Introduction to Aerodynamics

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Lecture No. # 35

Zhukov sky Transformation

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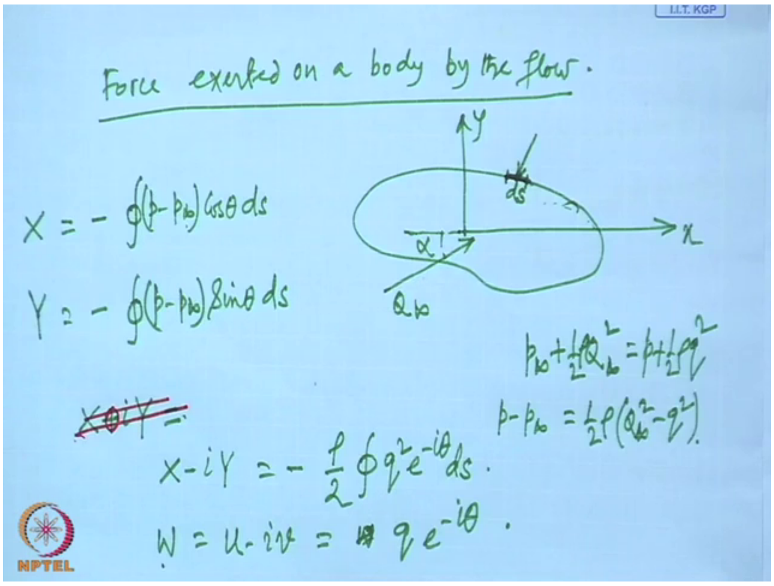

See this, your dx and dy will have different sign; when you consider this length element ds, your dx and dy have different sign, you consider whatever your order is direction of cyclic integration say conventionally theta this, changing from here to here. So in this case for this part of the length, the way we have shown here your x is decreasing, y is increasing. So, take that into account, and then say what is i theta ds.

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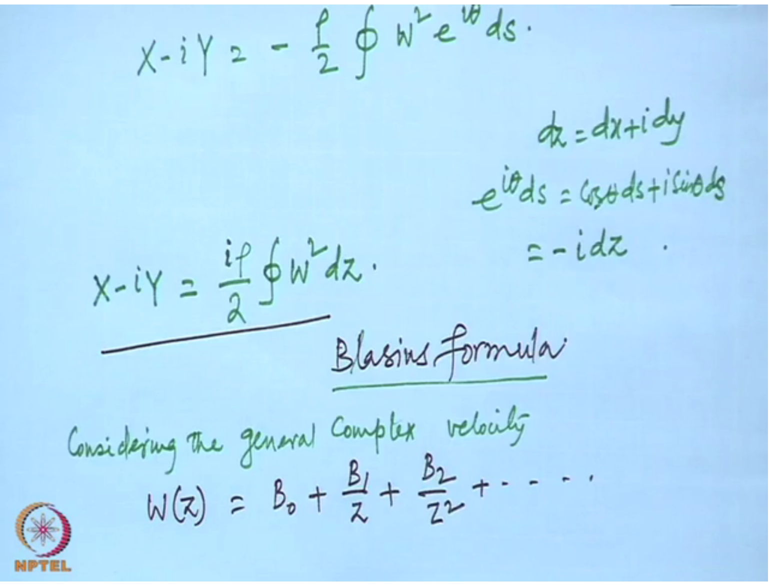

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d s minus only i d y

only i d y

What what is this becoming? Minus minus d x plus i d y what is that? So this is minus d x plus i d z sorry minus d x plus i d y what is that? Minus d x minus i d y, this is minus i d z and then we put it here, what we get then, x minus I y is equal to i rho by 2 w square d z. This gives the force exerted on a body, by two-dimensional ideal flow or two-dimensional potential flow, on a body force exerted by a two-dimensional potential flow general. This formula is known as Blasius Formula. A similar formula is also present there for moment but right now we will not do it.

Now, considering the general complex velocity, in any case the complex velocity of any flow problem as long as it is two dimensional ideal flow, will be given by this expression, some term may not be present in the particular case but, any other term cannot come, the terms which are here only some of them may come in a particular problem but nothing new they must be within this, must be a subset of this, so this is the most general complex velocity. So, assuming this complex velocity, what will be this force? First of all look, whether this function has any singularity, z is equal to zero is a singularity. And, we are interested only in a integral, over a closed control and z is equal to zero is within that control or origin is within that body. So, if you remember that, because of this integration according to that residue, theorem only what is required, the coefficient of one by z nothing else matters to us only the coefficient of z.

Now, anyway the function is not w but, w square, so think about w square, what will be w square and we are interested only the coefficient in 1 by z nothing else, because as you see from here after the integration, we will have only one by coefficient of 1 by z that is what will give us non zero integration, all other will give us zero, so we are not interested to know what they are, what we need is you square it and what is the coefficient of w square z.

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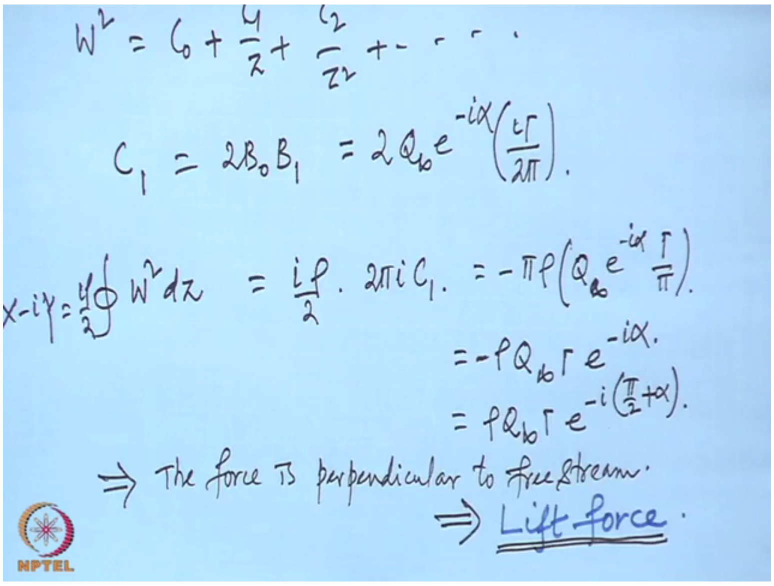

Let us say, W Square is c zero plus, c one by z plus, c two by z square plus so on and c 1 is equal to how much? When you square this, only 1 by z term will come because of that. Product of these two that is what is going to give us one by z nothing else is going to gives us one by z, each of these terms will be square or they will be multiplied by 2, so only these two will be multiplied and that will gives us square. So, coefficient c 1 is only 2 b zero b 1 and how much is that, b zero is what. Looking to this expression, for compress velocity b zero is the constant velocity present or uniform velocity present in the flow, which is q infinity e to the power minus i alpha, that is the only velocity which gives a constant velocity q infinity e to the power minus i alpha and b 1 by z, what basic solutions gives us velocity one by z, those which have complex potential log z, which has complex potential of log z, source and vortex.

Vertex the doublet, the way we have taken has no log z, so what will be d one in a general case and what will be b zero in a general, b zero in general case is q infinity e to the power minus i alpha and what is b one, b one, m by 2 pi and considering a clockwise vortex, it is minus i gamma by2 pi or m minus i gamma by 2 pi in general case. If you remember, that m usually the source gives a flow field which is spherically symmetric in all direction, so from physical ground we can consider or we can expect that source will not give any force, because it is not going to create any asymmetry anywhere and force will be created only if, the pressure is asymmetric, if there is no asymmetry in pressure, there will be no force.

Since source gives a spherical symmetry in all direction, the source will have no contribution and will neglect that part of source and we will take only that, i gamma by 2 pi. This force x minus i y is equal to i rho by 2 into w square d z, which we know according to our theorem 2 pi i c 1 residue theorem. So that makes how much pi square minus pi rho into q infinity into e to the power of minus i alpha. That is why, we have taken clockwise direction, that we have the negative sign there and in the force term, we do not have the negative sign that we have seen, when we discussed about that circular cylinder or you can write it rho q infinity gamma e to the power minus i.

Now what is this direction of this force? Looking to this expression itself rho q infinity gamma e to the power minus i pi by 2 plus alpha, if you remember q infinity e to the power minus i alpha was the free stream. Free stream was making an angle alpha to the x axis and that term is represented by e to the power minus i alpha m this force is now e to the power of minus i into pi by 2 plus alpha, so what it is, in which the direction of the force is, perpendicular.

This force is clearly perpendicular to the direction of the q infinity, we need not write even it is a real component and imaginary component and all that is not required, from here itself we can say that the this force is perpendicular to the free stream direction and which is by definition is the lift force and we can see again that it has now become general, that in a two dimensional ideal flow, the force can be only lift force, we cannot have a drag force a force in the direction of the flow is not possible of course, this is an unacceptable result, that force in the direction of the flow is zero, always which is not true is never true.

But that is what a two dimensional potential flow gives. The two dimensional potential flow gives only lift force. Now, with this in background let us now try to postulate some transformation, which we can apply and get the flow over an airfoil from the known flow about a circular cylinder.

Now, once again you know that whether we are in the circle considering the flow about a circle or considering the flow about an airfoil, at far away from the body, that is in the z plane and zeta plane, let us say in the z plane, we have the airfoil and in the zeta plane we have the circle, so both in the zeta plane and the z plane at far away that is at infinity, the flow is undisturbed the flow field is a uniform free stream, far away both in the z plane and zeta plane, q infinity is same, when z is infinity or zeta is infinity meaning, that once again the transformation function zeta as a function of z, cannot have higher power of z because, if we have z square z cube and so on, once again the velocity will not become same.

So, any transformation we may think of, any complex function we may think of, it cannot have z square z cube and so on and so the highest power of z in the positive side we can have only z to the power 1, in the negative power of z of course, we have no restriction, we can go to anything 1 by z, 1 by z square, 1 by z cube as in the case of complex velocity, in case of the transformation

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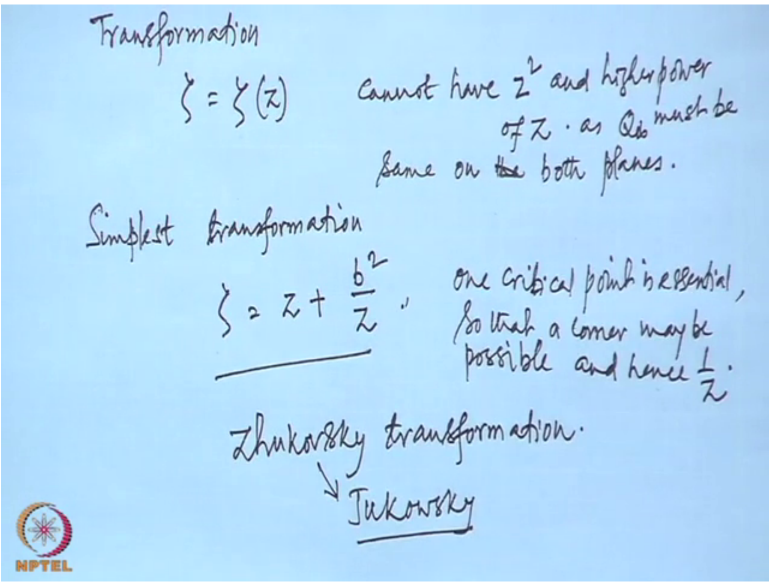

So, the transformation we considering polynomial or series as a transformation function we can have only zeta equal to some z plus 1 by z or 1 by z square, 1 by z cube anything we like. Now, the simplest will be z plus 1 by z, that will be the simplest transformation and this transformation or a similar type of transformation is called a Zhukov sky Transformation and that is what we will study. we must say that we could have even simplest only z y even b square by z, no we need it, because our airfoils will have a sharp trailing edge will have a trailing edge which is sharp or something nearly sharp.

While in circle, we do not have any sharp point, so we must have at least one critical point, if we just have z, which do not have critical point d z d f only at 0, for z we do not have any critical point on the surface of the circle we will not have, but we need at least one critical point, so that we can have one point which may become a corner, we need one corner, an airfoil has an corner a trailing edge.

So, that is why you must have one critical point on the surface of the circle and that is why at least this will give us a critical point, otherwise there will be no critical point. So, that is why this first term, hence it is simplest we are not considering second, third and other terms only first term is not possible because, that we will not get an airfoil type of body or we would not get anybody which has a corner but we need body with corners hence this is the simplest transformation. These are the two reasons, why all transformation is like this.

This is of course, not possible we cannot have z square or anything, here we could have one by z square and so on, but they will give more critical points and we are not interested in that, so we are taking this is the simplest case, and also we do not want to make it complicated and this transformation is called as Zhukov sky transformation, in some book you may find this spelling also, Jukowsky.

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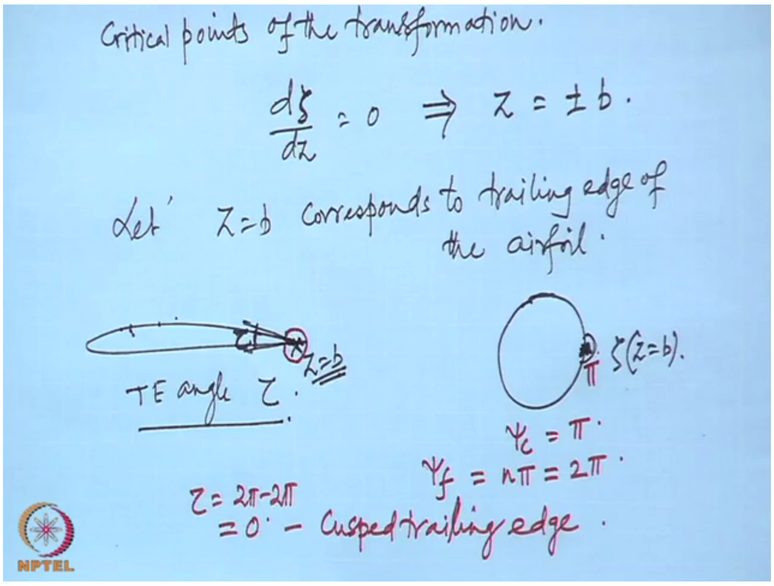

So, what is the critical point of this transformation? At critical points, what happened? d zeta d z equal to 0, so Z is equal to plus minus b. What happens to the angle at the critical point? Let us apply this transformation to transform an airfoil to a circle, then what will happen to the angle at the critical point, let us say one of this critical point coincides, with the trailing edge of the airfoil, let us say z equal to plus b this point corresponds to the trailing edge of the airfoil. So, what is the angle at the trailing edge, let us say these are the two tangent at the trailing edge and this is the trailing edge angle, we are calling the trailing edge angle to be tau, how much is this? This the point corresponds to z equal to plus b, on the transform plane this must also corresponds to some point on the circle, let us say this is the corresponding point, this is what z equal to b in the z plane and in zeta plane it corresponds to this, so how much is this trailing angle tau?

Consider two curve here that is, two segment of the circle, just on the two sides of this point, we will take those are the two curves and what is angle, contain by those two curves pi at any point on the circle, two intersecting curves of course, they are part of the circle only, in that sense, they are intersecting the angle made by them is pi, if this pi root have been are regular point, here also the like these points and these points they correspond to some of these points, so the angle is remaining pi, what happen here at the critical point, the angle will be multiplied by n times, where n is the lowest order of the non zero derivative.

So, what is the lowest order of the non zero derivatives? z equal to plus b, the second derivative itself is non zero, so let us call it the angle and this angle becomes this, the psi c equal to pi, then on the airfoil, psi will make a confusion do not think that it is a stream function angle, psi f will be n into pi and that equal to 2 pi, then how much will be tau, tau is 2 pi minus 2 pi, the trailing edge angles are zero.

So, by this transformation that is, from using Zhukov sky's transformation. only those airfoils which have zero trailing edge angle, they can be converted to circle or the circles can be converted to airfoils with zero trailing edge angle, the airfoils which have zero tailing edge angel they are called cusped trailing edge. Unfortunately, however all practical airfoils have non zero trailing edge angle, a cusped trailing edge then, the airfoil will look something like this.

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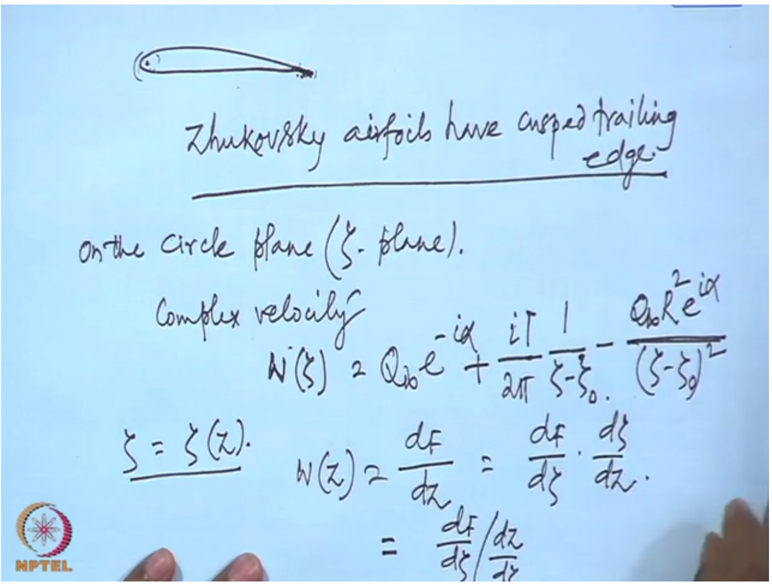

The two side's upper surface and lower surface are tangential linear the trailing edge. But this is not what the common airfoils looks like, in the common airfoils the trailing edge is small but finite angle, not zero. So, most of the or other all practical airfoils are having finite trailing edge angle, they are not cusped, this zhukovsky's airfoils they have cusped trailing edge. So this also you can say as zhukovsky airfoils are have cusped trailing edge.

Now, what essentially our transformation is, in the zeta plane we have the circle and we know the solution of the potential flow about a circle, which is given by a superposition of uniform stream, a point doublet which strength is given by the radius of the circle and if there is a lift, then a circulation, if there is no lift then circulation term is not there but, airfoils will in general give lift, so we are interested in lift, so we will consider that circulation, from the zeta plane we have a flow which is made up of a uniform stream, a point doublet which strength is known and a circulation which strength is arbitrary.

Now, this is the flow that we want to convert it to an airfoil, now essentially what is the conversation of a flow, a flow is a defected by a certain flow pattern, some streamlines what are they? A curve, so if we know the streamlines for the circle, we can convert those streamlines or transform those streamlines by this transformation and they will become the streamlines for the airfoil.

However, to do that see we have certain free parameters, what are the free parameters? As for as the circle is concerned, circle has two parameters, one is center and it is radius. So, given an airfoil, we have to choose the appropriate circle, meaning we have to choose the center of the circle and its radius. Radius is of course, we will given them linked to the strength of the doublet which will be related to the chord of the airfoil, if you remember, the line that join the leading edge and trailing edge. How to find that leading and trailing edge, that we consider the highest curvature and the radius of that curvature at the trailing edge side and the leading edge side, which will fix up the leading edge point and trailing edge point, leading edge is all right that is edge but, when you say which point, then it becomes a difficult question, but, which point? This part is leading edge part, we know this part is trailing edge part but, which point is the leading edge point and which point is the trailing edge point?

That is given by that the largest radius of the curvature in this and the corresponding radius of the curvature on the center of curvature and similarly, here also in this case of course, there is cusped trailing edge, so no question of curvature. But, in general case they are also that locate the center of curvature and join that two curvature and exchange it, wherever they intersect the airfoil surface, those are the leading edge point and trailing edge point but, anyway the chord is the line that joints those leading edge point and trailing edge point via straight line.

Now, let us say that how will the velocity v transformed? The streamlines for the circle, we can write the equation for the streamlines, you have already written and then each point we can transform and we get the streamlines for airfoil but, for the velocity how will the velocity v transformed? Velocity at a point in the circle is known to us the complete solution is known to us, then what will be the velocity on the airfoil complex velocity, complex velocity on the circle and complex velocity on the airfoil how are they related? On the circle plane which is zeta plane what is the complex velocity? The complex velocity is given by a free stream, a point doublet located at the center of the circle and a clock wise circulation, so the complex velocity due to uniform stream is we will consider a general uniform stream for circle, whatever the direction of the uniform stream, it is always towards a diameter but, the different diameter will represent different line, as for as airfoil is concern.

So, we must differentiate that, so we will consider a particular fixed x axis and the flow direction with respect to that particular angle, so the complex velocity is due to uniform stream, which is q infinity e to be power minus i alpha, the uniform stream contribution, plus a circulation i gamma by 2 pi, 1 by zeta minus zeta naught, once again the distance is measured with respect to the center of the circle, where zeta naught is the center of the circle, plus a point doublet located at the center of the circle and doublet strength is related to the radius of the circle, what is the complex velocity due to a point doublet? Minus q infinity r square e to be power i alpha by zeta minus zeta naught. The vortex strength was how much, the strength of the doublet was mu equal to 2 pi r square u infinity or if r is the center of your circle, then the doublet strength must be 2 pi r square u infinity.

So, that we have used q infinity r square and 2 pi on the nominator and denominator got canceled, here also there was on 2 pi, if you look back to the formula and e to be power i alpha has come because this doublet axis is along the x axis but, we have a consider, a flow which is in making an alpha angle direction, so there will be another alpha rotation for that, so this the complex velocity on the circle, what will be the velocity on the airfoil plane the z plane? Our transformation is zeta equal to or z equal to zeta z, what will be the velocity in the complex plane w z.

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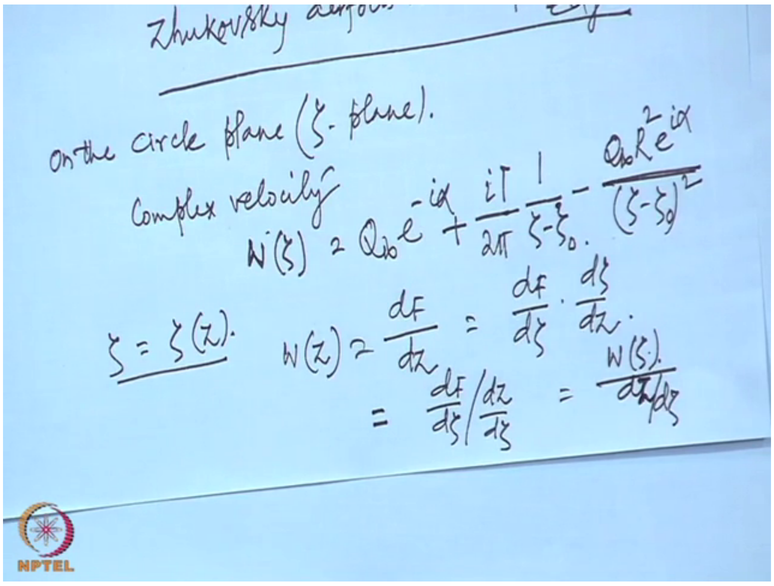

So, once we know the complex velocity at any point, we can find out what will be d z zeta at that point and find the velocity on the z plane or the airfoil plane.

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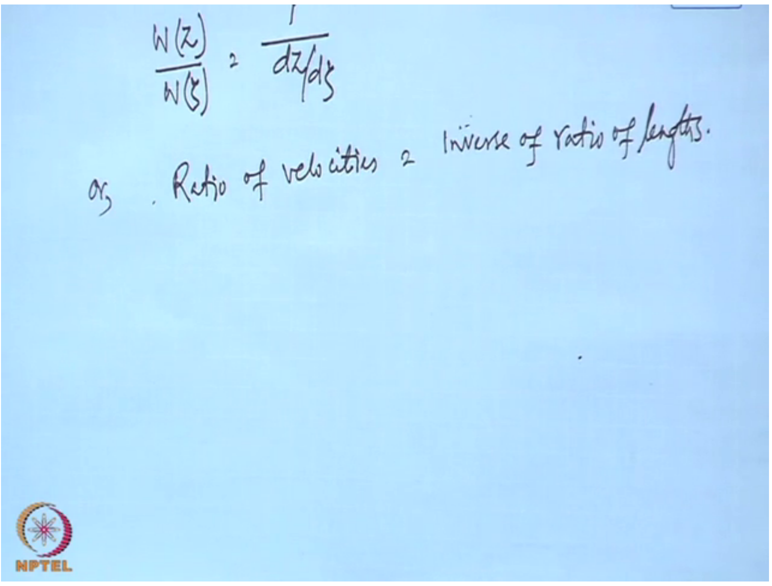

This is often expressed in this form also that… We will now apply this transformation to certain cases, choosing those two free parameters at r, we have already said that two parameters the center of the circle and the radius of the circle. And choosing the two, we see that what type of geometry we get, the flow field is of course, obvious once we apply that transformation.

Thank you.