Introduction to Aerodynamics

Prof. K. P. Sinhamahapatra

Department of Aerospace Engineering

Indian Institute of Technology, Kharagpur

Lecture No. # 20

Conservation of Energy and Energy Equation

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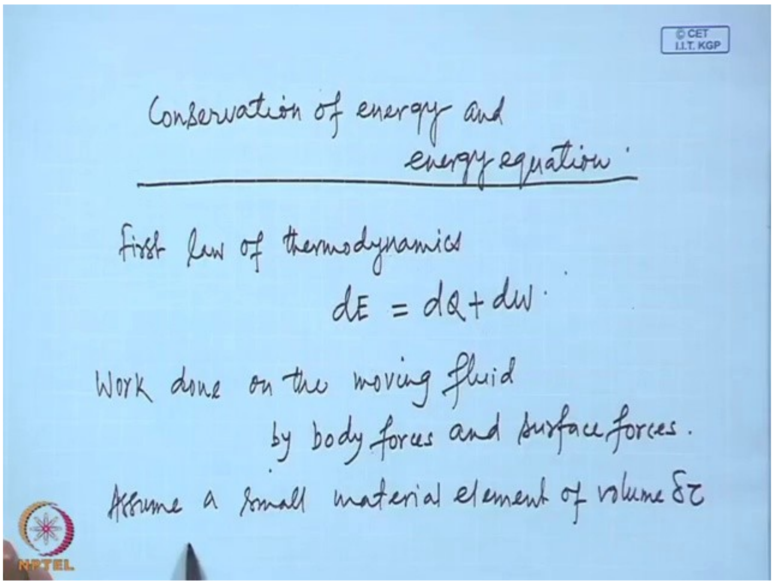

See this conservation of energy and energy equation as we mentioned that, this equation will be required when we have a considerable change in temperature in the flow. If it is not then of course, this equation will not be required and we have already mentioned that, most of this course or other this course fully and even the next course, we will consider incompressible flow without much change in temperature. So, this equation will not be required for this course as such. However, later on it will be required and used and you have to remember this equation without using it for the time being because, we I would like to complete this governing equation at this stage. It could have been left for later use or later application, and could have been derived later when you are going to use it, but since we are deriving the equation so, I would to complete the derivation at this stage.

As you know, the first law of thermodynamics usually we call statement of energy conservation. The first law of thermodynamics as it states that, the internal energy of a system changes due to the work done on it, and due to a heat added to it. So, if there is a system and you add heat to it, you would do some amount of work on it, then it is internal energy will change and the change in internal energy will be given by the sum of the work done on it plus the heat added on it. So using that conventional notations the first law of thermodynamics says, in mathematical statement we will write say, change in internal energy is equal to the sum of the work heat added plus sum of the work done; heat added to the system plus work done on the system. Of course in this application, we will consider that system is the fluid; the moving fluid, that is the system here.

Now as we have mentioned earlier that, the thermodynamical laws are derived or postulated for system in equilibrium; the system is in thermal and mechanical equilibrium, and we stated that the moving fluid is not in that sense a system in equilibrium in thermal and mechanical equilibrium is not in that sense in equilibrium. However, look into this rate of change in internal energy is sum of the heat added and sum of the work done. Now thus amount of heat added or the amount of work done, they are of course can be measured, and see whatever is sum if we consider that as the increase in the energy. So whether it is at system in equilibrium or not we need not be that worried; we can even consider that, even though the system was not in equilibrium, but it was passing through a successive stages of equilibrium, and we can use this relation even for non-equilibrium. Only perhaps change that we need is that while considering this energy, instead of treating it just an internal energy, we should treat it as the energy and moving fluid has energy other than the internal energy. Particularly, it is kinetic energy particularly it is kinetic energy. So this energy we can think of as the internal energy plus the kinetic energy.

Now, let us look to the different terms in it. Let us first start with the work done. The work on the system in this case, the system is as we say, the system is subjected to two different type of forces; the body forces as well as the surface forces and obviously both, if we considering consider a very small material element of fluid moving fluid then, both the forces will do some amount of work on it. So let us start this term by term. First of all, let us consider the work done on the moving fluid system. This will be by body forces and surface forces. Assume, a small material element of delta tau once again; so how much will be the work done by done on this material element by the body forces? See, the total body force rho f i d tau; work done is a scalar product of the velocity and force rate of work done.

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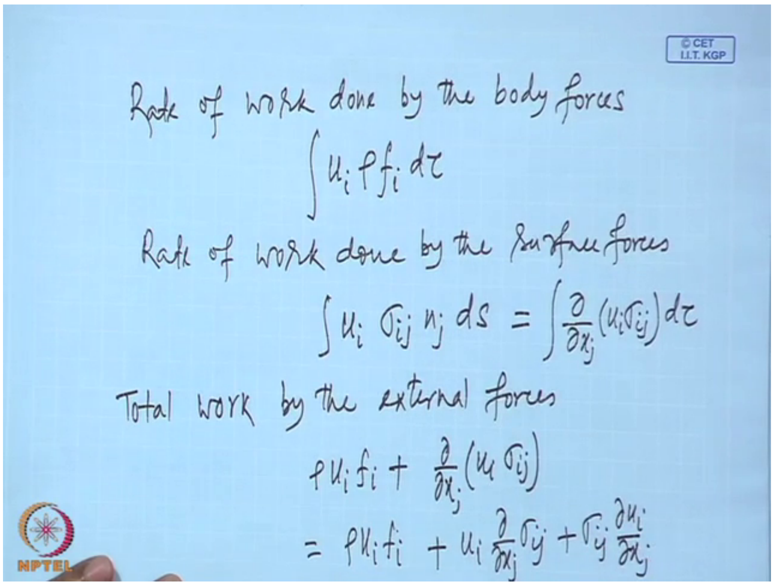

Yes how much is the total surface force? If the surface bounding this element, the surface of this element d tau delta s as we are writing then, the total surface force is as before sigma ij n j ds and again our scalar product with the velocity. So this will be u i sigma ij n j ds and if we add the two we get the total work rate of total work done by the forces. However, there is a difficulty in adding the two term; one is a volume integral; other is a surface integral. So once again, we have to express this surface integral by volume integral using that divergence theorem, and that is quite obvious, quite easy. What it will be? which again can be written as from the first two term, you see what it is, you can take out the u i from the first two term.

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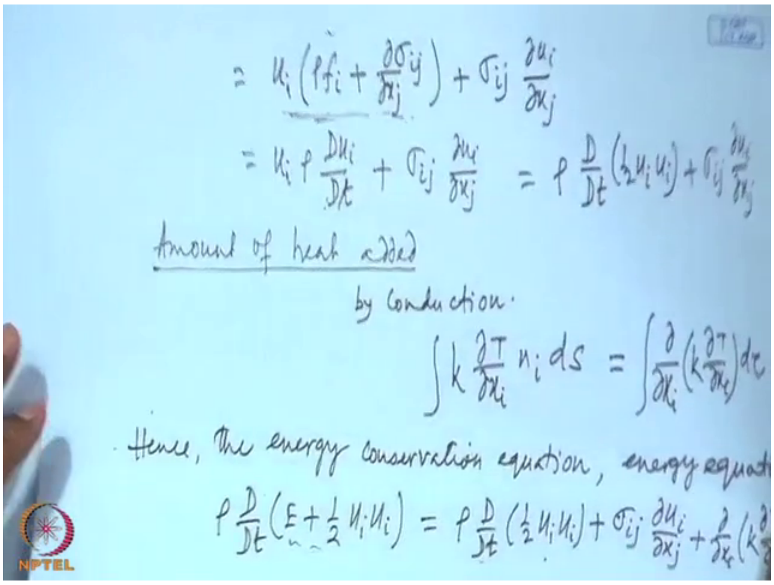

We can take out the u i the first term is the work done by the body force and let us before going that complete it. What is this rho f i plus d sigma ij dx j? Look back to Navier-Stokes equation, the most general form of the Navier-Stokes equation, where we have not replaced sigma ij and this is the right hand side or the force acting which is causing the change in momentum. So this term in the bracket is actually what is du i dt rho du i dt; the left hand side of the Navier-Stokes equation. So this is see, when this first term now can also be expressed as this; yes this is what? The entire term is of course, the rate of change of kinetic energy per unit volume; kinetic energy of the bulk motion not the kinetic energy of the molecular motion because, you are thinking in terms of energy equation where internal energy so, we should differentiate that; all of you must be well aware of that, what we call is internal energy is also basically a kinetic energy, but kinetic energy of the random molecular motion. It is not that; it is what is the flow velocity; so what we are calling flow kinetic energy.

So looking to this now looking back these two term, this first is from the body force of course, the work done by the body force or the volume force, and this is also a part of the work done by the surface forces. This is also another part of the work by the surface forces. The surface force work by the surface force has two terms in it the work by the surface force has two terms in it; these two and this one part has gone to this (( )). So, we can say that the volume force along with a part of the surface force is responsible for the change in the kinetic energy of the bulk motion, but what about what about this term? What it is? Think in terms of the element a small element du ij dx j is expressing at change in velocity across the surface across that material surface and sigma ij is the stress acting. See, it is basically expressing a deformation of the element; this work is equivalent to a deformation of the element and of course, it is not going to the kinetic energy of this molecule or sorry the bulk motion, and we can say that this essentially is changing the internal energy of the system. It is not affecting the bulk kinetic energy, but it is affecting the internal energy. So part of the surface force is instrumental for rate of change in internal energy, while the remaining part is along with the body force is instrumental in rate of change of kinetic energy.

Now as far as… So this is what about the work done, we have got the work done term. Now look to the heat added, how can be heat added to a system? Heat can be added to the system by conduction and any other form of heat addition at this we will neglect at this stage we will neglect because, this is the most usual and or most common flow cases that is the only thing which is important. Of course in certain cases, the heat by radiation convection is already can be taken care of by the other process because, all these are convective accelerations so, if there is convective term that convective heat transfer will automatically taken care of in the rate of change of the energy part because, you are considering convection by Eulerian description, you need not consider convection separately it is always included. Of course, even in Lagrangian case also it is included, but it is little difficult to see. In the Lagrangian case, the derivative is simply d d t partial derivative with respect to time. So you do not really see clearly that the convection is also part of it, but in case of Eulerian description because you express the material derivative is like in that way there is a convective term so, the convection is always included.

So, other form of heat addition like say heating effect through radiation or any other processes so that we are not going to consider here, I mean you are not considering extremely high temperature at which stage many other processes are involved that you are not going to consider. So, how much will be the amount of heat added? How much is the heat flux across a surface through conduction? How much heat can go out from a surface or come in or go out or come in whatever it is? On what it depends?

Student: Temperature gradient.

Temperature gradient temperature gradient fine and the rate of heat transfer is?

Student: Coefficient of thermal (( ))

Coefficient of thermal conductivity into

Student: (( ))

Gradient of temperature. So this can be written again say thermal conductivity; let us write k, the gradient and once again you can express it as a volume integral by using that same divergence theorem and there should be minus sign because of temperature gradient is positive so it will flow out of the system. But, we have taken that we are considering that heat added to the system as positive. So temperature gradient is seen from the side of system or towards the system? You are considering towards the system because, you are adding heat to the system that we have taken as the positive. If you look to first equation, we wrote for the first law of thermodynamics in which, we have that d e equal to d q plus d w both are both we have taken what is added to the system as positive. What you are saying is correct usually the this is known as Fourier's law of heat transfer Fourier law of heat transfer and the law is usually expressed in that form q equal to minus k grad t, but because of our this notation so we have made it positive.

So now, we can write the complete energy conservation equation energy conservation equation or in short energy equation. That first the thermodynamical relation which is written for unit mass, we will write it as a rate equation and we will also write instead of instead unit mass, we will write as per unit volume opposite side. Opposite side; so they get cancelled and see all these terms to get cancelled. So sigma ij xi ij is identically zero; so this becomes only sigma ij e ij; express or replace sigma ij by what we have already found sigma ij; sigma ij we have already found as minus p delta ij plus 2 mu look this first term, what is this? p e ij delta ij minus p e ij delta ij; what is e ij delta ij?

Student: (( ))

Yes, what will be p or delta ij e ij e ij delta ij? You know, delta ij is 1 only when i equal to j otherwise, it is zero.

Student: (( ))

So, it will become simply e ii; so the first term is minus e ii p e ii; what is e ii? Rate of expansion rate of expansion a dilation. Now, this entire equation is our rate equation so, if you forget this that rate, this is pressure and expansion what is that? What in thermodynamics is usually called as preview work or compressive work, compression work. So the first term gives that; more interesting is a second term; see, the second term can be written as can see it is 2 mu e ij e ij minus; what is one third? Again here also, there is e ij delta ij which will again become e ii and e ii in this case also, we can write as e kk; e ii or e kk writing this all the same. So this again become, it will be quite a long term if you expand, it is quite a long term, but as you can see

Student: (( ))

Why it is? How it is square? See already, you had du k dx another du k dx you are getting from e ij delta ij; e ij delta j is e ii and instead of writing e ii, if we write e kk is all the same, we can change any dummy variable can always be changed i e in e ii; i is a dummy variable it means e 1 1 e 2 2 plus e 2 2 plus e 3 3, the meaning of e ii is e 1 1 plus e 2 2 plus e 3 3 and meaning of e kk is same; e 1 1 plus e 2 2 plus e 3 3 and what is e? e i? e ii is du i dx i, so e kk is du k dx k; so that is what we have written du k dx k and d kk is same so, either you write e kk square or du k dx square anything and as you can see clearly, this second term can never be negative this second term can never be negative because, this e ij e ij contains this e 1 1 square e 2 2 square e 3 3 square which is or one the second term is simply one third of e 1 1 square plus e 2 2 square plus e 3 3 square, and the first term contains e 1 1 square e 2 2 square e 3 3 square also many other term which are also again square term, all the terms are square term. So that second time can never be larger than this, and since all the term present here are square terms, this second term can never be negative.

Now looking to this, what is what is this part of work? Work done by what? Looking from here, this is what is that second tangential stresses. This is part of the deviatoric stress containing only that part of the normal stress which sums to zero, plus the tangential and the tangential stresses. So, this is work by the this is work done by the tangential stresses or viscous shear stresses, and what we can see that this viscous, work done by this viscous shear stresses is always positive. That means, it is only one way process it is only one way process in which only the kinetic energy moves towards the internal energy that is the only similarity, and this energy since we cannot get back is basically loss. So, it is a dissipation; this term is called a dissipation term and usual just denoted by single symbol phi quite often; again do not get it confused with the velocity potential because, that is standard notation used conventionally phi for many things. So this second term is always positive and it is known as dissipation.

The first term is the conventional or familiar pressure volume or pressure sorry compression work, preview work, pressure volume work called as compression work and in this case, this is i useful work whatever work, we give we can get it back; whatever work we use while compressing, it we can get back it while compression. So here also, you see that we have not introduced any further unknown except this thermal conductivity. Only this internal energy is the only additional unknown or temperature which we set forth. So, there is no further unknown what we had earlier; three component of velocity, pressure, density, and temperature or internal energy now we have that. Along with the equation of state we now have six equations with six unknowns of curse additional parameter mu and k; the coefficient of viscosity and thermal conductivity which are usually function of temperature, and if they are provided as a function of temperature or set of equations is complete.

Now, we have as many equations as there are unknowns. However, as before we mentioned that when the flow is incompressible, the temperature does not come through the equation of state; the equation of state simply becomes density is constant. So there is no temperature, and mu is also a constant. So obviously, this energy equation is not at all required. In case, whatever small temperature difference or small temperature variation in is involved, if you want to find that, it can be done by solving the others set completely first; there is the combination of equation continuity and Navier-Stokes equations together; find the velocity and pressure use that velocity and pressure to solve this equation to get the temperature. That means, they need not be solved together, they can be solved independently because, if you see that in those equations then, there is no term which depends on temperature. The equation is completely decoupled from the energy equation and can be solved separately.

Student: Mu (( ))

Mu is then not a function of temperature; if the temperature variation is very small, say just about 50 to 100 degree type of temperature variation you are associated with in that case, mu can be taken as constant.

Student: (( ))

Standard value of mu can be taken in that and because even though, temperature mu is a function of temperature, but the variation is so small with temperature that over a range of 50 or 100 degree temperature, the variation in mu is the really negligible. So that type of small temperature difference is not affecting mu so, they can be solved separately the equations, and once that solutions is found the velocity particularly the velocity field because, to solve this equation you must know the velocity. This equation contains all velocity; these e they are velocity even here also, there is velocity. When you write this is in the full expanded form, there is velocity associated with it. So without knowing the velocity this equation cannot be solved, but the other equation can be solved without knowing the temperature.

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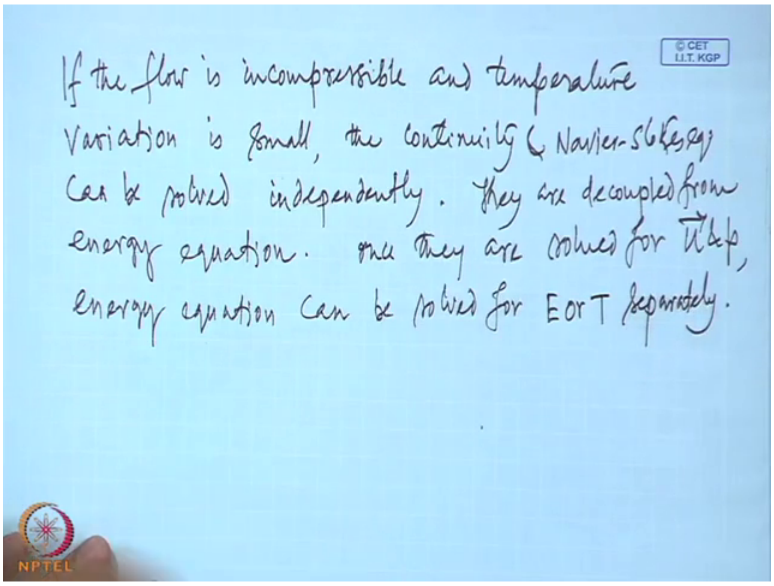

So this you can also summarize that, if the flow is incompressible and temperature variation is small, the continuity and Navier-Stokes equations equation can be solved. They are decoupled from energy equation they are decoupled from energy equation. Once they are solved for velocity and pressure u and p, energy equation can be solved for temperature separately.

We will consider one or two small examples such for solving this Navier-Stokes equation particularly, I think you have already solved one or two problem using these Navier-Stokes equation without even writing the Navier-Stokes equation or perhaps knowing the name perhaps, you have not heard of the name of these equations Navier-Stokes equations, but you derived a special form of Navier-Stokes equation for a special application and solved it. So will as we mentioned already that for the general case, there is no solution. For some simple problems; for some very simple cases some solutions are there and of course, most of them will leave for later, but one or two simple cases we will consider to look further solution of the Navier-Stokes equations.