Introduction to Aerodynamics

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Lecture No. # 38

Zhukovsky Transformation-Applications

(Contd.)

So, last time we considered Zhukovsky transformation for two specific cases, if a remember correctly, one in which the centre wise placed at the origin, and that parameter in the Zhukovsky transformation b was taken equal to R, which resulted in a flat plate airfoil. We considered another case, where again we placed the centre of the circle at the origin however, that two critical points Z equal to plus minus b, they are kept inside the circle, that is b was taken as less than R, and we found an ellipse whose fineness ratio was determined by the relative magnitude of b and R. We will consider few more cases, as specific examples and the first one that we will consider today is that the centre is now shifted from the origin along the say the imaginary axis, that is centre is no longer at the origin but it is shifted along the imaginary axis.

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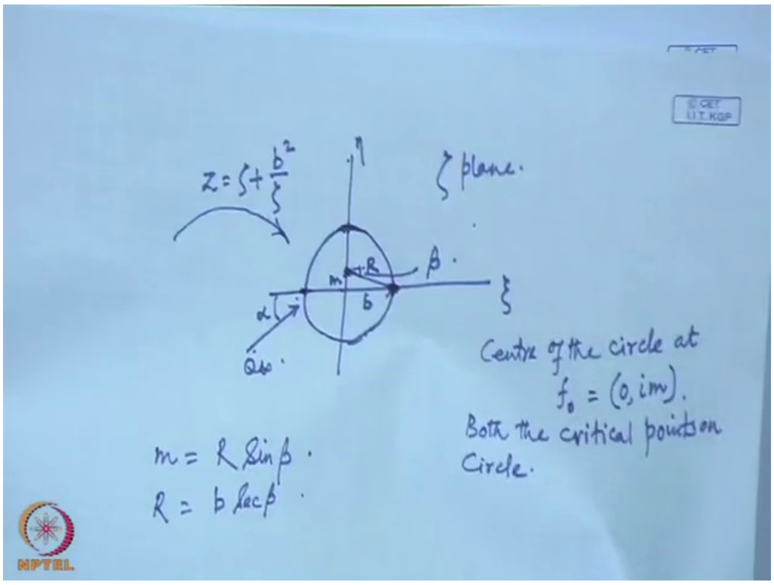

Let us, take that circle plane or zeta plane the centre is not at the origin, but somewhere here, this is the centre. However we make, such that the two critical points Z equal to plus minus b they lie on the circle, this is what is b, and we call this length say m. So, the centre is at and, if we remember the definition of the angle beta, this is the angle beta, then what will be m in terms of beta, m equal to R sin beta, and b or R and b they are also related, b sec beta. One thing is clear that since both the critical points are here lying on the surface of the circle, after transformation both will become two sharp corner, this point as well as this point they will become sharp corner or the resulting geometry will have two sharp corner.

Now, let us say, what is this transformed geometry, what is the transformed geometry? put this in the transformation Z equal to zeta plus b square by zeta, any point on the circle will be given by f zero plus R e to the power i theta substitute that in the transformation then what will be the points on the surface of the transformed geometry x and y substitute this and simplify what we will get is x equal to R cos theta into 1 plus this curve if you looked it this will be the maximum ordinate we will call it maximum thickness though it is of course, not thickness but we will call it thickness and you can see that this will be 2 m pi plus 2 alpha plus beta..

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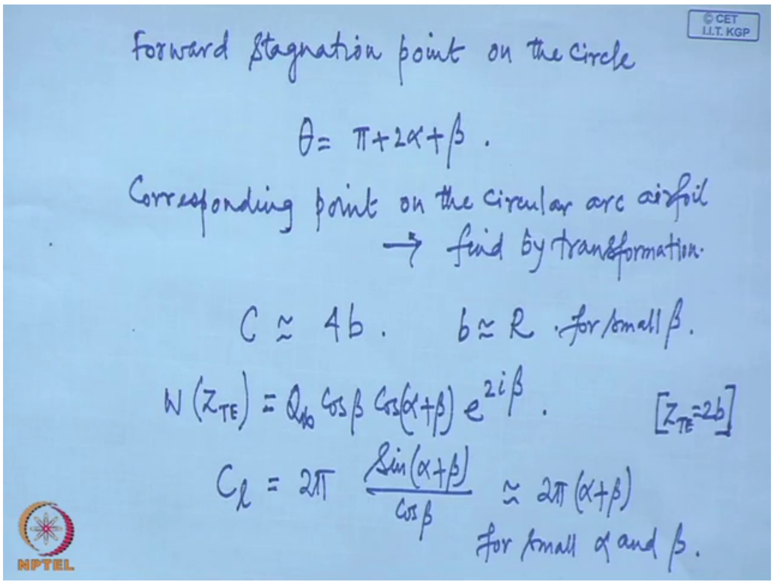

Assuming beta very small approximate it approximate the chord(consider one point first consider the trailing edge point which was at theta equal to minus beta and take beta very small what is the lift coefficient? the lift coefficient if you remember 8 pi R by C is not it? 8 pi R by C sin alpha plus beta by sin beta is not it? the relation this also we derived earlier 8 pi R by C sin alpha plus beta by sin beta or cos beta sorry not sin beta cos beta. and what happen for the other case the two lines are simply parallel this alpha zero is called zero-lift angle of attack, the angle of attack at which lift is zero for a flat plate this alpha zero is zero itself for this cambered airfoil it has become, for this circular arc airfoil it has become minus beta.

Now in terms of the airfoil or that is circular arc what is beta? For the circular arc beta is the location of the that where we have place the critical point but in the circle, if we talk it in terms of the circular arc airfoil what is beta? See beta is related to that maximum ordinate and how much is beta? Let us see what is beta? Beta is actually nearly approximately tan beta that is what, what is tan beta?

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m by

Student: (( ))

m by

Student: b

b, and what is b? b in terms of there is no R whereas, airfoil there is no R that R is only for our numerical help. So, after we get it then we must forget about R and all those things they do not concern us it is C by 4 then?

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R in terms of y max only it is a half of y max (m by b, b C by 4, b C by 4 and what is m? Half of y max, so substitute all those (what happened? Half of y max divided by C by 4, so how much is this?

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2 into y max by C. Now this circular arc we can also thought of it simply the representation of the camber line of an airfoil. It is the representation of the camber line of an of a camber airfoil, and in that case what is the y max by C? It is what is we called the maximum camber or called so, much of camber usually expressed in percentage. So, this beta is then so, that is what is the implication of the beta camber ratio, maximum camber ratio usually an airfoil is designated by the maximum camber ratio not the entire camber distribution, and what you see what the camber does camber from here we can see the camber shifts the zero-lift angle of attack camber changes the zero-lift angle of attack. If your airfoil has positive camber then the zero-lift angle of attack will become somewhat negative two, approximately two times the value.

So, if your airfoil has of say maximum of 5 percent camber usually it will be called that airfoil is 5 percent camber that implies that the maximum camber of the airfoil is 5 percent, 5 percent of the chord, then this gives beta is approximately how much? Two times of that about 0.1 radian. So, an airfoil with 5 percent of camber will have a zero-lift angle of attack nearly at minus 0.1 radian how does it help? It helps, because it allows you a wide, wider range of angle of attack, an wider range of angle of attack. We see that here the C l alpha curve is linear, C l versus alpha which is called C l alpha curve is usually linear but this linear behavior comes because of our linearization of the equation.

We considered our flow to be linear Laplace equation in real case of course, it is not but when the angle of attack is very small it behaves almost linear but as alpha starts increasing the flow behavior becomes little non-linear, and after sometime instead of increasing lift the lift will start decreasing, usually it happens for almost all these airfoils somewhere in the range of say about 15 degree for a finite wing it is little less also. So, the maximum angle of attack that you can use during flight is something if there are no camber is something between 0 to 10 degree or so that is the angle of attack that you can use 0 to 10.

If you apply camber and say that zero-lift angle become minus some 3 degree or 5 degree or so then you can use the angle of attack from minus 5 to 10, we are getting a wider range, and at a particular alpha you see that angle of attack the lift coefficient has increased, and airfoil which has no camber will give this much of lift at a particular value of alpha while at the same value of alpha a cambered airfoil will give higher values of it. So, at a fixed angle of attack camber increases the lift coefficient and in the general case you get a wider range of angle of attack to use.

The other inside the circle that is let us say this let us say we shift it by a very small magnitude a very small amount from the origin, this is the origin we place the centre of the circle somewhere say here, call the centre of the circle, we put the centre here, and then take one of the critical point on the circle. I think I should have taken it to this side. I think let us let us change it actually this should have come to this side we want to make it say b but minus b will be somewhere here this centre sorry, zeta zero which will taking So, that zeta trailing edge that is coordinate of the trailing edge is R e to the power i beta minus

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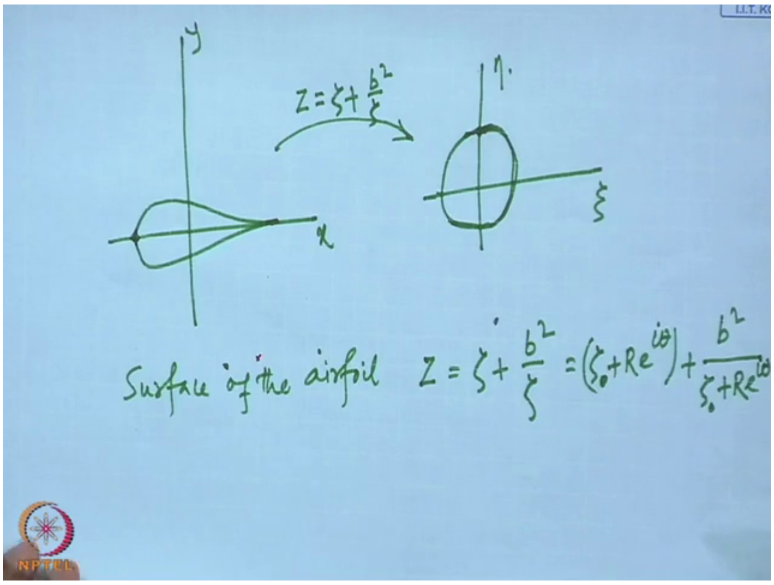

What is zeta? Zeta naught plus R e to the power i theta, zeta is zeta naught plus R e to the power i theta that is in general simplify and simplify and you will find that.

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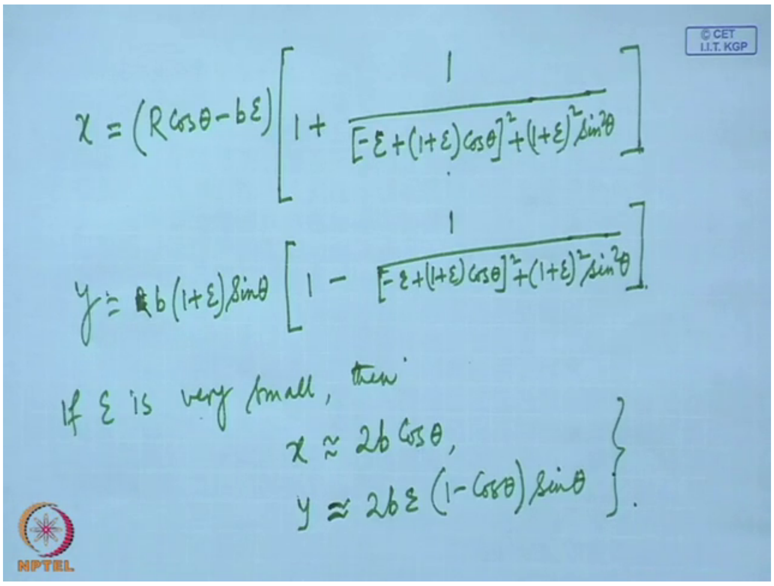

The transform geometry is given by x equal to R cos theta minus b epsilon 1 plus these expressions are of course, not very convenient to handle with but you can see that if you assume epsilon to be very small, then they can be approximated to (we already had a relationship between r and b.

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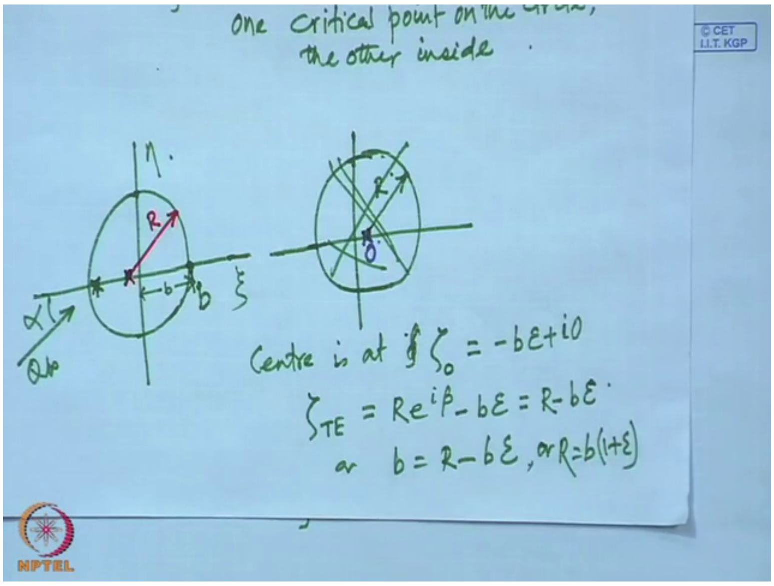

If you remember from here that this relation or it can be written as zero at or R equal to b into 1 plus epsilon.

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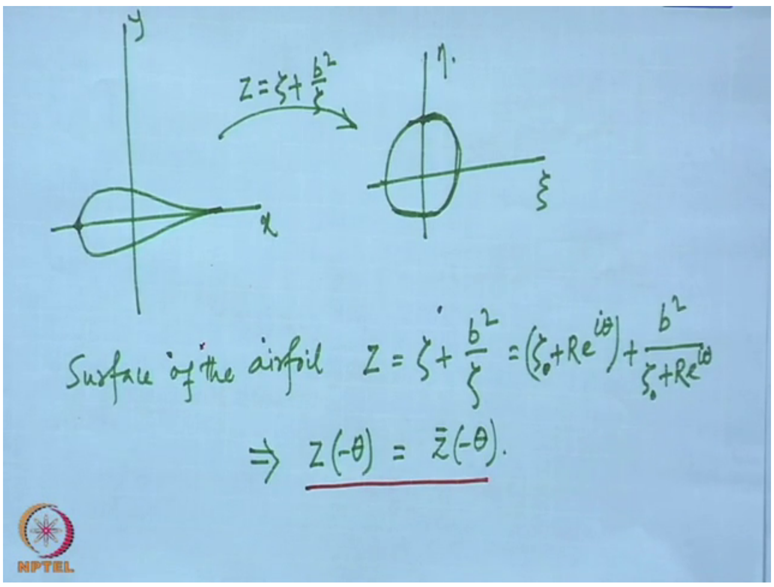

If you look to this relation for Z, you can see that from here agreed sorry.