Introduction to Aerodynamics

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Lecture No. # 34

Conformal Transformation (Contd.)

So, last time when we ended, we were discussing about the angle at the critical point; whether, we are looking for the question, what happens to the angle at the critical point of transformation? At all the regular points, we have seen and established that the angle is preserved; the angle at the jet plane and the angle at the zeta plane are same. The points where the transformation is regular, at the critical point of the transformation, we were looking to see what happens to the angle. How do we find the critical point of transformation?

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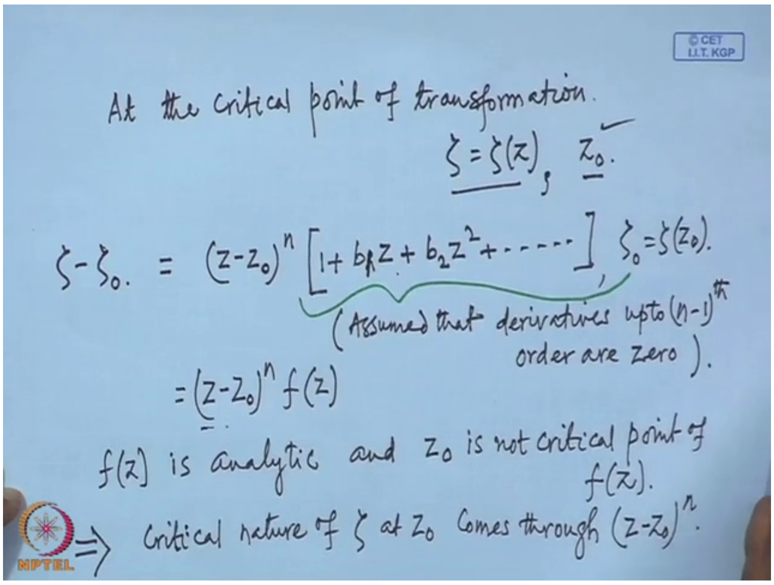

At the critical point of transformation, the transformation being zeta this is the transformation and this is the critical point and we have seen that we can expand, this function near the critical point as where zeta 0 is zeta at Z, Z 0, so it is a constant and we assumed we assumed that the point is Z 0 is a critical point meaning that the first derivative is 0 and in addition we assume that, all the derivative second, third, fourth up n minus 1 all are 0. Then, this part of the function, let us call that another function, 1 plus b 1 Z, plus b 2 Z square and this we are denoting by f Z.

Now, this f Z is of course, again analytic and it has no critical point because, you have already assumed that up to n minus 1th derivatives are 0 and nth derivative they are non-zero that is what we have assumed. So, the nth derivative is basically the first derivative for this function now hence that is not 0, so this part of this function that is, Z 0 is not a critical point, so we will write f Z is analytic, however Z 0 is a critical point for the transformers zeta, that means that criticality comes because of this part. So you can say that comes to So, if we consider 2 curve, that are intersecting at point Z naught in the Z plane and at point zeta naught in the zeta plane, then the angle between two curves Z naught in the Z plane and Z 0 in the zeta plane should be intercepted in that way, in one plane it is intersecting at z naught, in the other plane it is intersecting at zeta naught, so curve 2 minus curve 1 is f Z is a conformal transformation, so this angle is this part is 0.

So, what we see is that, in this case the angle has become n times, what is n the first derivative which is non-zero, it is of course, an arbitrary depending upon the function that we are choosing that is what is n. So, just to find out, what is the lowest order of derivative, that is non-zero at the critical point. Critical point means, the first derivative is 0, then we see that the second derivative is non-zero, or third derivative is non-zero and so on. Only we need the lowest. We will find out in some cases, what will happen to the angle?

Now, before we try to this apply this transformation as you said that our plan or our aim is to apply this transformation, so that an airfoil becomes a circle. You would like to apply the transformation in such a way, that an airfoil becomes a circle or vice versa, circle becomes an airfoil because, we are interested in flow over an airfoil. However, before we apply that transformation or look for some sort of transformation, let us consider since we have to consider everything in complex plane, let us look for the potentials, the complex potentials. So, what are the complex potentials? we have seen that our basic flows or the basic solutions of Laplace equation are a uniform source flow, point source, point doublet, point vortex. So, what are the complex potential for the basic flows or let us say what is the complex potential for our uniform flow or uniform system uniform stream lets us say in x direction?

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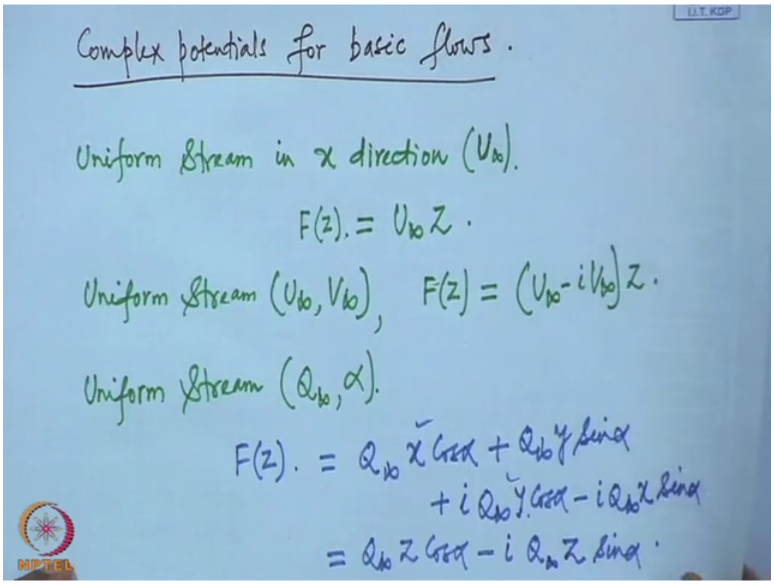

We will write some of these complex potentials complex potentials for basic flows, uniform stream in the x direction is; uniform stream in the x direction what will the complex potential? Complex potential is denoted by F. What is complex potential phi plus i psi for a uniform stream in x direction, what is phi? Uniform stream in x direction the magnitude of the flow velocity is U infinity, so what is the complex potential in the Z plane, phi plus i psi, what is phi for a uniform stream in x direction?

U infinity x, psi U infinity y, so what is complex potential U infinity x, plus i U infinity Z what is that U infinity Z. Let us say the undisturbed stream is such, that it has component y U infinity and V infinity, uniform stream but, it is not a along x axis, it has two component one along x one along y and the components are U infinity and V infinity. Then what it will be? Just go on adding, we want it to be expressed in terms of Z not x and y no longer x and y. For the U infinity component again this f(Z) will be u infinity Z only, so find what is for V infinity and then add it that will be easier, let us say the same velocity but, instead of expressing it in terms of component U infinity, V infinity we say that the uniform stream is at an angle alpha, you know that is rather perhaps more dear to us at angle of attack, the flow of an angle of attack. So, let us say at uniform stream the magnitude is Q infinity and angle alpha, alpha to the x axis. So, what will be f(Z) for the x component of the velocity Q infinity cos alpha phi is Q infinity cos alpha x, for the y component it is Q infinity y sin alpha, plus i psi, psi is what for the x component, plus i Q infinity y sin alpha or infinity y cos alpha and for this minus i Q infinity x sin alpha, then what do you get again? this together make Q infinity cos alpha Z or let us say Q infinity Z cos alpha and these two minus Q infinity Z sin alpha, so Q infinity Z cos alpha minus i sin alpha, what is that?

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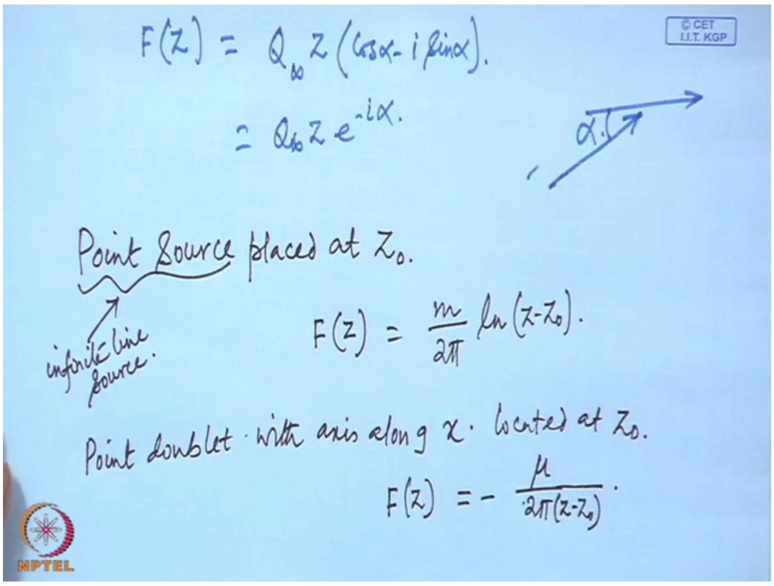

cos alpha minus i sin alpha is e to the power minus i alpha, so what is the positive angle of attack alpha, if this is the x axis and this is the angle alpha, which we call the positive angle of a attack, then is multiplied by e to the power minus i alpha. Of course, in complex notation, all the notations are expressed by e to the power i theta. Let us now consider this, a uniform, a point source placed at point Z naught, instead of origin we are placing the point source at Z naught, what will be the complex potential? Point source placed at Z naught; for point source what we have denoted the strength of the point source as m or sigma. So, point source placed at Z naught, what will be the complex potential f(Z), let us say forget about Z naught, think about origin, point source placed at origin, then what is the complex potential? Phi due to a point source is two-dimensional point source which is by infinite point source I mean now infinite line source.

As long as you are dealing with complex, it is all only two-dimensional flow because, it is not 3 dimensional flow, complex numbers are tuples flow so, we can use it only in 2 dimension, m by 2 pi log r, that r is the distance from of the point from the point source so, if the point source is at other location then it will be r minus something, say if the point source is located at x naught then it is r minus x naught and so on. And what is the stream function for point source, m by 2 pi theta, so m by 2 pi log r plus i theta, so what is that log of r is for i theta, that is log of z, if the point source is placed at Z naught, it will simply become log Z minus Z naught, so it is m by 2 pi log Z minus Z naught.

A source doublet located at point Z naught and let us say that a doublet axis is along x axis, I think we mentioned earlier, it is the line joining source to sink or sink to source, that is called the doublet axis, the direction is from sink to source but, the line joining source and sink, that is called the doublet axis. In a doublet the source and sink are infinitesimally distance away, the separation between them in approaching 0 but, they may approach 0 along x axis or along y axis or along some other line.

So, we consider doublet which has axis along x axis, the doublet which we considered earlier, the doublet which was required for flow past a circular cylinder, which has a doublet axis along the x axis, that is both source and sink are placed on the x axis itself, so, point doublet with axis along x and located at Z naught, remember the potential was minus mu by 2 pi cos theta by r, so what will be the complex potential, at that phi and psi that phi and psi you have already written many times the phi is minus mu by 2 pi cos theta by r, when it is located at origin, it will become minus mu by 2 pi, when you add phi and psi minus mu by 2 pi e to the power minus i theta by r or r into the power i theta which is Z, so minus mu by 2 pi Z, located at z zero z minus z naught, so this is this what it is the potential function is minus gamma by 2 pi theta and the stream function is gamma by 2 pi log r so, what is the complex potential minus i gamma by 2 pi log Z minus Z naught. and any function can be represented by an infinite series, then what will be the general form of that series in this case, for a flow problem? A general function is of course, A n Z a general series or polynomial is let us say A n Z to the power n, where n changes from minus infinity to plus infinity. Now can this be used to represent a flow problem, which is caused by somebody moving, through the fluid must be such that at far away that is at infinity there will be no effect of the body. Which is infinity boundary condition that a flow which has been created by the motion of a body through a fluid at rest. Then at far away the effect of the body will not be failed, the fluid at infinity will not feel that there is a body moving through it.

If that is so, what is the highest positive power for n possible in that expression A n Z to the power n, A n approaching minus infinity to plus infinity. The complex potential is represented by A n Z to the power n in general case for a n changing from minus infinity to plus infinity. But is it possible for a flow problem to have such a complex potential for a changing from minus infinity to plus infinity no minus infinity is possible but, plus infinity is not possible.

We cannot have n more than 1, where n more than 1 means we have say n, n is 2 means we have A 2 Z 2; A 2 Z square complex potential. What will be the velocity, complex velocity that will contain a two Z some constant into Z? And what does it mean as we go away velocity is increasing if the velocity contains an linear term in x means as we go away the velocity increases that means when a body moves through a fluid it effects will increase as the distance is increased, which is of course, not practical absorbed.

So, in the expression for velocity there cannot be any x or Z term. So, in the complex potential there cannot be any Z square or higher order term, is not possible. So, the most general complex velocity potential for a flow problem is the constant term that is Z to the power 0 that you are not writing because in the complex as long as potential there is a adding a constant does not make any difference. And looking to our basic solutions we can see that A 0 Z, this part of the potential must be due to the uniform stream that, uniform stream has a potential which is proportional to Z.

We will make as A 1 log Z and then, we will make it A 2, A 3, A 4 and so on. And the complex velocity will come as a constant term which, represents the uniform velocity plus or just to make a difference let us change the coefficients form. Then of course you must write here minus and so on. And just to avoid in this way you will write B 1 plus B1 by Z1 plus B 2 by Z 2 and so on. Let us now consider what will be the force that will act on a body, the force exerted on a body by the flow.

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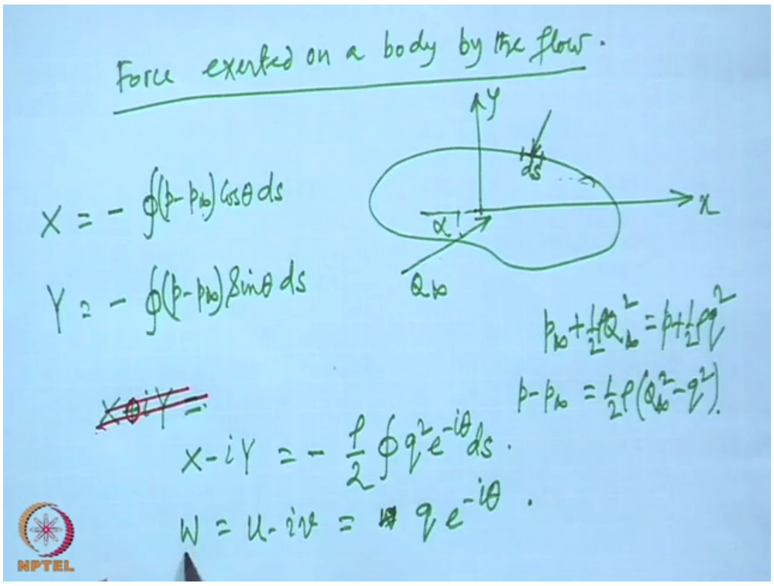

you replace p minus p infinity by the appropriate Bernoulli’s equation or you can make p infinity that, the contribution due to p infinity will be 0 or integration over this closed curve of p infinity cos theta is 0 p infinity sin theta is also 0. So these expression even can be written as simply p cos theta from there also, you can make it replace p by equivalent Bernoulli’s equation. Let us say at this point, the flow velocity is q then according to Bernoulli’s equation p infinity plus half q rho q infinity square equal to p plus half rho q square. So, what will be p minus infinity yes half rho q infinity square minus q square, out of q infinity square is again a constant. So when it is integrated over this closed curve again its, contribution will be 0. So in effect will remain as half rho q square then, we can write X plus iY, what is X plus iY, what is X plus iY, what is X plus iY e to the power i theta d s cos theta d s plus i sin theta d s. What will be d x d y it is not just arbitrary consider a d s length d s and then see what it is.