Introduction to Aerodynamics

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Lecture No. # 25

Non-dimensional Form of the Equations and Possible Simplifications

So, we have mentioned already that the Navier stokes equations which of course, are the governing equation for all fluid motions or extremely difficult to solve because of its non-linear nature and often the boundary condition is quite complex, non-linear as well. And in general, we do not have solution available for the Navier stokes equation. So, the next we try to see whether this equation can be simplified and then if it is, then under what condition whether some term from this equation can be dropped or can be neglected and to do this.

The straight forward or easiest approach is a dimensional analysis of this equation because if we express, the equation in a non-dimensional form we will be able to compare different terms present in it and if we find that one term is much, much smaller than the other terms you can drop that term. And with that view now, we will look for the non-dimensional form of these Navier stokes equations.

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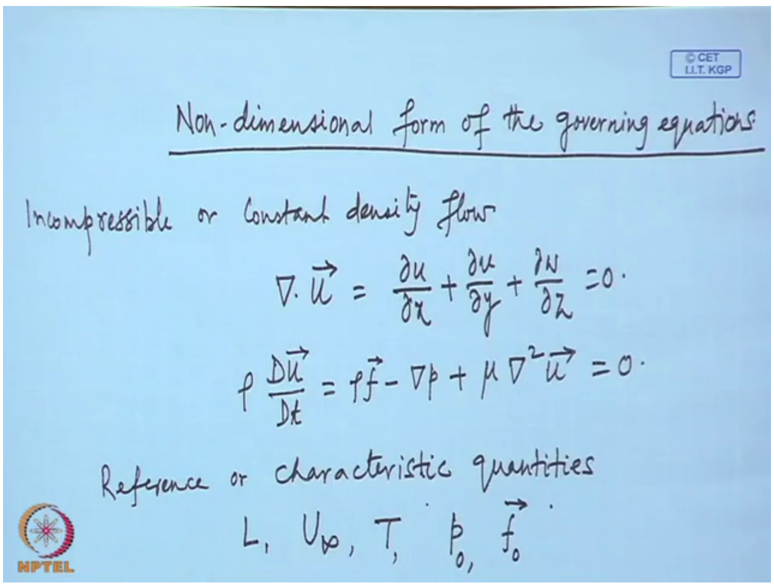

And if we consider incompressible flow (incompressible or constant density flow, you know the governing equations are the continuity equation, which gives divergence of the velocity field which is in cartesian system. You can write d u dx plus and the momentum equation which is to non-dimensional as these equations ok. We need not consider the energy equation as we have already mentioned that, the energy equation is decoupled in case of incompressible flow and these two equations can be solved independently. Once these are solved the velocity field can be used to solve energy equation in case it is required, I mean in case we are interested to solve it.

Now, to non-dimensionalize this equation we have to consider certain reference quantities or characteristic quantities. The characteristic quantities are reference quantities are of course, problem specific, but we can take some general cases like for the length. We can consider a characteristic length and this characteristic length, we mean that a length over which a considerable change in the parameters are absorbed say as an example, if we are consider flow around a circular cylinder then the cylinder diameter can be taken as the reference length or the characteristic length because if we consider a length of that amount, we will see that within that length a considerable change has occurred in all other parameters,

So, that will be taken as a reference length or characteristic length. In case of a flow over an aircraft, we can consider the wing chord as the reference length. So, references are characteristic quantities let us say for length, we will take only one length L that means; inherently we are assuming that the flow field has same characteristic length in all directions. See in a in a practical flow problem this may be different in different direction. As an example let us say that the problem that we talked about yesterday that flow between the two channels or say suddenly flow about a suddenly, started plate in that case the length of the plate in that direction there was no change.

So, that is not exactly the characteristic length rather the characteristic length is normal to the length in case of that flow between two plate, we can consider the spacing between the two plate as the characteristic length and in case let us say that there is also a pressure gradient in that problem, we did not use pressure gradient, but if there are a pressure gradient then there will be a variation in flow velocity, along the x direction as well and in that case the characteristic length in the x direction or in the flow direction and in the normal direction need not be same.

However, in this case we are considering the characteristic length in all the directions are same and it is L whatever, that L is L will be appropriately chosen in particular problem as far as the velocity is concerned. You see that most often we are interested in a problem where, there is a flow over a body or through a body just simply a flow that a fluid is moving that is not of any practical interest. So, usually there when the fluid is moving over certain some body or moving through certain body, that is what, is the interesting case that is what, we are interested in. So, the and when the fluid is moving over the body its velocity is continuously changing.

So, the characteristic length or characteristic velocity we will consider the velocity which it would have, if there are no disturbance. See if we consider an aircraft as we know that aircraft is moving with a certain, certain speed and in our aerodynamical problem, we study it that there aircraft is at rest. The fluid is moving with that speed and that we consider without the absence of the aircraft, that the air as if the air was moving with that speed when it was not disturbed by the aircraft, which we call the free stream speed or undisturbed stream velocity. So, this undisturbed stream velocity we can take as an characteristic or reference velocity.

So, for velocity we use our undisturbed stream and the undisturbed stream is usually denoted by this notation U with a suffix infinity that means, what is the flow velocity at infinity, where there is no disturbance. For time also you consider a characteristic time again the characteristic time is a time length of time over which considerable changes in the flow takes place. As an example let us say that, if the flow is somehow periodic then that period can be taken as the characteristic time because over that period, let us just take it for granted that the flow pressure or the flow velocity or the problem is such that, velocity at any point is varying sinusoidally then, the period of that sine wave we can take as the characteristic time scale because over that time a considerable change is taking place.

it is T similarly, for pressure also you take it like the undisturbed pressure or some reference pressure. Let us call it p 0 and for the body force also we take it a body force f 0, say as an example you can tell it this is if this body force is a gravitational force perhaps, we can take the gravitational acceleration on the surface of the arc. As the reference and we non-dimensionalize all other gravitational force in terms of that. So, with this characteristic scale we can express all non-dimensional parameters.

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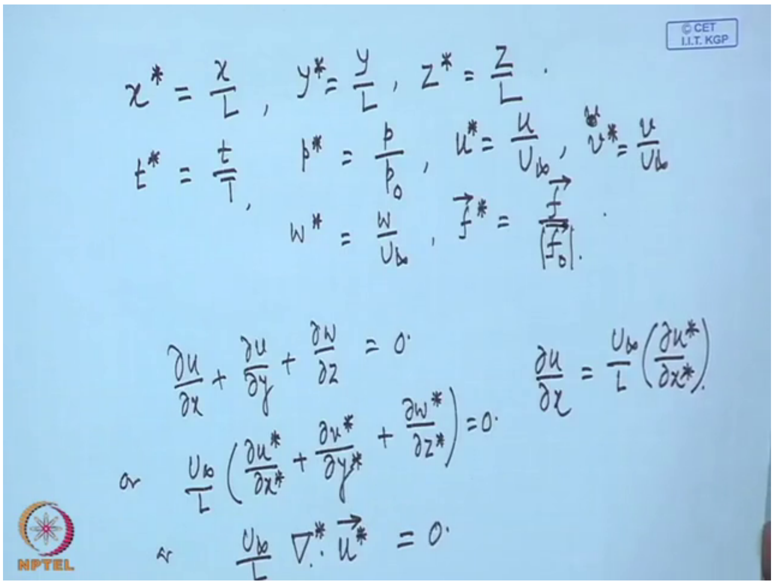

That is we will have now, the x non dimensional version of x. We will denote as by x star is simply x by L, this x star is non-dimensional x similarly, y star is y by L (Multiply throughout by L into rho u infinity square, multiply throughout by L into rho infinity square. The idea is to make the coefficient of these term as one coefficient of these terms as one.

Student: (( )).

L upon rho u infinity square .

So. What happen to the first term the time derivative term? The time derivative term becomes.

Student: M.

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We have made the coefficient of all these as one that is all, mu Laplacian u is dropped this is the governing equation or the momentum conversation equation for inviscid flow and this equation is named as Euler's equations.

So, inviscid form of the Navier stokes equation is called the Euler's equation. When the viscous terms are dropped from the Navier Stokes equation, the resulting equation is called the Euler's equation looking to this equation, how much simplicity we have simplification, we have achieved. Have you achieved a very great simplification to be precise really, no we have not achieved great simplification, it has become simpler that viscous terms as vanished equation is now, looks much smaller much neater of course simpler.

But the major difficulty in solving the Navier Stokes equation, which comes because of the non-linear nature of the equation, that still remains because this still remains non-linear and eventually that non-linear nature is not associated with viscous viscosity, then non-linear nature is associated with the Eulerian description of the motion the way we have defined fluid velocity we have defined fluid velocity at a point then consequently this non-linear term are present in the mathematical equation .

So, they will be present whether the flow is viscous or inviscid also see that there is another change very important change or very significant change. While the original Navier Stokes equations, the governing equation for viscous motion is a second order equation, the second order differential equation it contains that Laplacian operator which is second derivative.

So, the Navier Stokes equation was second,second order equation while, this is first order equation there is no second derivative term and then. What is the difference that in second order boundary differential equation or to solve second order differential equation? We need two boundary condition is we need two boundary condition, but for first order we can satisfy only one since, the actually problem this is just a simplification and approximation that, that is quite justified when Reynolds number is high.

The approximation is quite justified, but due to that approximation we have reduced our equation by one order, some second order to first order and that second order the two boundary condition that is of course, the realistic situation that actual situation that two boundary condition, but now we can satisfy only one the one of the boundary condition will remain unsatisfied. We cannot satisfy that What are these two boundary condition? Again considering flow over a body, the boundary condition is that there is no relative velocity with respect to the body. There will be no relative velocity between the body and the fluid, which needs both the tangential component of the velocity as well as the normal component of velocity will remain same or if the body is at rest both of them are 0.

The normal component of the fluid velocity on the surface of the body and the tangential component of the fluid velocity on the surface of the body, both of them are 0 assuming the body is rest otherwise; they have same magnitude these two boundary condition. We cannot satisfy the two only one we can and in an inviscid flow this is see out of these two that one is not penetrating and the other is not slipping obviously, that not penetrating one is seems to be more natural.

So, only that we can solve in the Euler's. Why? If we solve this Euler's equations the inviscid flow equations we can only satisfy, the boundary condition that the normal component of velocity or normal component of the relative velocity is 0, but the tangential component of the relative velocity 0, that we would not be able to satisfy. in that case there will be a slipping between the fluid and the body.

The fluid will slip over the body that condition cannot be satisfied. And this change is of course, a very significant change that one boundary condition, we would not be able to satisfy if we solve this equation or if we solve the flow as incompressible inviscid flow, we would not be able to satisfy that 0 relative tangential velocity boundary condition and consequently the solution that we will get, we will have some difference from the reality.

Any inviscid flow solution in which we can satisfy only one of these boundary conditions which is normal component of the relative velocity 0, that is what will be satisfied normal component of tangential velocity or relative tangential velocity, tangential component of relative velocity that will not be satisfied there we are slipping and as a result the practical solutions, will never be obtained even though it appears that this approximation is very well justified or Reynold's number is very large and so, we can really see that the terms is very small.

So, it appears that even though it is very small, it effects near the body is always significant on the body surface its effect is always significant. However we will come to that part later on there are techniques by which we can take the effect of it on the body, but treating the flow otherwise inviscid. What really happens is like this, this high Reynold's number flow, where we are neglecting the viscous term which inherently means that both the coefficient of viscosity as well as, the velocity gradients are small eventually whatever large the Reynold's number is this gradient is not really small on the body surface.

So, on the body surface itself or very close to that body surface even at high Reynold's number, the velocity gradient are large and consequently that viscous stresses are present near the body surface. They may not be present a little away may be it is present only say thinking about an aircraft, which is quite large may be within one millimetre or few millimetre from the body surface this effect is present, but after that it is not present anyway let us while get this.

So, we see that for a very high Reynold's number flow we can neglect the viscous effect however, the resulting equation still remain non-linear and which are again extremely difficult to solve and to be precise even the Euler's equations also have only limited exact solutions, even these equations cannot be solved for general boundary condition for this problem. The equation is still remain unsolvable for any given arbitrary body, the inviscid flow about that body is still unsolvable the equation cannot be solved except few simple cases.

So, we look for further simplification. The simplification must be justified it is not that we just go on dropping, we will drop only when it is justified. As the viscous term we have dropped because it is justified. Now, to do that let us go back what we discussed earlier. We said that the in general the velocity field fluid velocity field is super position of three contribution velocity, field associated with an isotropic expansion velocity, field associated with rigid body rotation plus a velocity field associated with no expansion, no rotation which includes even a uniform flow.

Now, in a case in incompressible flow now, we have seen that there is no expansion when the flow is incompressible or the fluid is incompressible then rate of expansion is identically 0, that is the continuity equation an incompressible flow cannot have any expansion. We will now, see that if the flow is inviscid under certain conditions it will become irrotational as well that means, there will be no rigid body rotation.

So, if the flow is incompressible inviscid and then it becomes irrotational then the velocity field is simplymade up of a velocity field, which has no expansion, no rotation that is just a pure solenoidal irrotational field, which we earlier denoted by that small v and that will now, be the complete velocity field and just the condition of continuity and condition of irrotationality showed us that this velocity field satisfies.

The Laplace equation we see that this Euler's equation changes to the Laplace equation, if the flow is already we have started incompressible and inviscid and if you can see that show, that this is irrotational as well then this flow will become a potential flow for which the boundary condition again will be divergence of u equal to 0 that is the only divergence boundary condition only governing equation.