axis.

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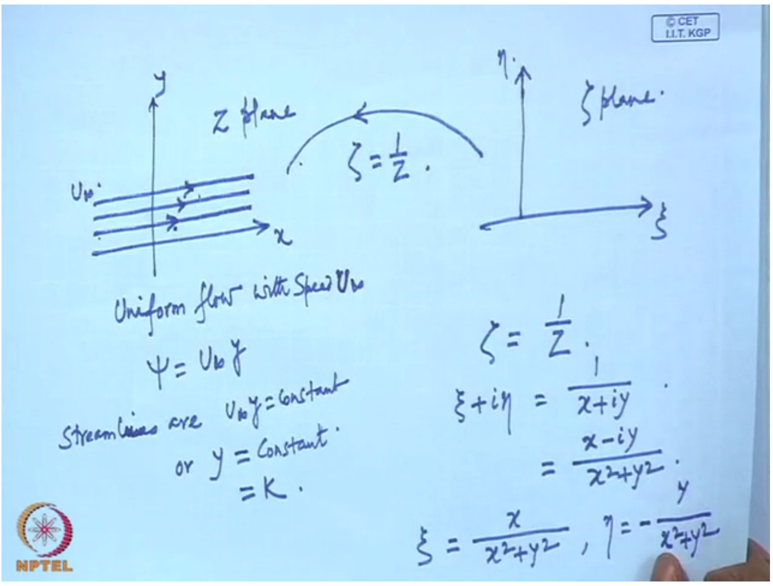

So, in the z plane we have the streamlines we can. So, so only a few streamlines with speed u infinity. So, the streamlines are what the stream function is u infinity y and the streamlines are given as u infinity y equal to constant or simply y equal to constant. We apply a transformation zeta equal to 1 by z apply the transformation zeta equal to 1 by z. if eliminate x what do you get simple from the second relation find x put in the first what it is. In xi square plus eta square. Including minus plus sign if you want Xi square plus eta square Plus. Plus In the eta by eta by k square eta k square

What is it hyperbola

Student: Ellipse

This is ellipse

Student: Circle

This is a circle

Student: Circle

So, on the xi plane or sorry on the zeta plane these uniform flow streamlines are circle. Circle with radius and center what circle with radius where Minus eta by 2 q circle with radius. Do that minus 2 by k circle with radius how much or center center at 1 minus eta by 2 k minus 0 u minus center sorry center is 1 by 2 0 Minus eta by 2 minus 2,Eta by 2 k,

Student: Minus 1 by 2 1 by 2 k that is

Yeah it should be 1 by 2 k and radius

Student: 1 by 2 k

So, for each value of k you will get a circle try one more this time again uniform stream, but along the imaginary axis a uniform stream along the imaginary axis and the transformation is 1 by z square. Transformation is simply z square, uniform stream along the imaginary axis or along y axis transformed by zeta equal to z square.

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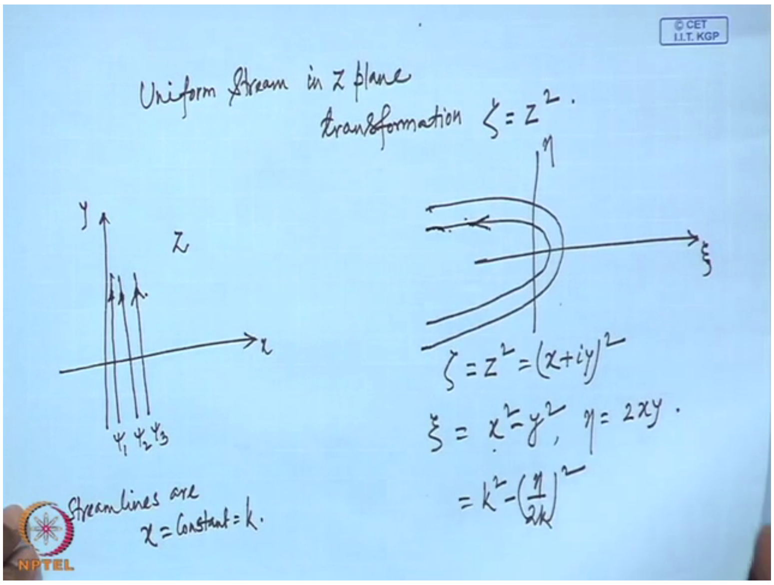

In the streamlines are x equal to constant. So, what will happen to the streamlines on the zeta plane?

Student: Parabola

Parabola xi is x square minus y square y zeta by 2 k from here in eta equal to 2 x y if you take x equal to k. So, y becomes zeta by 2 k and x square is k square. So, this becomes k square minus let us try now with Now try this problem develop a zhukovsky airfoil of chord 152.5 millimeter thickness to chord ratio of 15 percent thickness to chord ratio 15 percent camber 5 percent. Consider that airfoil is developing full zhukovsky lift; consider the airfoil is developing full zhukovsky lift at 10 degree incidence, in a wind of 18.3 meter per second. Draw few adjacent streamlines that drawing part of course, you will not be able to do here that the rest.

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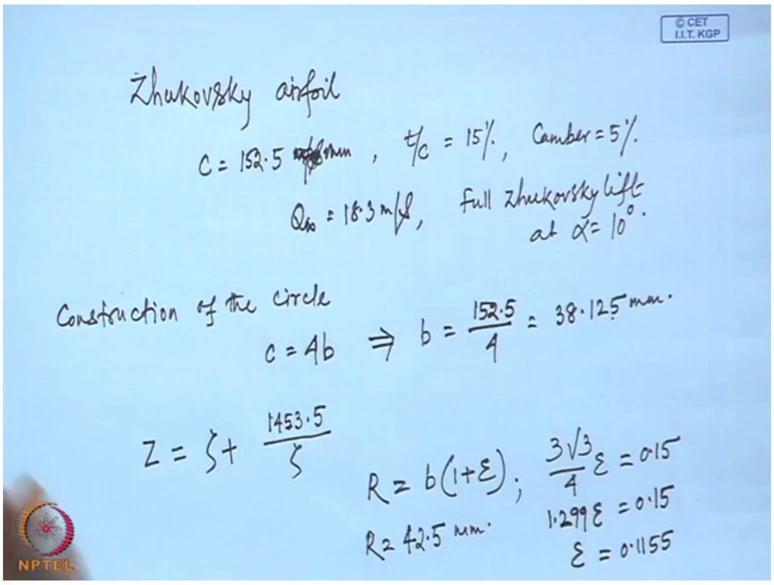

How will you proceed how will you proceed how will you proceed what will be the steps of this problem, any suggestion what we have discussed earlier, since we have not in a position to solve flow over the airfoil directly. We have to transform this airfoil to a circle what we did by zhukovsky transformation. So, the first thing is we have to find the circle, along with the parameters of the transformation. Then we have to get the we know the solution for the circle, we can construct the solution for the circle, and then transform it back to airfoil. So, what do you want to know we want the radius of the circle we want the center of the circle with respect to the origin, we know we would like to know where will be the critical points. So, these are the things that we have to first find out. Once we can do that we can write the solution for the circle which then can be transform back to airfoil.

So, chord is how much chord is related to chord is approximately 4 b. Where b is the parameter in the transformation zhukovsky transformation. So, chord is 4 b. So, that gives b to be and the zhukovsky transformation becomes z equal to zeta plus b square by zeta b square becomes 1453.5 this is what is b square, but see we have we should notice these things, we have used millimeter for b imply that now this z zeta they are all so have to be in millimeter. What is the radius of the circle, R equal to R is related to b. b into 1 plus epsilon R is b into 1 plus epsilon. Epsilon is at unknown to us, but we have another relation maximum thickness what it is maximum thickness 3 root 3 by 4 epsilon

We have 3 root 3 by 4 epsilon is maximum thickness is how much is that 0.15. This 3 root 3 by 4 you can remember this value is 1.299, that is value is taken 1.299 0.15 that gives epsilon equal to 0.1155 Now, we have b we have epsilon we have radius actually it becomes 42.253 that let us say take 42 5. Now flow over circular cylinder is represented by a uniform stream a point doublet placed at the center of the circle. And if it is lifting a circulation, the doublet strength is given by the radius, or the radius of the circle determines the strength of the doublet. What it is the doublet strength?

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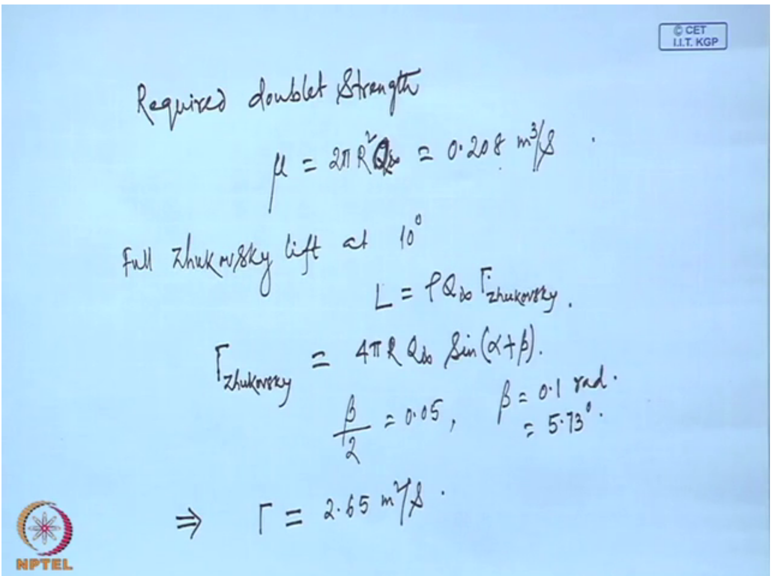

That equates doublet strength how much. What is the relation 2 pi R square q infinity? What we are writing capita l Q or small Q capital Q no, but in a do not want system not the something different can we make it is given 0.208 meter cube per second. Now, in this case there is a circulation also, because the problem says that this airfoil develops this airfoil as camber, and this airfoil develops whose zhukovsky lift at 10 degree. Now how much is the full zhukovsky lift at 10 degree. Now much is this is what is given by the kutta condition, how much is that 4 pi what 4 pi R q infinity.

Student: Sin alpha

Sin alpha plus beta. Now alpha is of course, given 10 degree how much is beta. Beta is, Beta is camber is given 5 degree camber is given 5 degree or so, camber is related to beta how much it is, Beta by 2 is the maximum camber. So, we have beta by 2 equal to how much

Student: 5

Not 5 percentage

Student: 5 by 100

0.05. So, beta is how much 0.1 radian that equal to approximately 5.73 degree. So, now we can put that value. This will give the circulation to be 2.65 meter square per second. So, we now have the complete flow for the circle. The flow on the circle plane is now given by and uniform stream of 18.3 meter per second a doublet strength of 0.208 placed at the center of the circle and a circulation of 2.65 meter square per second placed at the center. So, we can now write completely the complex potential on the circle plane.

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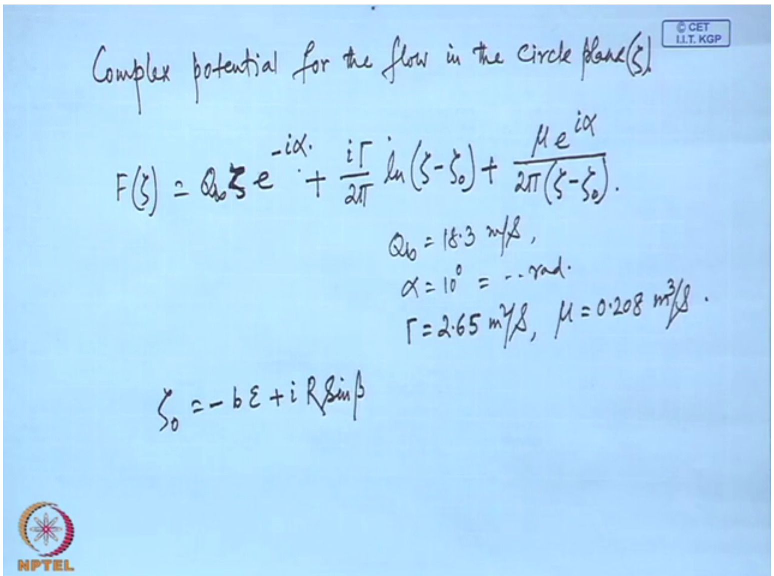

Yes complex potential due to the uniform stream complex potential due to uniform stream at 10 degree angle of attack. What it will be forget about those numbers complex potential for the stream at an angle of attack alpha. You have written it earlier you got to remember that now q infinity, Cos i cos I, complex potential directly

That will content both the potential function as well as the stream function. And it is potential. Q infinity, Q infinity z sorry not z zeta q infinity zeta e to the power minus i alpha where alpha is 10 degree expressed in gradient. Remember in this expression these alpha are in gradient, into sin infinity plus due to the circulation i gamma by two pi log we have the values for q infinity gamma mu alpha. You already have q infinity 18.3 meter per second alpha equal to 10 degree equal to some radian, gamma equal to 2.65 no 2.65. What we still did not have not done is zeta not center of the circle, but we have found all the components of zeta naught. What is zeta naught location of the center minus b epsilon plus alpha beta

I r's R beta actually i R sin beta you can do it R beta also no problem, because the beta is really small. So, once you put it the complex potential on the circle plane is complete. You now have complete potential function on the circle plane, both potential function as well as the stream function. You can take out the real and imaginary part split it or you can leave it in the complex potential form itself and then transform it back to the airfoil plane. I mean, after this what we did for are the first two problems that sort of things we have to do. We have to now transform back to zeta to z using that zhukovsky transformation once again.

So, that of course, will be quite involved just a substituting this. So, it will be better to try it numerically you may try that even plotting and all numerically neither I would accept that you will complete that complete the transformation and plot the streamlines, but what I hope that you are a following how to handle the problem.

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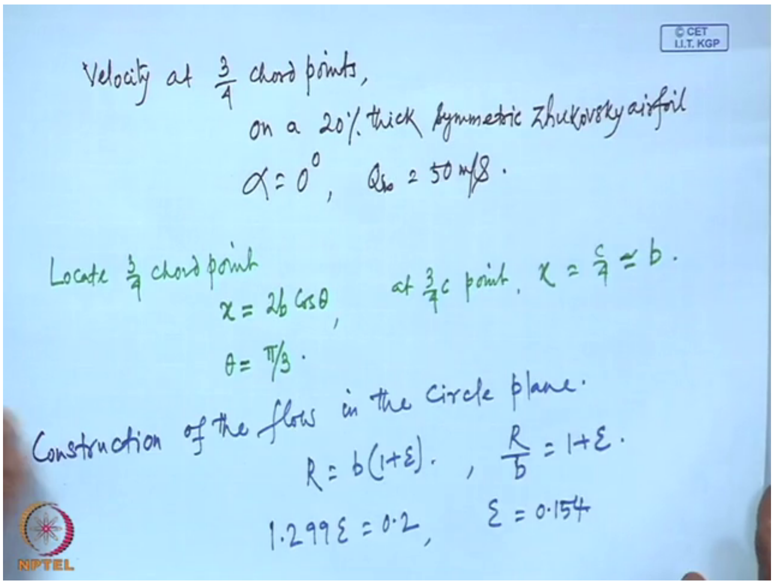

Let us consider one more, find the velocity at the three forth chord point on a symmetric zhukovsky airfoil of symmetry zhukovsky airfoil 20 percent thick. Velocity at three fourth chord point on a 20 percent thick symmetric zhukovsky airfoil, also given alpha equal to 0 degree and the free stream is 50 meter per second. See there are 2 3 chord see forth chord point one on the upper surface one lower surface; however, in this case it is given the symmetric airfoil and the flow is also at 0 degree angle of attack. So, the upper surface and lower surface will be same.

So, for locating only one is sufficient. So, where is the three forth chord point, where is three forth chord point. Three forth chord point is three forth see from the leading edge. So, from the origin where it is origin at the center at the middle. So, it is

C by 4 from the origin. So, how much is that what is that point corresponding theta corresponding theta. You can use that relation x equal to 2 b cos theta and c is 4 v. So, this is approximately b. So, how much is theta

Pi by 3. That is the location of the point, but before that once again we have to construct the circle and the flow on the circle. So, we will have almost the similar, approach what we had earlier. What do you need to find we need to find the doublet strength and the circulation? For doublet strength we need the radius of the circle, but the doublet strength we need the radius by the circle. How is the radius of the circle R equal to b into 1 epsilon no? What is 0.2 epsilon is not 0.2?

Since the chord here is not exactly specified since the chord here is not exactly specified we will not be able to write how much is the chord and how much is equal to b and how is equal to R specifically. So, we will keep it as the ratio R by b is 1 plus epsilon and epsilon can be found from the thickness. How much is the epsilon 1.299 epsilon equal to 0.2 that gives epsilon equal to 0.154. So, we have R by b equal to 1.154. How much is the circulation 2 pi R q infinity sin alpha plus beta.

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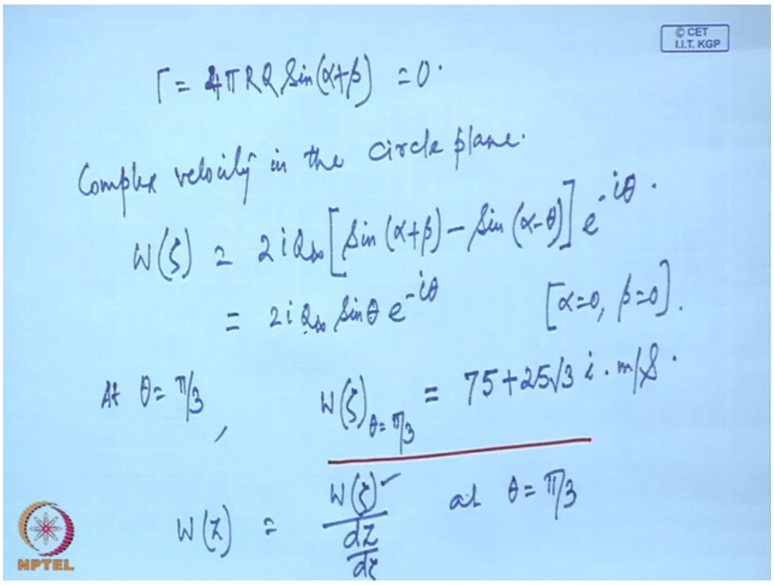

The circulation gamma equal to 2 pi or 4 pi

Student: 4

4 pi R q infinity sin alpha plus beta how much is this is equal to 0 with for this case alpha is 0 beta is 0 in symmetric airfoil. Beta is related to camber alpha is given as 0 and since this airfoil is un cambered symmetry beta is 0. So, for this case there is no circulation it is only a uniform stream plus a doublet. Now what is expression for the complex velocity on the circle plane?

We can write it in terms of q infinity and mu and this, but we have other expression also if you remember the complex velocity on the circle plane. If you remember we had this type this expression we earlier derived 2 i q infinity sin alpha plus beta minus sin alpha minus theta in to e to the power minus i theta. Now in this case we now have all the values, we are interested at theta equal to pi by three, or for this particular case alpha is 0 beta is 0. So, the first term is 0 here it becomes plus sin theta only. This gives how much 2 i q infinity only sin theta e to the power minus i theta. We are not interested in velocity at all the points we are interested only velocity at the three forth chord point which corresponds to theta equal to pi by 3. So, at that point what will be the velocity complex velocity on circle plane? Then put this value q infinity is 50 w is zeta corresponding to theta equal to pi by 3. please check this Please check this calculations simple q infinity is 50 theta is pi by 3 please check this calculations

Now, this is the velocity at the theta equal to pi by 3 point on the circle, but we do not want to know the velocity on the circle we want know the velocity on the airfoil. So, we have to find out what will be the velocity on the airfoil. We have w z equal to w zeta d z d zeta of course, everything at theta equal to pi by 3. So, this w theta pi by 3 already we have found we have to find out what is d z d zeta at theta equal to 5 by 3.

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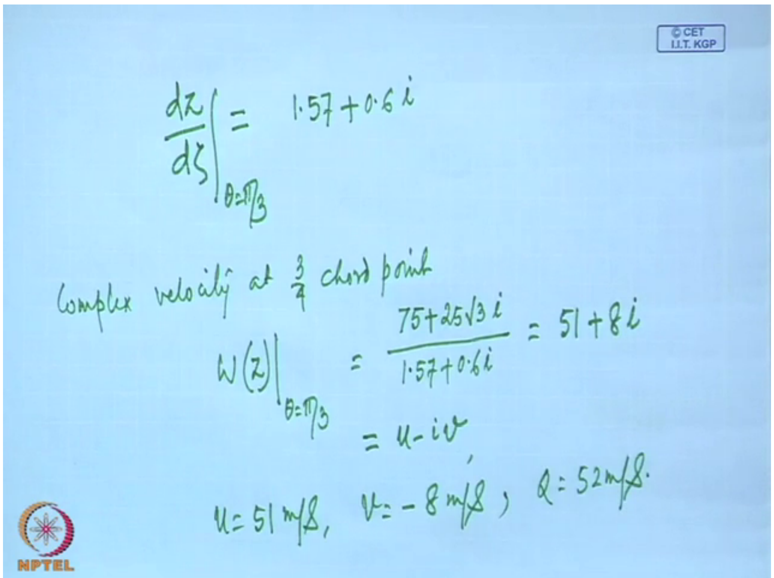

I will just give you the final result for d z d zeta, but please check this at theta equal to pi by 3. What I obtained is something like 1.57 plus 0.6 i. So, finally, the complex velocity at 3 chord three forth chord point what I obtained once again please check this numerical calculations.

How will we get d z by d zeta?

Student: D z d zeta

We do not have d value even have the d value. It will not be require see it is d z d zeta is what 1 minus d square by zeta square and it will contain only b by r. So, b by R we have perhaps i show that a details calculation also tomorrow for one case and remember this complex velocity is not u plus i v it is u minus i v. So, what we have is u equal to resultant speed is of course, i about 52 meter per second angle also if want the direction of the flow in terms of angle that also you can find.