Introduction to Aerodynamics

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Lecture No. # 39

Zhukovsky Transformation - Applications (Contd.)

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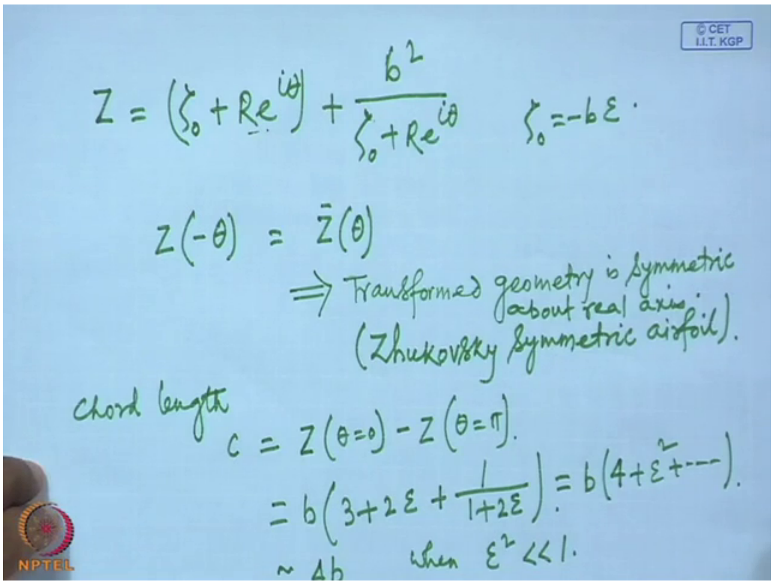


Now, in this case since Z is zeta naught plus R e to the power i theta, and what is zeta naught what is zeta naught minus b epsilon, so there is no imaginary part in zeta naught, there is no imaginary part in zeta naught. So, imaginary part comes only here due to R e to the power i theta, and this gives Z minus theta is… And what does it mean, what is the meaning of this? Z of minus theta is complex 1 to conjugate of Z theta. What is the meaning of this?

Student: (( ))

Say consider only a simple case R e to the power i theta, for this also this replication apply this relation, what is R e to the power i theta? It is a circle, and for this also if we say Z equal to simply R e to the power i theta of a complex function, then for that function also this relation holds Z minus theta is complex conjugate of Z theta, now since Z theta is a circle, what this relation? What is the implication of this relation? What do say about circle? For what type of curve do you expect this relation to hold?

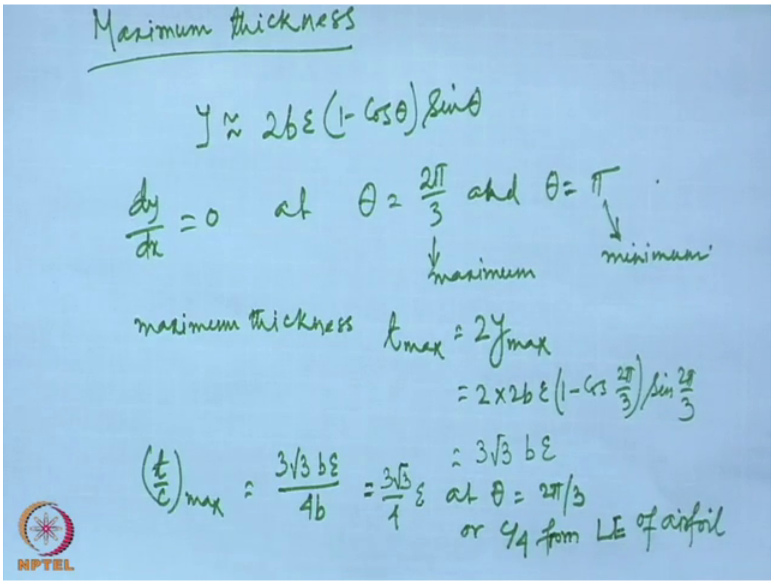
Student: (( ))

Symmetric. So, it implies that the transformed geometry in this case or whatever we are calling transformed airfoil that is symmetric. So, the transformed airfoil is symmetric. So, in this case what we the geometry is a symmetric zhukovsky airfoil, symmetric zhukovsky airfoil, what we have shown earlier like this a symmetric zhukovsky airfoil, see this also you can very clear easily see from the approximate relation x equal to 2 b cos theta, y equal to 2 b epsilon 1 minus cos theta sin theta, you can see that for theta and minus theta, the y values are simply plus and minus, the transformed geometry is symmetric about x axis, symmetric about real axis.

How much will be the chord length of this airfoil agreed, at theta equal to 0 it has only real value, at theta equal to pi also it has only real value, and the difference between the 2; the 2 x coordinates that is what is the chord length, and how much is it? Substitute all these expressions for x and y that we wrote earlier, you can do it, and the result will come as b into 3 plus 2 epsilon by 1 by 1 plus 2 epsilon, you can expand this in power series or apply your binomial theorem to find this 1 plus 2 epsilon to the power minus 1, and if you do this; this will come as b into 4 plus epsilon square plus still higher order term, and if we say that epsilon square itself is small then this is equal to 4 b. So, the chord will be 4 b. What will be the maximum thickness? please find out maximum thickness? If you want you can use the approximate relation?

The exact relation may be little difficult to handle, but you can use the approximate relation to find what is the maximum thickness? Where it is and how much it is? how much?

(Refer Slide Time: 11:58)



Use that simple relation y equal to 2 b epsilon into 1 minus cos theta sin theta, use that, yes use this itself, find simply where is y maximum, dy dx equal to 0, where it is?

Student: (( ))

at theta equal to

Student: (( ))

Pi by

Student: 4

Pi by 4 you are getting maximum

Student: 4 pi by (( ))

4 pi by 3

Student: 2 pi by (( ))

2 pi by

2 pi by 3, and but theta equal to pi will give the minimum, at theta equal to pi y is 0, this gives maximum and this eventually gives minimum, if you want you can check with second

derivative, but by inspection itself we can say that at theta equal to pi it y is 0, also at theta equal to 0. So, at theta equal to 2 pi. And what is the maximum thickness then? How was is the maximum thickness? How much is it? yes C by 4

C by 4 from

Student: (( ))

C by 4 from where

Student: (( ))

From the origin or from the center of the airfoil, also we can see it is C by 4 from the leading edge of the airfoil. So, the maximum thickness of the zhukovsky airfoil is at quarter chord from the leading edge, the maximum thickness of zhukovsky airfoil is at quarter chord from the leading edge, at the leading edge, of course, the thickness is 0. So, from 0 to the maximum thickness is increases within a distance C by 4, and then again it becomes 0 from C by 4 to at c.

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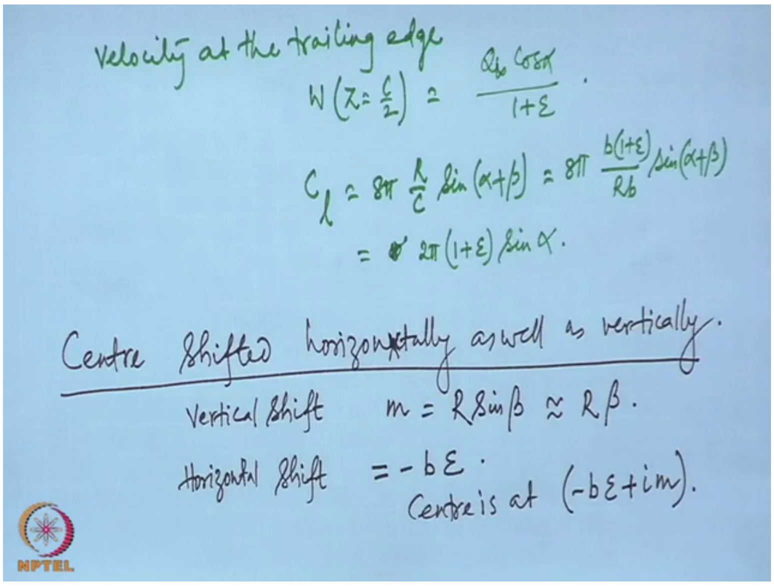

The other parameters like the velocity at different points, the trailing edge is cast that is already known, we have velocity at the trailing edge, how much? compare to the flat plate it is slightly reduced, for the flat plate the velocity at the trailing edge was simply q infinity cos alpha, it is q infinity cos alpha divided by 1 plus epsilon, epsilon is usually small because the airfoil thickness is of the order of say 10 percent, t by C maximum is of the order of point 1, at any other point of course, you can find using that xenon formula, velocity on the circle divided by dz df, the complex velocity on the circle divided by dz df is the complex velocity on the airfoil. How much the lift coefficient?

Let us see what is the effect of thickness on the airfoil lift? 8 pi a by C sin alpha plus beta, 8 pi a by C sin alpha plus beta or R by C, 8 pi R by C sin alpha plus beta, beta is of course 0, R is how much? b into 1 plus epsilon, R is b into 1 plus epsilon, C is 4 b, sorry beta is 0 in this case, so it is 2 pi into 1 plus epsilon sin alpha, so thickness marginally increases the lift curve slope, thickness of the airfoil as long as it is quite small increases the lift curve slope marginally.

Now, once again we will come back to the general case that we considered in the beginning, the general case we considered in the beginning; however, the center is located neither on the real axis nor on the imaginary axis, but somewhere else, but that result also you can get it by combining these last two results, where you have shifted the center along horizontal and along vertical, a combination of the two will give the general shift, and then of course, you can see what will be the general result of the transformation, because of the horizontal shift we find a symmetric airfoil with certain thickness, certain maximum thickness, because of the vertical shift we find no thickness the airfoil is basically a line, it becomes a line, but it is curved line, shifted with respect to the real axis, or shifted with respect to the horizontal axis.

So, if now combine this two that is if we shift the center both vertically as well as horizontally, what is the shape we are likely to get, this is what is the last case we will consider under Zhukovsky's transformation, eventually we have considered this in the beginning itself. What it will be? Will it be a symmetric geometry? No the symmetric geometry is available only when the center is lying on the real axis, as long as the center is lying on the real axis the transformed geometry is symmetric, whether it is center and coincides with the origin or center is away from the origin, but if it is on the real axis then we get a symmetric airfoil symmetric geometry. If the center is shifted vertically also we see that we have got something a curved which we called a camber line. So, when you shift both what we will get is a camber airfoil, airfoil which is no longer symmetric about its chord, rather it is symmetric with respect to that mean line or camber line.

So, if we shift the airfoil both and as before we consider this vertical shift, what we considered vertical shift last time? m equal to R sin beta, and which is approximately R beta, and horizontal shift or you can call it the center is at the center is at minus b epsilon plus i m, that is the location of the center. Point on the circle that corresponds to the trailing edge of the air foil, point on the circle that corresponds to the trailing edge of the air foil is at an angle minus beta, is at an angle minus beta and critical point minus b is placed on it.

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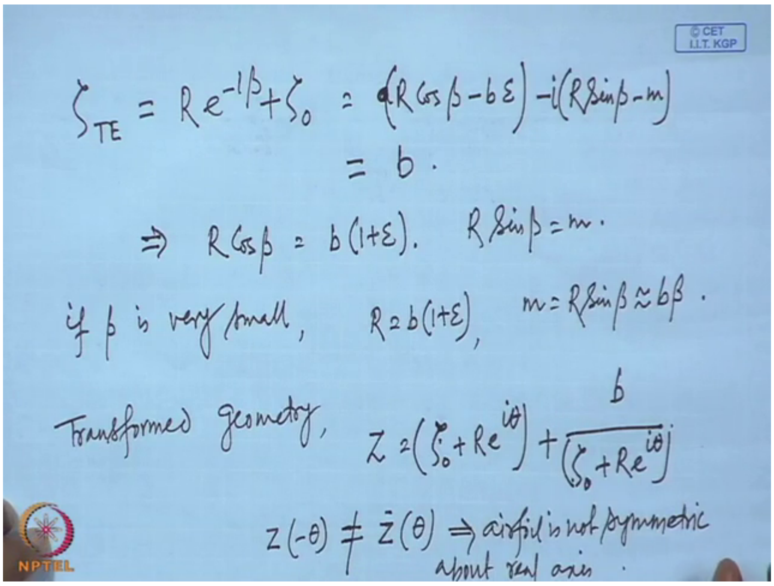

So, how much is that? Point on the circular plane that corresponds to that trailing edge point as before is zeta trailing edge, be careful about the interpretation of this point zeta trailing edge actually circle has no trailing edge, this is the trailing edge of the air foil which corresponds to the point on the circle, it is not trailing edge of the circle or circle has no trailing edge, how much is it sorry R cos beta minus b epsilon isn't it, and this will sit at the critical point, this is what is our requirement. the transformed geometry once again you can get since zeta naught itself quiet an imaginary quantity in this case this relation will not be satisfied, or the geometry will not be symmetric about the real axis or x axis. sorry looking to the earlier case for the symmetric air foil case we see that the x has remained the same, the chord length has remained the same, the expression for y it has the same first part, but; however, it has been added a term 2 b beta sin square theta, 2 b beta sin square theta, which is positive always, when you use this if you imagine a symmetric air foil then on the y coordinate there is a positive addition on the upper as well as on the lower surface. So, the lower surface is moving towards the chord line and the upper surface is moving away from the chord line, or the chord line is no longer a symmetric line of the air foil, or the air foil is not symmetric about the chord line, rather we have another line which is in between about which the air foil is symmetric, which is the mean line.

The thickness of this air foil can be obtained as thickness upper surface minus the lower surface, upper surface is for theta equal to 0 to pi, the lower surface is from pi to 2 pi, since it is sin square. So, irrespective of the value of theta this will give a positive numbers, so while you subtract this part will get cancelled, only this part that will become double.

Once again you can find out what is the t by C max? t by C is this, 4 b is C, t by C equal to epsilon into 1 minus cos theta sin theta, 4 b is C, and you need not do it again we have already done it, we can take that result from there t by C max is 3 root 3 by 4 epsilon at quarter chord, at quarter chord which is C by 4 from the leading edge, the mean line can be obtained by how? how to get the mean line? how to get the mean line?

Student: (( ))

Yes the mean line about which the upper surface and lower surface asymmetric, the line about which the air foil is symmetric, yes how to get the mean line? or the camber line?

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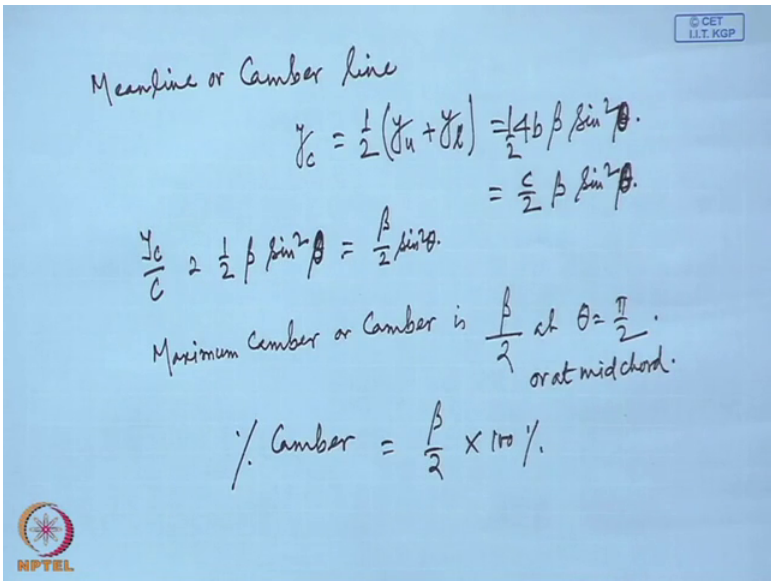

We will for the time being call it say mean line or camber line, say y C camber line y C is how much? It is the middle of the upper and lower surface. So, half of y upper plus y lower, and how much is that? How much it is? and y upper and y lower what happened? The first term gets cancelled, the first term gets cancelled, only the second terms remain for both upper and lower and it becomes; that means, it becomes doubled, 4 b beta sin square beta isn't it, it was 2 b beta sin square beta, so it becomes 4 b beta sin square beta, or into half into half remains, 4 b is again C, we can it and we write it as C by 2 since our interest is always by C with this. So, maximum camber or what actually called camber maximum camber, how much and where? this is not sin square beta this is sin square theta no?

This is not sine square beta sin square theta yes maximum camber is; obviously, beta by 2, your sin square theta maximum can be 1, and where it is? at theta equal to Pi by 2, and where is that? On the air foil where is theta equal to pi by 2

Student: (( ))

Y axis y axis

Y axis that is at the mid chord

Student: (( ))

At the mid chord, from leading edge, usually is given in terms of percentage camber, percentage camber is beta by 2 into 100 percent, so for zhukovsky air foil as far as geometry is concerned we have seen 3 salient features, the trailing edge is cast, the maximum thickness is at a quarter chord, and the maximum camber is at mid chord, what we have called as zhukovsky air foil which are very easy to obtain from circle, that characterized by cast trailing edge, that is trailing edge angle is 0, it is maximum thickness is at quarter chord, and its maximum camber is at mid chord.

Which are of course, not very good characteristics for a real air foil, usually a cast trailing edge is not desired even it is very difficult to manufacture, compared to a fiat angle airfoil, extremely difficult to manufacture, and also as far as a overall flow characteristics are concerned, an airfoil which has a maximum thickness at so early at quarter chord, you see it is increasing its maximum within 25 percent, then decreasing towards the 75 percent, remaining 75 percent, and so that is also not a very a good or desired characteristics or desired property of an air foil similarly, maximum camber so behind at mid chord is also again not dear, usually the air foils would like to have all these things at around something near about 35 to 40 percent in that area, maximum thickness is not at 25 percent 35 to 40 percent, maximum camber is also somewhere in that range 40 percent or so, that is what is preferable.

So, in that respect Zhukovsky's air foil are not a very good candidate for practical air foil to be used in air craft however, but only that type of air foil this Zhukovsky's transformation can give, there are some other type of transformation or rather there are many transformations which can give air foil which are more most suitable, I mean having trailing edge angle non 0, having thickness somewhere little behind, or having camber little ahead, even you can construct transformation where you can have many parameters by which you can fix all these as much as you want, I mean you can take those as parameter like I want the maximum thickness to be at 35 percent, I want the maximum camber to be at 38 percent, I want the trailing edge angle to be 2 degree, all these things can also be specified of course, the transformation become complex.

However we remember that Zhukovsky's transformation the simplest transformation, but not the only transformation, there are many other transformation which can be used and we can get different type of geometry or any type of geometry that we want of course, as your requirements becomes more stringent the transformation also becomes more complex; however, as we have already mentioned that this Zhukovsky's transformation or any this complex transformation which is a good technique to find flow very easily is restricted only to 2 dimensional flow, you cannot extend it to 3 dimension.

So, for other practical purposes also we must need to look into some different type of approaches, which are valid say for 2 dimensional any given type of airfoil to any 3 dimensional shape. So, there are many such methods and some of those methods will be covering at different stages of this course, may not be in this particular course, but its continuation courses, what would like to point out one more thing that we have seen that in all these theory, whether we use this Zhukovsky's transformation or any other complex transformation or any other complex method, which can solve the flow for any given geometry, the result that we are going to get will always be lift unacceptable a realistic value drag 0, that is true as long as we are restricted to that equation laplacian phi equal to 0, the drag force that we are going to get is 0, no matter whatever complex method or whatever sophisticated method that we try to solve it, we are not going to get drag equal to 0, unless you bringing something else.

So, what we would be rather doing we will try to do a little bit of course, again we leave something of that also for our later course, that what is technique by which we can get something of the drag also, this gives a lift which is quiet acceptable, quiet acceptable what we have found that lift is more or less lift linearly varies with angle of attack as well as well as angle of attack is small that is quiet acceptable, if the airfoil has a camber then the 0 lift angle of attack will be somewhere at minus 2 times that camber, that is also realistic, again that lift coefficient C C l equal to 2 pi into 1 plus epsilon sin alpha plus beta, 2 pi into 1 plus epsilon alpha plus beta for the general case that is also realistic, it gives very good approximation of the lift coefficient.

So, as far as lift is concerned we can we are satisfied more or less, that whatever we wanted we have we are getting good lift, what we can get in an experimental measurement what we are getting is very close to it, within may be 8 to 10 percent, and the lift coefficient for airfoil or room type of geometry is rather high, and so within 8 to 10 percent of this is quiet alright, but the drag coefficient is always 0. So, we will try to see that what is the reason, why we got drag equal to 0, and how to avoid that situation, how can we estimate drag? at the very rudimentary concept of that we will try to focus, but before that we will of course, complete this transformation by certain numericals. So, perhaps next two or three lectures we will dual on some numerical examples and then we will move to look for non 0 drag.