Introduction of Aerodynamics

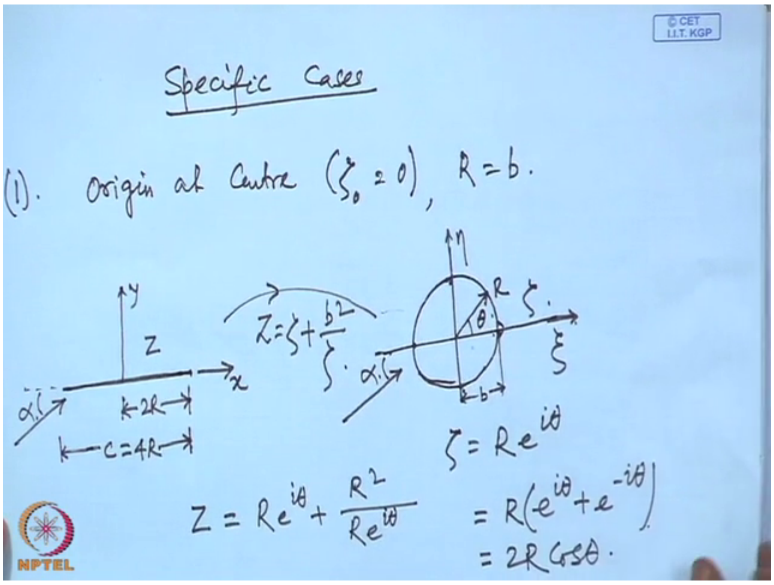
Prof. K. P. Sinhamahapatra
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 37

Zhukovsky Transformation-Application

So, we will first start with the case where we placed the origin of our coordinate system in the zeta plane at the center of the circle itself, and we take the parameter b to be equal to the radius and

(Refer Slide Time: 00:37)

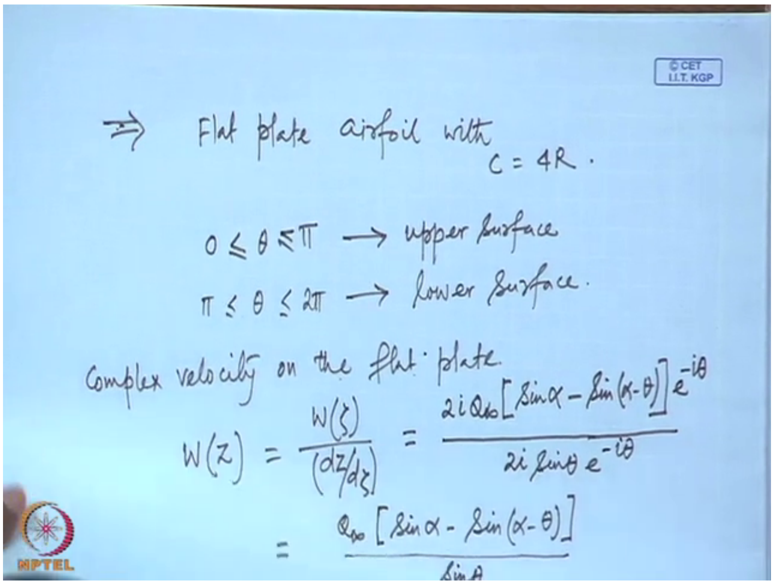


let us say the flow is coming at an angle of attack alpha to this. Now, what will be the transformed geometry? What is the transformed geometry? First of all, you can see that in this case, this and this are the two critical point plus b and minus b; both the critical points are on the surface of the cylinder or on the circle. Each of this critical point will be transformed to a corner; that angle pi between the two curves intersecting at the critical points will become two curves with included angle zero. Angle between the curves two pi and the included angle by the curves is zero as we have seen.

So, this point as well as this point will transform to a sub corner, but what will be the geometry? See in this case, this zeta is Re to the power i theta then, what is Z? which is 2R

Cos theta. There is no imaginary component that means, this transform the transform body is lying only on the real axis in the x z plane just a straight line just a straight line or we will call a flat plate. So in this case, the transformed body is a flat plate airfoil a flat plate airfoil obviously no thickness. The resulting body has zero thickness; how much is the length? This point which has transformed to this point this length is 2R. All other flow properties that is the flow velocity on the transform plane that is on the Z plane. That is flow velocity over the flat plate airfoil can again be obtained using those formula that we have already developed. Velocity on the circle divided by that dz d theta pressure can be obtained then using the Bernoulli's equation lift of course,

(Refer Slide Time: 06:30)



we have already derived that a by a by c; how much is the lift force? Chord equal to 4R; this corresponds to the upper surface of the flat plate where the circle… These are the two stagnation point that we have. This is the stagnation point that we have fixed at the filling trailing edge and the other stagnation point will be see here, the forward stagnation point will be here and the rear stagnation point we have fixed it here.

Now, let us say the complex velocity. We already had that expression; this we simplified last time with the value of circulation; how much was that? What was the expression? Or, you can write that expression 2iQ infinity; it was sin alpha plus beta minus sin alpha minus theta. How much is beta here? Beta is zero; beta is the angle that the line that joints center to the trailing edge point or the center to the critical point with the x axis. In this case, the center itself is on the x axis so that beta is 0. So this become sin alpha minus sin alpha minus theta into e to the power minus i theta.

Sin.

Yes.

Sin minus p alpha plus beta.

Yes, sin alpha plus beta.

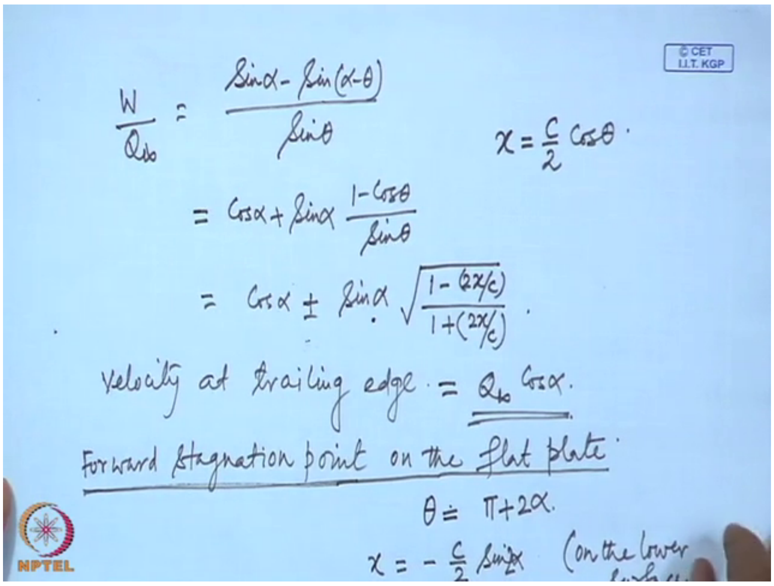
Student: (( ))

Minus sin alpha minus theta.

Student: Yeah, sin beta is zero.

Beta is zero beta is zero here and how much is dZ d zeta? So this gives velocity on the surface of the flat plate at any point. Again we see that, this complex velocity also has only a real component the complex velocity has only a real component meaning on the flat plate plane the flow velocity is simply along x axis. That is or tangential to the flat plate which is as it should be because, the normal component of velocity by boundary condition is zero which was satisfied on the circle.

(Refer Slide Time: 12:30)



Most often this expression is written in this, how it becomes then sin alpha in terms of x, if we use x instead of theta because that might be more convenient to write this as a function of x instead of a function of theta then, what it will be? Theta is the coordinate of varying over the flat plate from 0 to 2 pi instead we want to use it x or x by c; how is x and theta related? x equal to 2R Cos theta 2R Cos theta not R Cos theta and 2R is c by 2 we will forget about this R; c by 2 Cos theta.

So this how much is the velocity at the trailing edge? We know for zhukovskies airfoil, trailing edge is not a stagnation point at the stagnation. At the trailing edge, there is a finite velocity even though the corresponding point on the circle plane has zero velocity that is using this figure. See, this point the critical point has transformed to this trailing edge point; this is the point that has become the trailing edge point. At this point, we have forced the velocity to be zero; we have made it a stagnation point, but this is not a stagnation point on the airfoil surface, which we have already derived yesterday, which would be the velocity at the trailing edge.

So, velocity at the trailing edge; how much? You remember, the velocity at the trailing edge we derived in general is, Q infinity cos alpha plus beta. In this case, beta is zero so velocity at the trailing edge is Q infinity cos alpha. You get you can get it from these two relation, but not from this. So, is there any stagnation point on the flat plate itself? So the circle had two stagnation point; one is at the… One is at this point and one is at this point. The forward stagnation point is here, and the rear stagnation point is here. We have seen that the rear stagnation point is not a stagnation point on the transform plane or for the airfoil. What about the forward stagnation point? Will it remain a stagnation point? If so; where? What will be the forward stagnation point? On the flat plate where will be the forward stagnation point?

Student: (( ))

Where it will be?

That the origin is (( )). At the origin. At minus R. At minus R at minus R. (( )) x equal to. x equal to?

Student: (( )) minus 2R.

x equal to minus 2R, I mean on the flat plate; you mean to say at the leading edge; at this point itself.

Student: No sir.

Then?

Student: At R.

At R? Here, in the middle why it should be? is it become u 0 there?

Student: (( ))

If you say x equal to R, which is c by 4? R is c by 4.

Student: Minus c by 2.

Minus.

Student: 2R.

Minus 2R; no, how can it be? On the… think in terms of the theta itself first; where will be the forward stagnation point? Where the circle where the stagnation forward stagnation point is? At theta equal to… that is not that this at what point which point?

Student: (( ))

Theta equal to?

Student: Pi plus alpha.

Theta equal to pi plus alpha then, where is that on the flat plate?

Student: (( )) minus 2R cos alpha.

Minus 2R.

Student: Cos alpha.

Cos alpha.

Student: No, w is not coming zero exactly (( ))

But, w should come zero if it is stagnation point.

Student: (( ))

So, it is not at pi theta equal to pi plus alpha, but where it is? You know the expression for w either, you can make it zero, you can find out at what theta it will become zero; at what theta this will become zero or.

Student: (( ))

Where is that point going that theta equal to pi plus alpha on the circle? Theta equal to pi plus alpha on the circle; where it is? What is the point it is being transformed to? Is that again theta equal to pi plus alpha? And let us say forget this perhaps, this is easier; w equal to zero for what value of theta? Yes, solve this.

Student: Sir, zero all (( ))

Theta equal to.

Student: (( ))

Pi plus.

Student: (( ))

Two alpha.

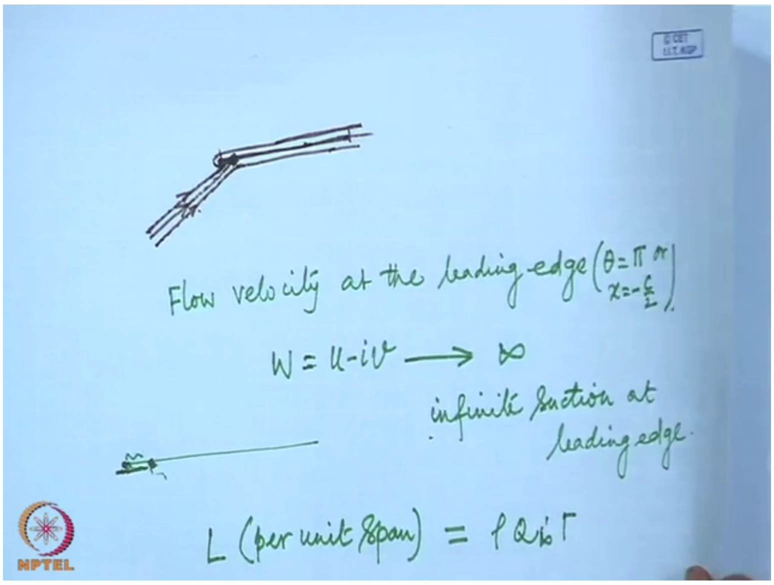
Student: (( ))

Theta equal to pi plus 2 alpha.

Student: (( ))

Or in terms of x, it will be sorry sin 2 alpha; in terms of x, it is minus c by 2 sin 2 alpha. This is obviously on the lower surface; look to then say what will be the velocity?

(Refer Slide Time: 26:08)



I mean, the flow pattern. First of all, think that this is what is our flat plate; the flow is coming at an angle alpha; the forward stagnation point is somewhere here on the lower surface. Let us say it is somewhere there. It is on the lower surface; it is not at the front point; it will it will be at the front edge when alpha is zero; for any non zero alpha, it will be on the lower surface assuming positive alpha.

Then, think about say two streamlines adjacent to this. This is of course, becoming this the body itself and another streamline which is here is following parallel to this body. Another streamline which is ahead of it; going here then, it is turning and again becoming parallel to this. So, these are the streamlines flow streamlines or the flow will be like this. The inviscid potential flow about a flat plate airfoil will be like this. This streamline will go here and then, will turn back and go round over the upper surface. What is the velocity at the leading edge? Can you find out what will be the velocity at the leading edge? Leading edge corresponds to theta equal to pi; the leading edge of the airfoil corresponds to theta equal to pi or where x equal to minus c by 2; you can use either. Yes.

Student: (( ))

The complex velocity u minus i v, how much it is at the leading edge?

Student: (( ))

Q infinity cos alpha Q infinity cos alpha.

Here also you are getting Q infinity cos alpha. It is little bit strange; see, w equal to Q infinity sin alpha minus sin alpha minus theta by sin theta, and how much is sin theta?

Student: (( ))

Sin pi.

Student: Zero.

Zero then

Student: (( ))

If you take limit then it is a?

Student: (( ))

So pressure at the leading edge, how much it should be again infinite.

Student: (( ))

Pressure should be zero; why?

Student: (( ))

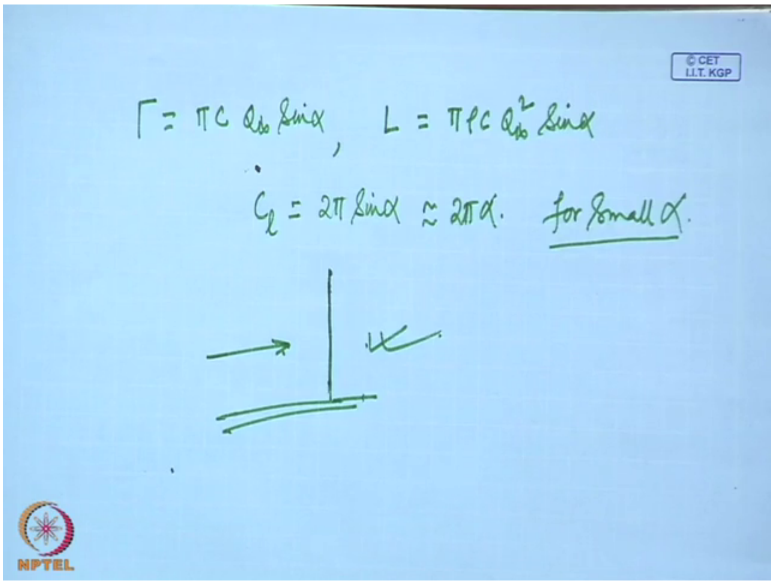
That means, pressure also you can define; velocity is not defined; pressure is also not defined. Meaning that, at the leading edge point at the leading edge point and infinite suction peak is created an infinite suction is created at the leading edge or the leading edge is basically a singular point in this case. Now, we see that is there any justification it of course, this is not correct in a if we consider any real flow problem even flow about a flat plate at a the leading edge is not undefined. Usually, a viscous flow that will be a stagnation point and or the flow velocity they are zero there also, and pressure will be some finite pressure the pressure will be some finite pressure, but in the in this inviscid flow, if you look to this figure you see at this point which is the stagnation point from here then, as we move to this side which is part of upper surface. This part of this the front part of this leading flat plate, which is ahead of the stagnation point. This is the stagnation point and this part on the lower surface flow, velocity is increasing from zero to some something. At here it is becoming something undefined. You are calling it infinite in the sense that this is going on increasing, the flow velocity here is zero and then gradually increasing and here it is undefined. It is you are saying very large because, it has very large (( )).

So, there is a very quick or very high acceleration in the flow velocity flow over this small distance. This is of course not a very large distance. Over this distance, the flow has accelerated very rapidly to a very high value. Consequently, the pressure here is also; as the velocity increases pressure decreases, the flow pressure there decreases. So, if when the flow increases tremendously, we can say the pressure has decreases tremendously or infinitely minus infinity, and we are calling it suction so our of course, you can never verify it experimentally because in experiment, we will never get this ideal inviscid irrotational flow. You will not be able to create that ideal inviscid irrotational flow in a in an experiment. So, you can never verify this experimentally, but what you can verify that, what happens to the pressure or velocity near at the leading edge. If you do that experiment with some air foils, you will see that at the leading edge the pressure is really very small really much lower compared to the pressure at any other point, and flow velocity is quiet large near the leading edge. And, as the thickness of the air foil decreases, the suction near the leading edge also increases; that is pressure decreases as the thickness goes on decreasing thickness of the airfoil. So in the limiting case when the thickness is really zero, we can except that the pressure is approaching to very small value or the suction there is very large infinite. But of course, this is not real, but a very large suction near the leading edge is a reality.

While you will be doing some experiments and see, we will measure the pressures. We will see that there is a very large suction near the leading edge, and that suction increases as your angle of attack alpha increases as well as your thickness decreases; for thinner air foil the suction is higher. Similarly, at higher angle of attack the suction is higher. How much is the lift force per unit span? lift force per unit span? You have already derived the general formula for lift force or the lift force is rho Q infinity circulation. How much is the circulation here for the flat plate?

Student: (( ))

(Refer Slide Time: 37:38)



Let us write it at this stage and then first calculate circulation. How much is the circulation?

Student: (( ))

Yes, pi.

Student: (( ))

Pi c Q infinity sin alpha. So the lift force you can straight away write

Student: Two pi sin (( ))

Two pi

Student: (( ))

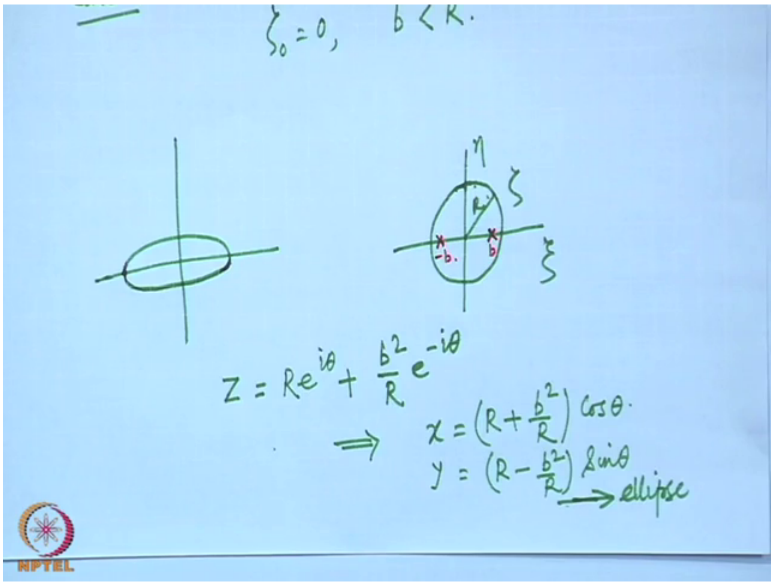
Sin alpha 2 pi sin alpha. And see in practical aerodynamics, the angle of attack is usually quiet small; look into an aircraft most of the time the aircraft flies at angle of attack which is somewhere very close to 1 to 2 degree except leaving landing and takeoff where the angle of attack is little higher of the order of some 12 to 15 degrees in that range.

Most of time or say 95 more than 95 percent of its total flight time, the angle of attack of an aircraft is usually of the order of 1 to 2 degree which is quiet small. And for such small angle of attack, we can make an approximation which is 2 pi alpha A very remarkable result with such simple theory; the lift coefficient that is given this two pi alpha comes quiet close to the lift coefficient of an airfoil.

Next, we will consider another case. Next case that we will consider of course we can find, what we have found here flow along the length of the plate flow along the length of the plate. We can find even flow which is normal to the plate; you may try that how to get it; flow normal to a flat plate. That is the plate is like this plate is like this and flow is coming like this. For such a situation for such a situation, this inviscid ideal flow is not a very good approximation. It is very very poor approximation, but anyway it can be done using these transformation and you will look, how to get this; I will consider… I am not doing it here; you have to look for it how to get it.

The next case that I would like to consider is again the centre of the circle is placed at the origin itself, but the radius of the circle is larger than b or that parameter b is smaller than the radius of a circle.

(Refer Slide Time: 42:39)



Let us say, this is what is b; b is less than R; this is plus b; this is minus b; that is the critical points are now inside the circle. There is no critical point on the surface of the circle. So, the transformed geometry will not have any sharp corners the transformed geometry will not have any sharp corner at each and every point on the transformed geometry, the angle is same as it is on the circle; that is again pi. What is this body now? Z equal to zeta plus b square by Z; zeta is Re to the power i theta on the surface of the circle. What it is? What is this? Yes.

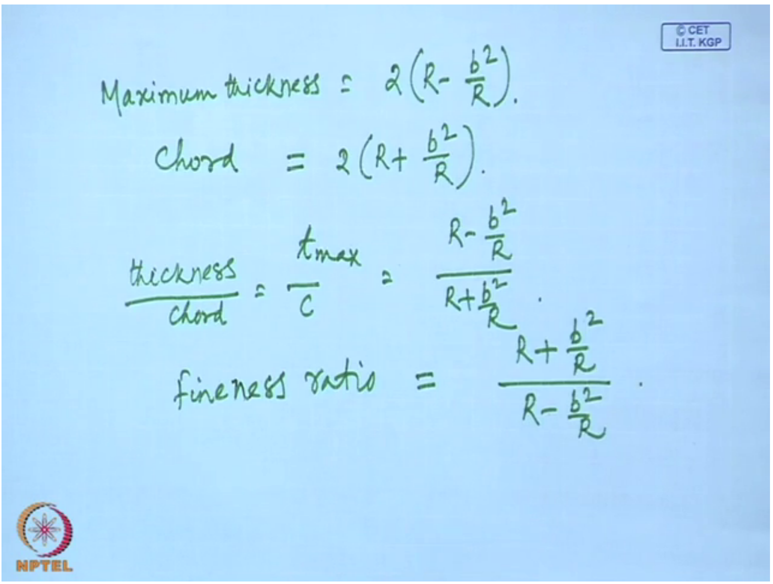
Student: (( ))

What is this geometry?

Student: (( ))

So, this is an ellipse with synonymous axis of R plus b square by R and the minor axis is half of the minor axis is R minus b square by R. We will not call it major axis and minor axis rather, we will call thickness and chord.

(Refer Slide Time: 47:11)



So, the thickness or the maximum thickness of the transformed geometry and in analogy with the thickness to chord ratio for an air foil here also, you can have a thickness to chord ratio which is also called a fineness ratio. That is by thickness you have you of course mean t max and in case of ellipse instead of this t max by C, this fineness ratio is a more preferred parameter which is the inverse of this quantity.

So let us say, you are asked to find the solution or the flow solution for an ellipse. Can you say what will be your steps or how you will proceed to it? See, you will be given this either flat plate or ellipse or some other air foil that we will be getting, whether the things that will be given to you not that circle that is what your job is. Let us say in this case, we are showing that we have something which is in elliptical shape, and we want to know the forces acting on it, and the it is also told that the force is only due to pressure then, what will be your steps? What will you do? Any suggestion from anyone, how to proceed?

Student: (( ))

How to proceed Let us say, you are given that you have we have an elliptical or let us say, an elliptical column and a wind is coming to it at a facing such an angle, at such a such an speed; find the load that is acting on this column due to this pressure so that ultimately, you are going to construct that column. How you are going to use this knowledge that we know the solution of the flow solution for ellipse provide from the known solution on circular cylinder how you are going to use it? Because, that is what is that is what is your problem. No one is going to ask you, we have a circle and in that centre we have a doublet of this strength of a circulation of this strength and find this pressure on this circle no. The problem is this.

Let us say think in terms of aircrafts only. Someone say that the landing gear of an aircraft are elliptical in cross suction and then, the aircraft is running on the ground at so that at such a speed, the flow is coming to this landing gear at such an angle of attack to find the pressure acting on this landing gear. So that you will be able to find, how much it should be; it is; or what type of material you should use; what would be the strength of that material. So, what? How you are going to proceed? You know all these; how much is the flow velocity; you know all these formulae, every everything.

Student: (( ))

Um

Student: (( ))

Um

You need to convert R and v into c and (( )). We will convert R and v; where are we getting R and v? When we do not know R and v.

See, as far as the flow over column; let us say these column itself in front of you of course it is not elliptical, it is rectangular is concerned.

Student: (( ))

Where is it is R or where is b? We are converting R and b into the terms of (( )). So knowing t by C or the fineness ratio of the ellipse since that ellipse is known to you, you see that is the reason see in our transformation. However, if you look to the other, you are showing it this way that transformed from Z to zeta not zeta to Z. Here also, it is if you know if you know about the ellipse; let us assume that you know the fineness ratio of the ellipse that will give your b by R. First of all, if you know the R, you know what is its doublet range for the

Student: (( ))

Flow over circle and applying that vector condition, you can also find what would be the circulation then, you can write what is w zeta and once you can write the w zeta, next of course everything comes straightway. So, the velocity and any other of course, you can find here. Once again in this case, since there is no critical point on the circle, you cannot satisfy kutta condition and you cannot say what is circulation. So, this flow will either be treated as null lifting; there is no circulation or there will be some other condition given from which you can find what is circulation. Just as it is, we cannot apply kutta condition because, there is no sharp corner on the ellipse; on the ellipse, there is no sharp corner and hence, you cannot find what would be the rear stagnation point on the cylinder or the circle for a given case. That is essential to have a sharp corner where we can satisfy the kutta condition trailing edge at the corner.

For an ellipse, there is no trailing edge corner. So, you cannot satisfy kutta condition. We cannot say what is circulation or how much easy it is. So, it is it will remain unknown as in case of original cylinder. However, it will be usually treated that it is non lifting like circle is usually cylinder is usually non lifting; elliptical cylinder will also be treated as usually non lifting. There is no circulation or something else must be given something else must be given which will fix the value of circulation. Unless those are given, you cannot fix the value of circulation in this case. We will consider few other specific cases before we move down to something.