Introduction to Aerodynamics

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Lecture No. # 23

Exact Solution for Simple Problems

Now, these exact solutions for Navies stocks equation see the case for which exact solutions are available. The most simple of them is for unidirectional flows for unidirectional flows. There are many flow problems where the flow is in only one direction, and let us call it that is the x direction. It need not be a straight line the direction need not be a straight line it can be curved also; that means, we can have a x which is curved. And further in many situation, these unidirectional flows are found to be independent of that particular direction. That is, if the flow is in x direction, then is found that this flow is independent of x the flow is in the x direction, but then it is independent of x.

Whatever change is taking place only in a particular cross section in y and z, and very and approximately this type of flow you always find say in a pipe. If we exclude the end part of the pipe, that is where the flow is entering the pipe, and where the flow is going out of the pipe. If you exclude those region, some part just immediately after the entrance, and some part just before they exit. Then in the major part of the pipe, the flow is almost like this, that flow is in only in one direction, and in particularly it is independent of that direction.

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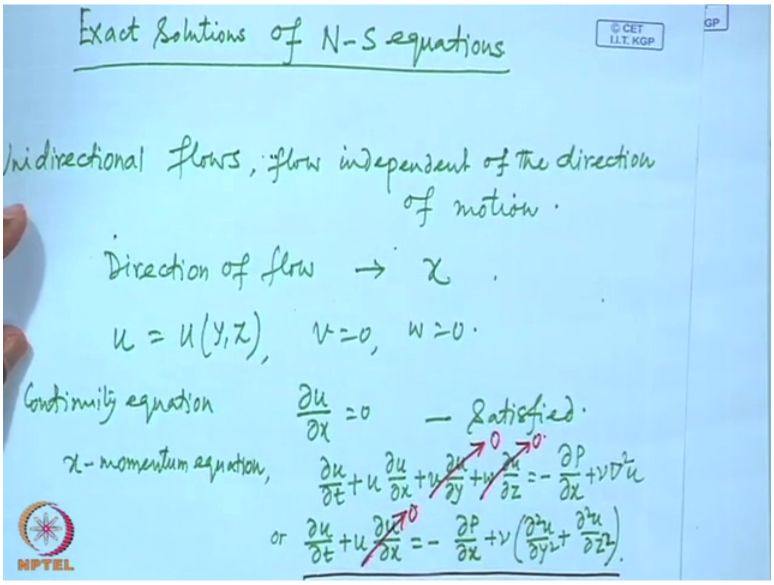

So, assume for flow which are unidirectional flows So, unidirectional flow that means of course and that is the direction we are treating as x the direction of flow itself. We are calling as x, let us call that x that means we have only x component of velocity. The y and z component of velocity are 0 for these problems and the y and z component of velocity is 0 and the x component of velocity is independent of x meaning, we have the x component that is u is function of y and z only and v equal to 0 w equal to 0 the three component of the vector velocity u are like this.

Now, let us see what happens to the equations, then what happens to equation then. The continuity equation is now, very simple d u d x is equal to 0 there is no v no w. So, the continuity equation the divergence of u is simple d u d x equal to 0 which is already we have considered. So, the continuity equation is basically automatically satisfied by this, anyways still write continuity equation and this is satisfied by default.

So, you really do not need this equation this is already been satisfied. The momentum equation, let us write in component form now the first x component of momentum equation x, component of momentum equation we will call it x momentum equation. The x component of equation if you look to this equation, the x component of momentum equation will be u will that rho we will take it to the right hand side d u d t plus u d, u d x plus v d u, d y plus w d, u d z equal to that modified pressure in terms of that modified pressure plus nu Laplacian of u.

Nowhere look here, in this term and these terms are 0 this term is 0 this term is 0, v and w are 0. So, only these remain and in the Laplacian also only one term will remain. So, this will sorry one term not two terms will remain Laplacian of u is d 2 u d x 2 since d u d x is 0 d two u d x 2 is 0. So, this is we have d 2 u. So, this is the equation that we have.

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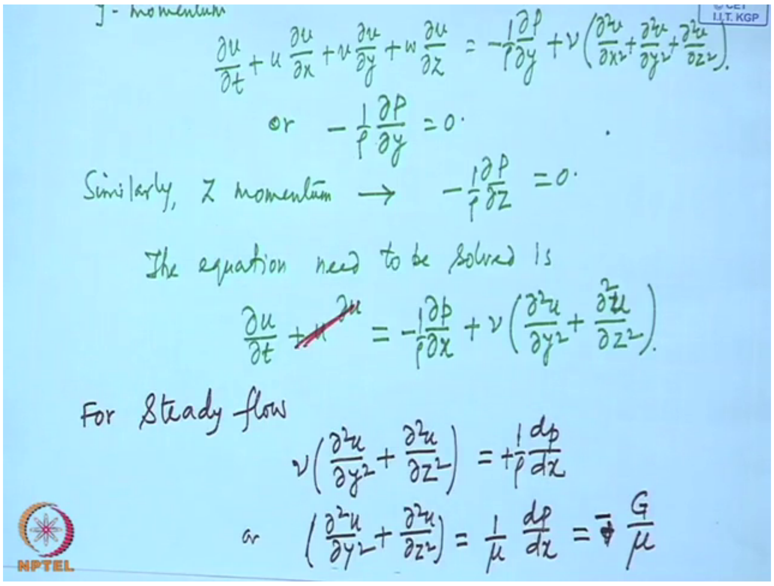

Now similarly, if you write the y and z component of momentum equation, what is it let us write. Once all the entire the left hand side is in all these equations they contain v. So, all of them are 0 all the terms are 0 this 0, this is 0, this is 0, this is 0 and this all 0. So, it gives only and similarly, the z component, z momentum, z momentum, will also come as z momentum will give that is that modified pressure is not changing in the y and z direction. So, the practically the equation that we now, have is this the equation that we need to solve that x momentum equation d u, d t plus u by the way this terms is also 0 sorry, sorry this i have not written 0 d u, d x is 0. So, this is also 0 this you so, this term is not there.

Since pressure is now, a function of x and t only pressure is not function of y and z. we will write it also if the flow is steady then the left hand side is 0. If the flow is steady then this left hand side is also 0 and then the pressure becomes only a function of x and this can be written as ordinary derivative (Say since, the flow velocity u is independent of x that implies that this pressure gradient d p, dx will also be a independent of x, p itself is not a independent of x, but d p dx will be independent of x, because if d p, d x is not independent of x then u cannot remain independent of x.

So, this d p, d x is independent of x and in general we can write d p, d x, if we look to this equation this d p, d x can be written as a function of temperature time alone and here it is just a constant. So, we write that constant as I am sorry or you know let us that make it minus where g is d p, dx.

G is d p dx.

G is minus d p dx.

Minus.

Which will be a function of time for an unsteady flow, but for a steady flow it is a constant. So, finally, this is the equation that we need to solve for this problem .Now, you see this equation is linear that is the basic idea. We have made the equations linear which are originally non-linear, because of that acceleration we have assumed a case or we have approximated some flow problems. So, that the convective term is basically vanished and the flow has become linear.

I think this equation we have solved earlier, I do not know whether you have written equation in this form or not, but of course, you derive the equation from by considering free body diagram for that particular problem and then obtained this equation. And I think you solved pipe flow in your school physics no might be add a I thought that you did something like that anyway may not be.

Now, we will apply this first of all for a common problem, which is classically known as

Hagen- Poiseuille flow problem Hagen- Poiseuille flow, which is equivalent to or which is a similar to flow in a very long pipe or long duct, but in which the end effects are not considered that means, this solution will not be valid for near the entry and near the exit.

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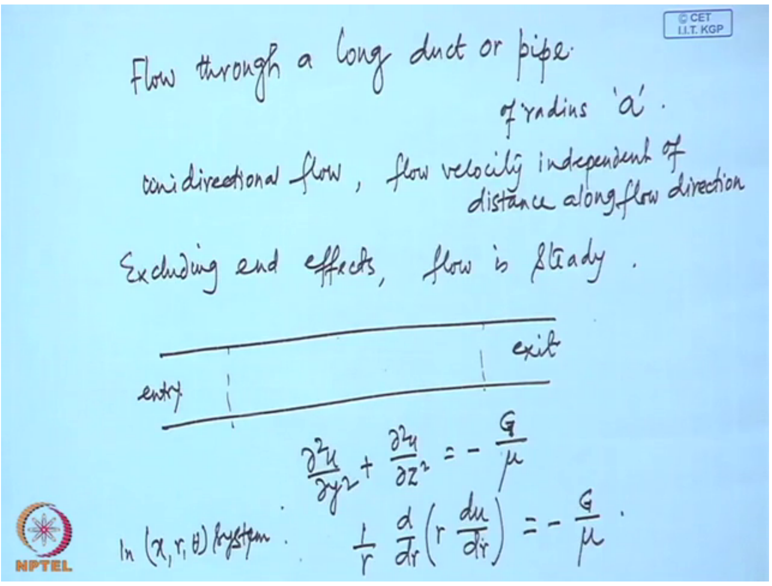

So, flow through a long duct or pipe of radius say a, so this is a unidirectional flow and flow velocity independent of distance along flow direction.

Excluding end effects flow is steady near the entrance and the exit flow never reaches a steady condition, but after certain distance from where the flow started, after certain distance the flow reaches a developed state and is practically time independent. So remember that the solution that we will be getting is not valid near the entry and near the exit. So, if you have it is very long duct. So this is entry some portion near the entry and some portion near the exit is not valid in this entry and this exit part this equation does not hold.

The equation of motion for this is the Laplacian operator in the cross sectional plane. The Laplacian operator this left hand side is the Laplacian in the cross sectional plane. Now, see this y and z or x y z is not a very good or appropriate coordinate system for this for this type of problem the co-ordinate appropriate coordinate system is x r theta x is of course, again the x direction. So, the cross section is represented by r theta. Now, from our physical observation or from general sense we can say that this flow when is a developed is basically Axisymmetric that whatever, the theta is in an direction any particular theta direction the flow will be same in all theta direction there cannot be any theta dependence.

So, what is the Laplacian operator now, the Laplacian operator will be in terms of r and theta and since theta is independent. So, the Laplacian operator will contain only r derivative and in x r theta system. This equation becomes 1 by r d d r, see we need not even write partial derivative there is no variation in x that is already the case the flow is independent of flow direction and there is no change in theta direction also, only change in r direction. So, is ordinary derivative the Laplacian operator in r and theta would have contain another term for theta derivative, but that is 0.

Now, can you solve this equation this ordinary differential equation with the right hand side is a constant. So, it is very easy to solve or very easy to integrate.

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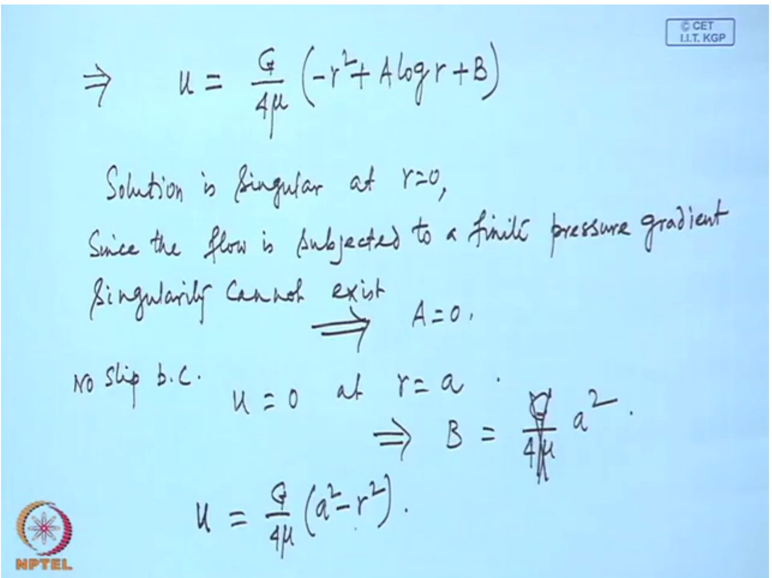

The integrated result is that is, if you integrate that equation twice you get as u equal to g by 4 mu minus r square plus a log r those a and b are two constant of integration those you have to now find using the boundary condition. Now, what boundary condition we can apply here, if we consider this duct of radius a then where the flow is in contact with the duct that is the wall and at that there will be no slip. So, you can say u equal to 0 when r equal to a, any other boundary condition.

We can say here really know that is the only one boundary we have then two unknown constant. How do you get two value for the second one? Look that this term the second term a log r becomes singular at r equal to 0 logs 0 is not defined. So the second term becomes singular at r equal to 0. The velocity is there undefined, but in a real practical problem how we are getting this flow we are applying certain amount of pressure difference between two points in the flow.

So, obviously you are applying a certain finite amount of force. If you look in terms of a dynamic problem you are applying a finite amount of force in this flow. Applying a finite amount of force there is no physical reason that why should they are be a singularity somewhere. So, the singularity cannot exist this solution gives a singularity at r equal to 0, but our physical reasoning says that there cannot be any singularity in this flow how is it that possible? That is possible if this term does not exist if this term is not there that means a is 0 if this term is not there then this singularity is does not arise so, this a is 0.

What we can write that and then you have no slip boundary condition u equal to 0 at r equal to a and then, what do we get we get b equal to g by 4 mu a square this gives b equal to g by 4 mu. And hence the finally, we can write that solution to be u equal to g by 4 mu, you can see that is a parabolic distribution or a Paraboloid we consider the entire pipe with.

Student: (( ))

Why.

Student: (( )) instead of a square that is g by four mu a square a is what (( )).

Sorry I.

In the initial solution b is inside bracket and b is again g by 4 mu a square so inside the bracket before small (( )).

But no no there is b of for for b we have already taken out g by 4 mu. So this b will not have this I just put that value of r equal to 0 and wrote this, but this g by 4 mus is already taken out. So, the flow velocity within the cross section is distributed as a Paraboloid. Paraboloid of revolution or if you just consider a one particular value of theta it is a parabola and it has a maximum at the axis r equal to 0 and it become 0 at the wall.

Now, it is quite often the average flow velocity is an important quantity. We would like to show that if there are no variation across the pipe, because most often we consider that the flow velocity in a pipe we call only one velocity. In that case it should be the average flow velocity. Now what would be the average flow velocity you can find an average distributed average distribution, but we will find the average in a much simpler way or we will look to this how much is the volume flow through this pipe. What is the volume flow rate or volume flux? That is the total volume passing a particular cross section, sees if you are analyzing a pipe or designing a pipe you should know that how much volume this pipe will allow hm.

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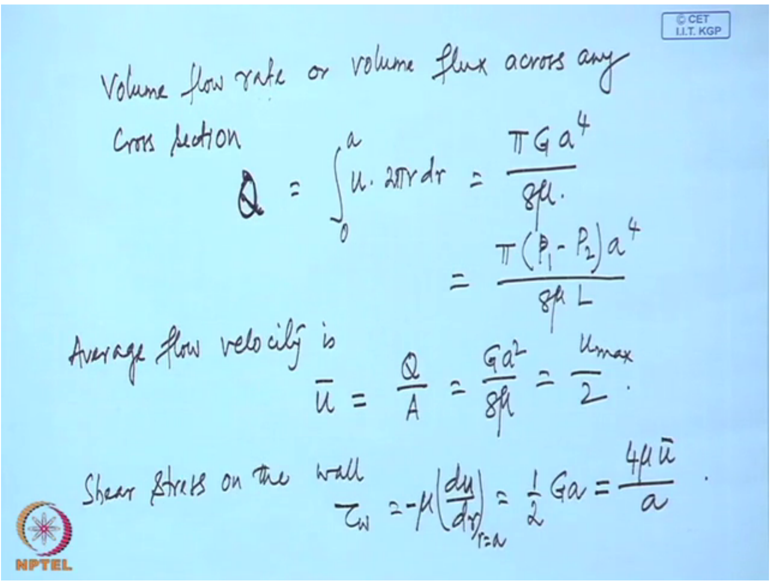

So, the volume flow rates how you get the volume flow rate.

Student: (( )) area into (( ))

Tell me how much is that volume flow rate or volume flux? Most often it is written as q discharge capital q sorry it is capital q how much is it u area is 2 pi r d r integration from 0 to a will come as pi g a, to the power 4 by 8 mu. So, you can instead of writing this g, we now can write it in terms of some pressure or the modified pressure to be precise and that g can be written as see, if we consider that at two point a two station one and two the distance between that is length l then the pressure gradient is p 1 minus p 2 by L. So, this can be written as if we consider a length of l of the tube and the pressure at that 2 point is a p 1 and p 2.

Student: Sir (( )) modified pressure or (( )).

Modified obviously.

Student: (( )).

Modified pressure so, as an example see if you are considering a pipe which is inclined and then length you should take the effect of gravity gravitational force also within that modified pressure do not forget that. This pressure itself contains that gravitational effect or the body force effect we're not writing them separately, but that body force is present within this pressure itself. So, this is just not simply just fluid static pressure plus the effect of body force that is also within pressure. So, there might be a something where there is just think about a situation like you might have seen that from some dam or something the water is coming out and you are not applying any pressure difference. It is open to atmosphere on an inclined surface the water is coming out both these exposed to atmosphere.

So, it is the pressure is basically atmosphere at all the points. So, there is practically no pressure difference, but there is a difference in modified pressure, because there is a difference in the gravitational effect, gravitational force and then this average flow velocity can be defined as we will call it simply u bar is q by a.

So, what is that what that becomes…

Student: (( )).

Hmm

Student: P 1 minus p 2.

Or in terms of g at least we will write g, because that is shorter to write in terms of g it is g square by 2.

Student: (( ))

G square by eight mu.

Student: Eight mu.

G square by 8 mu can you write that as u maximum by 2 you can see from that solution u equal to g, g by 4 mu a square minus r square when r is 0 you get u max maximum u velocity which is g by 4 mu a square. So, it is that half of that the average velocity half of the maximum velocity. What is the shear stress acting on the wall shear stress on the wall. How much it will be? In this case the expression will be mu into or minus mu into d u d r in general this shear stress is tau r theta wall shear stress is tau r theta.

So, it will contains a theta derivative also if you write the strain rate for r theta, but since theta derivatives are 0 it is coming as d u d r only stress on the fluid is mu d u, d r stress on the wall is minus mu d u, d r. And how much is this comes as this d u, d r of course, is at r equal to a the d u, d r is to be evaluated at r equal to a, because we are finding stress on the wall and this comes as half of g a often expressed in terms of average velocity 4 mu u bar by a.

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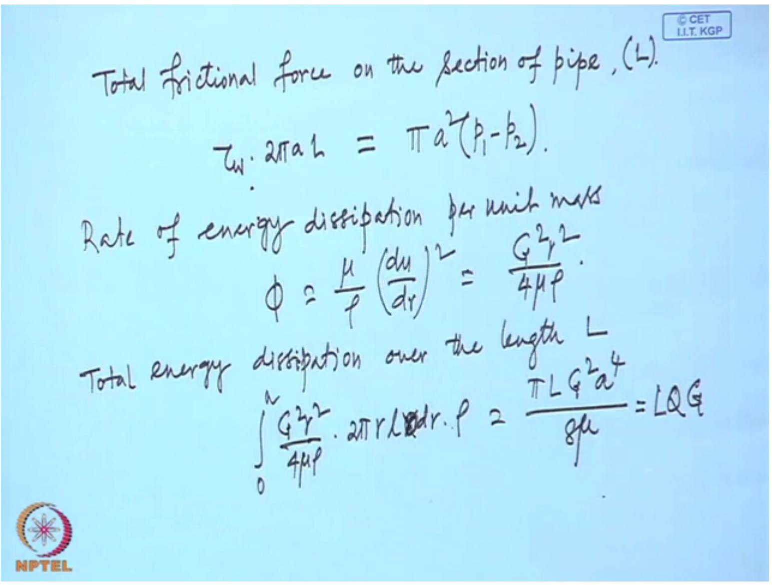

So, total frictional force on that section of the pipe. How much will be total sectional frictional force on the section of pipe. This shear stress is to be multiplied by the area on which it is acting and area of that on which area it is acting.

Student: (( ))

The curved area and that area will be how much two pi a into l.

So, this is that how much is that then substitute and you can see this will be something pi. The rate of energy dissipation is another important parameter and it is also required if you want to solve a temperature field. Once the velocities is known, you can solve for the temperature. And then of course this term will be required rate of energy dissipation say per unit mass or per unit volume. Let us say per unit mass this is the term pi is this becomes.

So, total energy dissipated over this section of length l this has to be integrated over the length this nothing actually by mistake I wrote the act wise 2 pi r l d r dissipation over the length. So, it becomes volume. So, you have to multiplied a one more rho. The first term we wrote here for unit mass so, another rho is for that to make it unit volume and of course, integrate from it and this will come as pi l a 4 in terms of the other parameters l q g . So, this much of energy is lost over the length of l and to maintain the flow of course, you have to supplement it that is being supplemented by maintaining that pressure difference. Some non dimensional parameters are used to express this a frictional effect in this flow and they are define in this way.

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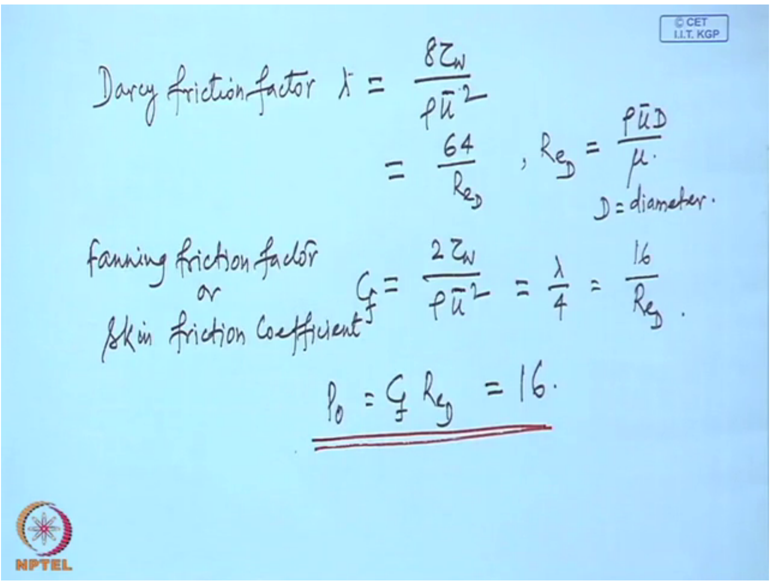

One is Darcy friction factor Darcy friction factor which is we will denote by lambda and is defined by this is a popular parameter in hydraulics and other field machineries hydrodynamics, but not used in aerodynamics, but anyway this is practically a hydraulic problem not an Aero dynamical problem.

So, we will be using this Darcy factor this is the definition and how much it will come just say substitute those values and see? It will come to be this 64 y r e d where this r e d is a Reynolds number based on diameter. The Reynolds number based on diameter in case, the pipe is not circular then this diameter is to be treated as hydraulic diameter another friction factor is defined which is again in hydraulic they call it fanning friction factor or we will call it skin friction coefficient.

This skin friction coefficient is of course, a very popular parameter or in aerodynamics this definition is say or most often we will call it tau w by half rho u square tau w. That is wall shear stress by half rho u square and obviously from here it will be one forth of this lambda this is by definition this we will write as c f equal to lambda by 4. In this case and often this product of this skin friction coefficient and this Reynolds number based on diameter this r e d the product of these two is called Poiseuille number. Poiseuille number p o is this c f into r e d and this of course, in this case is becoming 16.

As you can see this skin friction or this skin friction coefficient or the wall shear stress is an extremely important parameter in aerodynamics. If you look back to our original discussion in the first and second lectures we said that our main aim is to find the force that is acting on the body as the body moves through a fluid or alternatively if fluid moves over the body, obviously that the force may come from different sources that we also talked at that time. One of the source of this force is definitely this frictional force or the viscous effect, which is the wall shear stress of course, that is not the only force there will be other contribution from the other type of pressure distribution and all, but this is also one of the major component or important component and then particularly for say commercial vehicles or transport vehicles, the drag force is predominantly comes from this wall shear stress.

So, if you think about a transport aircraft then the drag force that is acting on that transport aircraft is mostly from this wall shear stress or fiction viscous effect and obviously see that that determines how much engine power you should have. So, that you can take your aircraft and hence how much fuel you need to consume. What type of engine you are going to have? How much fuel you have to consume? So, finding that wall shear stress on the aircraft is of course, a very important or fundamental task of aerodynamics. Before we switch to some other topics we will solve a couple of more such problem where we can solve the Navier stokes equation mathematically. Then of course, since we are interested in certain type of problems however, Navier stokes equation cannot be solved mathematically or analytically we will look for alternatives.