Introduction to Aerodynamics

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Module No. # 01
Lecture No. # 03
Fluids and Forces in Fluids

We have discussed so far that, the main interest in aerodynamics is to determine the forces that are acting on a body, which is moving through it. And we had mentioned that, to do this, what we need to know is, the detail flow about the body, the complete velocity distribution, pressure distribution and once you know them, we can find all the forces and movements that, eventually will act on the moving body.

Now to do that, the first thing that we need to set up the equation of motion of the fluid or the flow. And this necessitates that, we should have some knowledge about the basic properties or defining properties of fluid. Now, it is well known to you that, the defining properties of the fluid is the is by which a fluid can be deformed. If you compare to a solid, the fluids are deformed by a relative motion with respect to another, even subjected to a small force.

For a solid, if we apply a small force any deformation that may occur is negligibly small, while in case of a fluid, it is not necessarily small. Of course, there are certain fluids which show the dual behaviour that is, it behaves to some extent like solid and to some extent like fluid.

And of course, we will not include that type of fluid in our discussion as an example say, jelly, paint, peach, they have both dual this dual behaviour; sometime, they behave like a solid, in some respect they behave like fluid. Like, peach as such is behave like a solid, but if a large force is applied on it for a long time, then it shows a fluid like deformation; same thing about paint also, it usually does not show liquid behaviour, but if you shake it or brass it, it flows with the brass.

So obviously, we will not discuss or not consider these type of fluid for our discussion, because mostly see we are interested in air. And of course, it is not necessary to discuss only air, because there are many fluids including both gas and liquid, (()) have almost similar behaviour. So, we call this to be simple fluids.

And we can define them in such a way that, if we apply a force on this fluid, then the tendency of this applied force to deform it, is not possible without a change in volume; that means, fluids are those substance, which cannot withstand the tendency by an applied forces to deform it, in such a way that, (( )) the volume unchanged, this you can of course note as a definition of simple fluid.

That, fluids are those substance, which cannot understand sorry withstand the tendency by applied forces to deform it in such a way, that will (( )) volume unchanged. And we will see, what this definition actually means when it come to the forces that are acting in a fluid. Right now we will say that, this does not mean that, the fluid cannot resist any deformation, the fluid can resist deformation, but the it cannot prevent the deformation; and once the deformation occurs along with the resistance also vanishes, so this will consider the definition of simple fluid.

And you see, the two most common fluids in nature, air and water are quite easily explainable by these definitions. And of course, we will not even use the terms simple fluid all the time, we just go on saying fluid, but meaning that we are excluding those special type of fluids sometime called thixotropic fluids, which has dual behaviour.

Now, as far as these two different phases of fluid, the fluids are either liquid or gas. Now, as far as this two different phases of fluid, the distinction lags mostly or in what is known as the bulk modulus, the bulk modulus of elasticity. The liquids usually deform much less compared to gases that is due to a small force, the specific volume or density of the gas changes much more than the liquid.

So, in any situation, where the pressure change is large, then you see that this gas and liquids, they will behave differently. However, if the pressure changes are not large, then in the behaviour of gas and liquid are almost same. In that situation, we do not need to consider any special property of gas, when the pressure changes are not very large, then there is no difference as far as, dynamics is concerned of liquid and gas.

Can you say some situation, where the pressure changes are large from your say, daily experience? No. One very simple example is the atmospheric flow, particularly if you are dealing with weather or meteorology we need to consider the airflow in almost the entire atmosphere. And in this case, over the atmosphere over the great height of atmosphere, the change in pressure is quite considerable.

Another situation occur that is, in very high speed motion very high speed motion not just say 50 meter per second or 100 meter per second, but much larger than that, in that type of situation also the change in pressure is appreciable; and in that type of situation, the the special property of gas that is, large deformation need to be considered separately. But in most other situation, the behaviour of fluid in general, that is both liquid and gas are almost similar as far as the dynamics of the fluid is concerned.

And most of the time during this course, we will confined our self to that type of situation, where the pressure changes are not much larger. And any discussion that we make are applicable to all fluids, both gas and liquid. Now we know that, all substances are made up of certain molecules. And we also know that, there is enough space between the two molecules, the molecules are not very closely packed that is, it is not that all molecules are in contact, there are enough space between any two molecules. And what happen to the force between these two molecules, just think about two molecules in isolation, any idea, how the force behave with distance?

Student: (( ))

Inversely proportional to the

Student: (( ))

That is for very large distance let us say, the distance yes

Student: (( ))

No, not for sometime for some distance

Student: (( ))

Now, let us say, if the distance is see we are talking about now, a distance which is fraction of nano meter fraction of nano meter like say, 10 to the power minus 10 meter or even perhaps smaller, which is small in the molecular scales itself. We will define two scales, which are molecular scale and another is macroscopic scale. A microscopic scale, which is the molecular scale, when you talking about sizes, distance or everything as of molecules and the other macroscopic distance, which you commonly observe.

If the distance is say, fraction of nano meter of the order of 10 to the power minus 10 meter or little smaller. Usually, the force between two molecules is a very strong repulsive force usually; this can also be very strong attractive force, if there is a possibility of electronic exchange between the molecules let us exclude that case.

So, in other case we have a very strong very strong repulsive force between the molecules, as the distance increases increases in that respect in that 10 to the power minus 10 scales as it increases at a certain critical distance, the intermolecular force between these two molecules drops to 0; then, again with further increase the force now become attractive, instead of repulsive.

And in that distance say, when the distance between the two molecules remain say, within about 10 to the power minus 9 or 10 to the power minus 8 meter, that is a nano meter to few nano meter, these force decreases or these force varies with distance to the power of minus 7 or minus 8 and then, when it comes to macroscopic scale, what you are saying that, the forces behaves inverse square, so this is what is the variation.

Now, what is the usual molecular distance in say fluids? The usual molecular distance is say, ten times or of the order of ten times, it is not exactly ten times, because the distance is not a fixed distance it varies from molecule to molecule; the distance is of the order of ten times the critical distance, in case of a gas. The distance between two molecules on an average is ten times the critical distance between the molecule the critical distance is, the distance at which the intermolecular force drops to 0, which is of the order of say, 3 to 4 nano meter usually, 3 to 4 nano meter.

So, the average distance of molecules between gases is about 30 to 40 nano meter. In case of liquid and solid, it is of the order of that critical distance. In case of liquid and solid, the distance is of the order of critical distance. Now, when we see the fluid as a bulk as a bulk then, now think that we want to have a plot of density versus distance, if we have a plot of density versus distance, how will the variation look?

Student: (( ))

The density will decrease, why?

Student: (( ))

It is not exactly so. See this, when the distances are very small you know the density is concentrated in the nucleus of the, the mass is concentrated in the nucleus of the molecules. So, when you are thinking of distances which are say much less than this 30, 40 nano meter, which we say the average spacing within that distance. In some position you might find some molecules, in some position, you may not. And the density is violently non-uniform, it is violently non-uniform.

Now, if we are considering a volume, which has a diameter or characteristic size greater than this, then you see at any instant this volume will contain a large number of molecules, it will contain a large number of molecules and consequently, the density will reach to more or less a uniform value. So, for a very small, if we consider say instead of now just a distance, what we call a sensible volume? If we consider a very small sensible volume, then for extremely small volume, the density variation is violently non-uniform.

However, when the volume reaches to certain large value certain large value again, this is quite small even in the macroscopic sense it is quite small say, may be even 10 to the power minus 15 or 10 to the power minus 12 cubic centimetre; even that type of volume, will contain a large number of molecule and the density will have more or less a uniform behaviour.

Now think, if we are using certain instruments to measure any property I took density as an example, but this holds for any other property density mass or specific volume, they are all almost related. But, if we think about the velocity or any other any other property of fluid or flow, all of them behaves in the same manner that, when the sensible value volume is extremely small, small in the macroscopic sense or in the molecular scale, then the variation is violently non-uniform. But, when it is small in the macroscopic sense, but extremely large in the microscopic or molecular sense, then we reach to a more or less uniform value.

So, when we consider the bulk of fluid for our dynamical study we see then that this particle structure of the fluid is practically of no importance. Because see, what will be interested when we call, so the density at a point or velocity at a point we will be thinking of a sensible volume, which is small in the macroscopic sense, but still extremely large in the molecular sense, in the molecular scale, it is extremely large.

When we say a point actually by that point we will mean simply a sensible volume of the size of say 10 to the power minus 12 or 10 to the power minus 15 centimeter cube, not smaller than that. So, as far as the dynamics of bulk fluid is concerned, that particle nature of the fluid is not important or will not come into picture usually, except in very very special cases. So, most of the time, we will not even think of these molecular structure of the fluid, we will just consider the fluid is a continuous medium, where density mass or any of these properties are distributed continuously. And point in our discussion, we will have that meaning, that the point is not a point in the molecular scale, but in a macroscopic scale.

This hypothesis we know that already used for solids also, in case of solid also there is this situation is there. Though of course, the sensible volume is much smaller, because the molecules are more closely packed, but there also, it is actually in the molecular sense it is not continuous, the mass distribution is not continuous, there also some space between two molecules. But in solid mechanics also, we consider the same hypothesis that, the matter is a continuous distribution of mass.

So, this hypothesis, which seems to be quite natural, in the macroscopic sense we cannot think of. Even our daily experience shows that, things are everything is continuous, the mass is continuous, the velocity is continuous. So, this is known as continuum hypothesis. And any mechanics that uses this hypothesis is all together called continuum mechanics.

So, all solid mechanics and fluid mechanics can be collectively called continuum mechanics. And then, any difference that comes in solid mechanics or fluid mechanics is mainly due to the relative ease of deformation of fluid, because a portion of fluid can be much easily deformed than a portion of solid or the relative motion over a portion of solid is negligibly small, when compared to the relative motion of different portion of fluid.

So now, we will consider the forces that act in a fluid. Now, forces in general are of two types. The forces we will define them to be are in generally of two types, there are certain forces called long range forces, which acts when the interacting elements are a large distance apart. Think about a fluid or even a solid, there are two portion of the say two element of solid or two portions of these fluids, which are interacting, one is exerting certain force on the other.

Now, there are some forces, which can exert even when the distance between these interacting elements is large. Of course, as the distance increases the force decreases, but they still act, even at a very large separating distance, the force is still present. You are very familiar with the most common example of long range force, which is the force due to gravity, also this force has a tendency that it acts over the entire volume, all molecules or all elements will instead of molecules we call elements, which is not exactly the same, molecule is a molecule, element is a freed element is a very small fluid element, which may contain a infinite of infinitely large number of fluid molecules.

So, it acts over all the elements or over the entire matter, this force acts over the entire matter, consequently these forces are also called as volume force or body force. So, they are called long range forces or volume forces or body forces. And the most common example is, the force due to gravity. Any other volume forces or the long ranging forces you are familiar with?

Student: (( ))

Yes

Electrostatic force

Magnetic force

Magnetic force let us let us call it electromagnetic force, any other? There are the certain fictitious forces is it not, certain fictitious forces particularly, if we express the motion in terms of a set of accelerating (( )).

Student: (( ))

Yes, if we express the motion with reference to a set of accelerating (( ))

Pseudo forces

Pseudo forces or fictitious forces as an example the centrifugal forces, there also body force or volume force. The other kind of forces are, short range forces or surface forces, they act only when there is a direct mechanical contact between the interacting elements. And of course, the nature is the transport of momentum by the molecules and that is why the distance is to be of the order of that molecular distance; that is when the distance between the interacting element is of the order of molecular distance, which you can say is indirect mechanical contact, only then this force acts, this also you are familiar with.

Student: (( ))

Whether you call the the normal reacting force, that is the nature of that and the friction comes off as a result of it. When two rigid bodies are in contact see, this force exist even the friction comes only when there is a relative motion otherwise not. But, the normal reaction that is there, whenever two rigid bodies or two solid bodies are in contact, so that normal force is actual actual force which comes, because of in case of a solid; see, the the molecules, which are vibrating they penetrate, one molecule from one body to other body, so that is why that distance need to be so small.

If the distance is larger than that what we call let us say, of the order of the critical distance in terms of molecular scale, if the distance is larger than that, then this force will not exist, this force acts only when there is a direct mechanical contact. Now, as far as expressing the volume forces or body forces, so we write.

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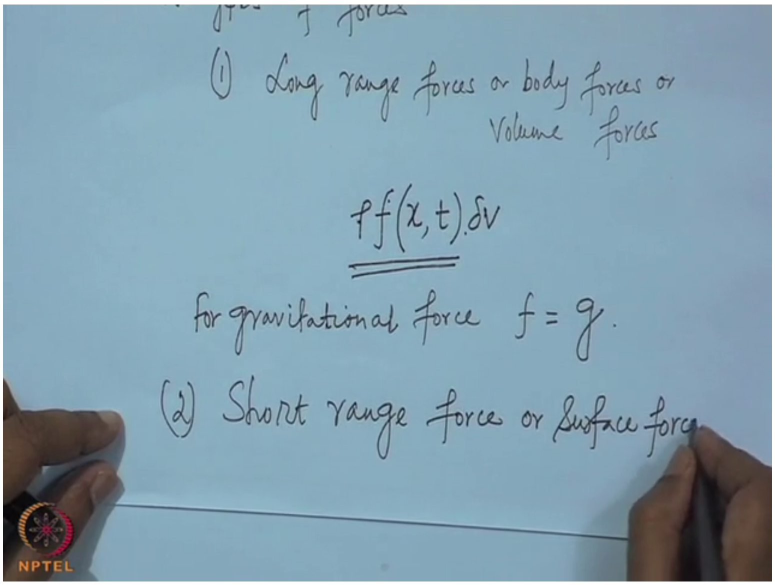

So, volume force or body forces or long range forces say, two types of forces, please make note even when I am not writing, it is not that what I am writing only that you will write sense. In that case earlier also (Refer Slide Time: 34:34), you should denote it as a (( )) sense that, this is the position vector. So, that means this element we are considering, which has a position x at time t, this is we are just denoting it, where this is what we have denoted it.

Now, let us say that let us say that in this case, this is what is the force acting on this element and the normal is in this direction, that is this part of the fluid to which this normal direction points, this part of the fluid let us say, this is only fluid, outside also fluid and inside also fluid and this is an enclosing area, then the fluid which is outside that is exerting a surface force on the fluid, which is inside and let us say, that is what this T n x t. What we have written here, then what will be the force that is, this inside fluid is exerting on the outside fluid? yes yes

Student: (( ))

Negative of this fine, so on the other part

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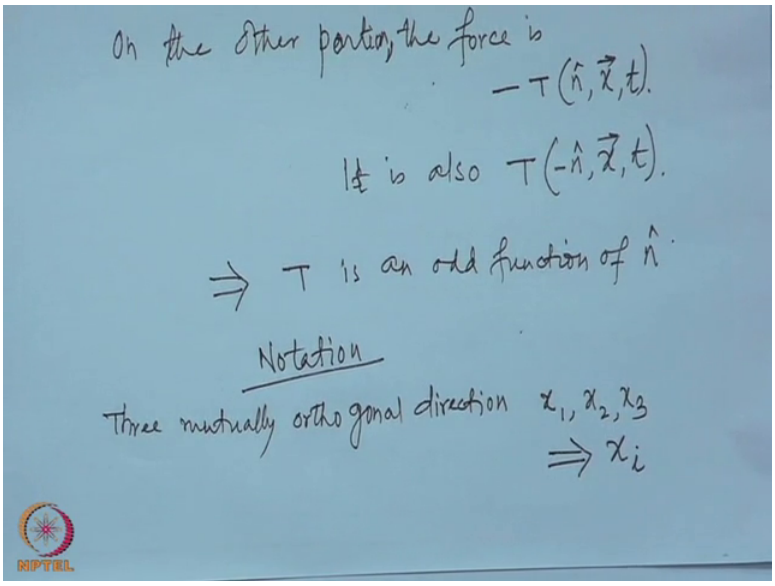

T by the definition of this symbol cannot we write that, it is also this. If we change the direction of the normal in the other way that gives the same force. And what does it mean mathematically? That, this surface force is an odd function of the normal, the surface force is an odd function of normal no sigma sign is required.

So, as an example see this, this is a term (Refer Slide Time: 42:54), this is one term where this i is repeated it means, so whenever this term is there it means we do not need to write that sigma j equal to 1 2 3 that is not required, this automatically means this. Now, look to the second expression that I have written a j b j equal to a 1 b 1 plus a 2 b 2 plus a 3 b 3. What does it imply, see a j is what? Is a vector a, b j is another vector b. So, what is this a 1 b 1 plus a 2 b 2?

Student: (( ))

So, this is what is vector dot product, so dot product is written as simply like this. We will define two special term or special notations sorry before that, we should talk about vector is, vector needs one index, vector has three components, so one subscript needs. But, there are certain quantities, which have more than three components. You have already come across, stress strains which have force per unit area and since both has force has its own direction, the area also has its orientation.

Consequently, the resulting term has this two in them, but if they have a component in a particular direction also you need to specify the area on which they are acting and that area is specified by its normal. So, you need to specify two particular directions, two express stress. So, these types of quantities are called tensors.

And tensors are represented by two subscripts the tensors are represented by two subscripts writing any tensor you want. Whenever there are two subscripts attached to your letter, it represents a tensor. And the stress and strains are the familiar example that you have already come across.

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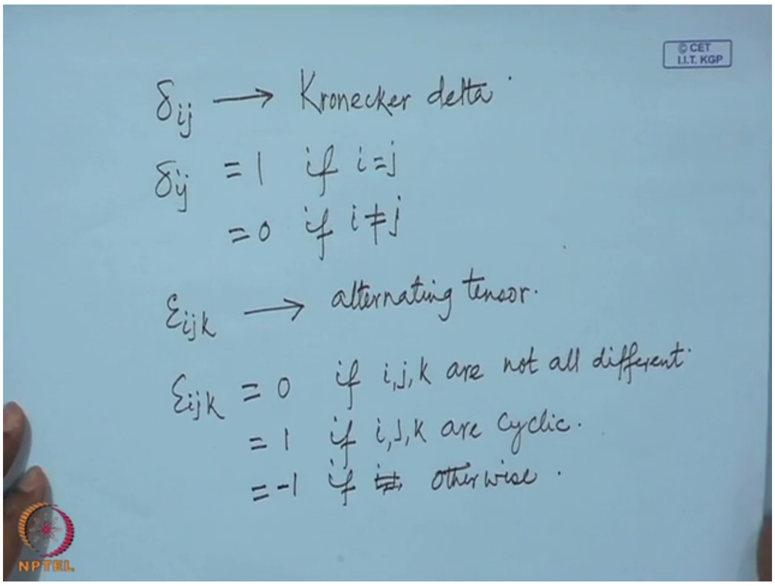

We will define two special tensors, one is a second-order tensor known as delta i j, known as Kronecker delta. See as you can see that, i j if we are considering three dimension, then i can take the value 1 2 3 and j also can take the value of 1 2 3. So, this sigma i j has nine elements sorry delta i j has 9 elements, delta 1 1 delta 2 2 delta 3 3 and so on.

Now, this delta i j has a special meaning that, it has a value equal to 1, if i equal to j when i and j are same, if we represent this delta i j mathematically this also can be represented by a matrix having 9 elements or 3 by 3 matrix. So, i equal to j means the diagonal elements, so the diagonal elements are the 1 and all other off diagonal elements are 0. So, in the matrix representation, this is simply the identity matrix.

The other one is, epsilon i j k called the alternating tensor as you can see this third-order tensor, it has it needs three subscript to express it. Third-order tensor meaning, in three dimension, it has 3 by 3 by 3 that is 27 element, total 27 elements and here also, the elements have three assigned values, it is like that, epsilon i j k equal to 0, if i j k are all not, that is all of them has to be different, then it is 0.

Even, if two of them are same, it is 0, when three of them are same of course, it is 0, when two of them are same, any two of them are same, it is 0. So, for nonzero value, all three must be separate different and it is 1, if i j k are cyclic. When they are different and cyclic different is already, because the first condition gives if not different they are 0. Now, if when they are different, if they are arranged in a cyclic order, its 1; if it is not, it is minus 1 you can write if otherwise, if not cyclic. And you can see the where it is required in this index notation, this alternating tensor comes to represent say, the what we call the cross product of two vectors or the vector products, so to represent vector products we need alternating tensor.

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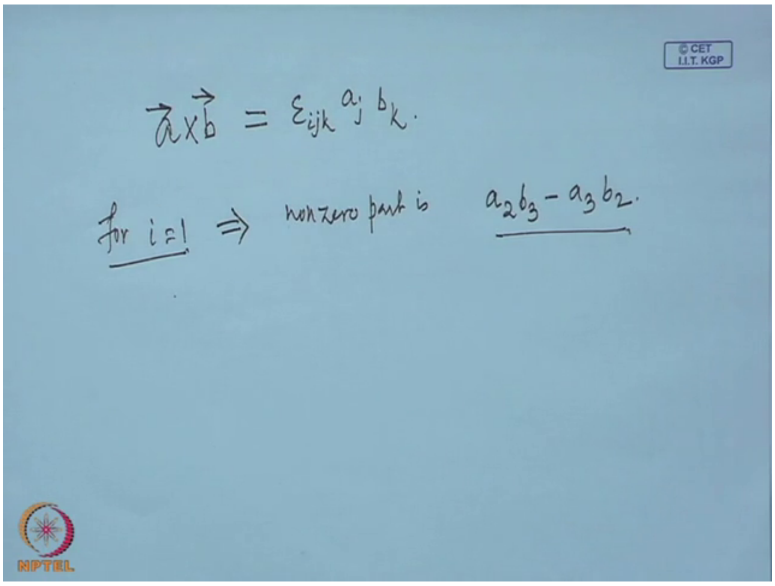

So, that is two vectors denoted by a cross b is in index notation can be written as, epsilon i j k a j b k epsilon i j k a j b k and you can, if you try to expand this i j k you can see that, for each i you will be getting the three component of this left hand side vector. I want to take an example let us say, the i for i equal to 1, what do we get for i equal to 1? First of all you think, what are the nonzero components, looking to epsilon i j k if i equal to 1, then j and k cannot be 1 because that means 0, for any value of j and k 1 it is 0.

So, there are two possibilities that, j equal to 2 or 3 and k equal to similarly, 2 or 3. Now and of course, they have to be different, j and k are not same, because if j and k again become same then, epsilon i j k will again become 0, so when j is 2 k is 3 and when j is 3 k is 2. So, these are the two possible, two possible nonzero case j equal to 2 k equal to 3, which makes a 2 b 3.

And in that case (Refer Slide Time: 53:58), what is its epsilon 1 2 3 which is cyclic, so 1 2 3 that is positive. So, the nonzero component is a 2 b 3 minus a 3 b 2, is it not, the i th component of a cross b, similarly you can check it that this is. So, a vector product is denoted by this notation (Refer Slide Time: 54:38), epsilon i j k a j b k.

One must, one thing you must remember that, in a those indices which are repeated, they are also called dummy indices and dummy indices can be changed at any time. So, it it is not wrong that, if one step you write d u i d x i and in the next step, you write d u j d d x j, it is all right, because both means the same.

Of course, these are the basic introduction of these notations, index notations. And anything extra, if required, we will explain whenever it comes, but with this now, we will go back to that surface force that, we were discussing earlier, but I think we will wait for few minutes for that.