Introduction to Aerodynamics

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Lecture No. # 18

Equations of Fluid Motion - Navier-Stokes Equations (Contd.)

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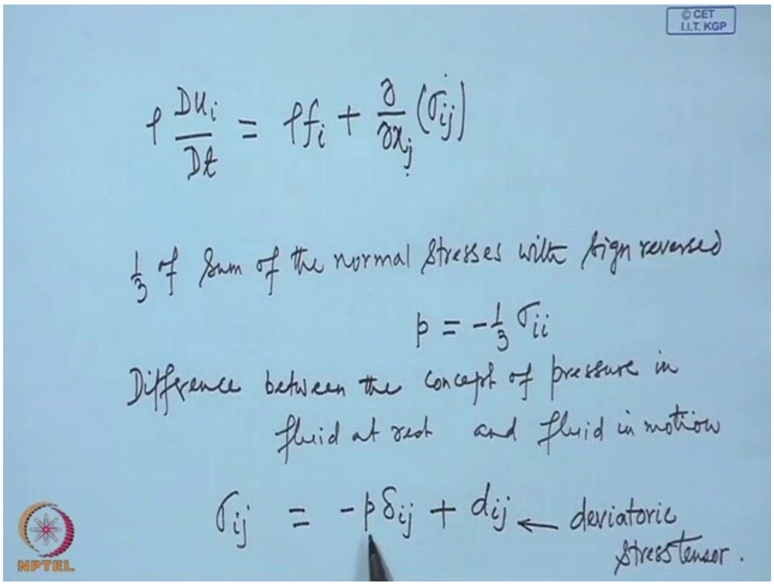

So, the general form of equation of motion that we have obtained is given as rho Du Dt to rho f i and divergence of the stress tensor, the divergence in the direction of the normal to the plane on which the stress is acting, that is the divergence with respect to the j component. Now to proceed further with these equation, obviously we have to know something more about the stress tensor, now to look to this stress tensor that what this stress tensor is likely to be. First of all think about that what this stress is, this stress is a reaction of two adjacent fluid elements, mutual reaction of two adjacent fluid elements, and represents momentum flux across the interface of two adjacent elements, the momentum flux across the adjacent part of the two elements, across the surface of the two adjacent elements.

Now, obviously this is local behavior, so we can expect that some local description of the fluid or the state of the fluid will perhaps be related with this stress, at least when the fluid is at rest we have already seen that this stress is given simply by the local pressure. When the fluid is at rest we have already seen that the stress can only be a compressive stress which we have called the pressure and it so happens that that is the pressure defined as in thermodynamics, considering mechanical equilibrium, thermal and mechanical equilibrium. Now when the fluid is in motion obviously we cannot expect that it will be the same, because the momentum flux is likely to be influenced by the fluid motion, when the fluid is at rest the momentum transfer across any surface it just by simple the molecular motion, but in this case we have another motion, the motion of the bulk material.

So, the stress is unexpected to be the same as the stress at rest, also while discussing about the stress on a in a fluid at rest, we saw that the pressure is the only stress and the pressure is also equal in all direction, that is only the normal stress exist and it is independent of the direction in of the normal, whatever the direction of the normal is the stress will remain the same which is known as the pressure is same in all direction when the fluid is at rest. Now when the fluid is in motion we cannot expect that only the normal stress will act and also that the normal stress will be independent of the direction of the normal, eventually none is true, when the fluid is motion fluid is in motion the stress is not only normal stress. And the normal stress that is present that is also not same in all direction, I mean, if the direction of the normal change, the normal stress also change, these are of course, observation.

So, we do not have that simple thermodynamic concept of pressure. When the fluid is in motion, however still when the fluid is in motion still that compressive effect of the pressure or something equivalent to that is still present. So we can try to redefine or get some quantity which is equivalent or analogs to that pressure, which may not be pressure but which is analogs to pressure. And then we can call it pressure, or rather that is what is the practice followed, in a when the fluid is in motion we define such a normal stress which is the average of the three normal stresses considering any particular set of axis. Of course, this is a tensor property; we have already mentioned about it, that the some of the diagonal which is called the trace of a tensor is an invariant.

That means if we change our axis system, the elements of the tensor will change. But the change will always be such that the some of the diagonal will always be constant, and eventually in when the fluid was at rest that one third of that trace or the some of the diagonal that is what the pressure was. In this case also there will be an one third of that trace when the fluid is in motion, that stress tensor is also have a invariant trace that is some of the diagonal, and one third of that we will call it again a pressure, one third of the some of that diagonal term we will call it again the pressure, but we should remember that this is not the same as it was when the fluid was at rest.

The magnitude may vary. However the basic concept itself is different, in when the fluid was at rest this one third of the some of the diagonal element is independent of the direction of the normal. Whatever is the direction of the normal it will remain the same, but when the fluid is in motion it is not so, it depends on the direction of the normal, and even it can it can be shown that this one third of the some of the diagonal will be same. If we take an average over all possible direction of the normal, if we take the average of the sorry if we take the average over all possible direction of the normal component of the stress that will also be the same as the one third of the stress, or one third of the some with the diagonals.

We can also define it as the average of the normal stresses acting on a say small spherical element centered about the point the point at which you are interested. So this quantity these averages of the one third or one third of the average of the normal stress we will continue to call it pressure. But this certain difference that this is not the pressure in thermo dynamical sense, it is just basically a mechanical definition, and they might be of course, might not be some difference with the pressure, what we call the pressure or what is the thermo dynamic concept behind pressure, is not the same as the one third of the diagonal some of the diagonal elements of the stress in a fluid at motion. But we will continue to call it pressure, but now meaning that it is simply the one third of the some of the normal stresses, sign reversed the sign reversed, but it is not the pressure as we define in thermo dynamics.

So, with that so we remember this differ one third of some of the normal stresses with sign reversed. If we just look superficially as a mathematically you will say that in case of fluid at rest again this pressure was defined as one third of the some of those diagonal elements, or some of the normal stresses with sign reversed, and we are defining the pressure here also with that, that is the similarity. But the main difference is behind the concept, the pressure is basically a thermo dynamically quantity and is defined in thermo dynamics.

I think you have done little bit of it in your kinetics theory gases, and that is the same pressure when the fluid is at rest. But when the fluid is in motion, this one third of the normal stress or some of these normal stresses is not the same quantity, but because of this mathematical similarity we have we will continue to call it pressure, and quite often will even forget this distinction, but remember that there is a very important a conceptual distinction between the pressure in as define in fluid at rest and as define in fluid in motion.

Now, then the fluid the stress tensor at in fluid at motion can now be written as this pressure plus some other part of stress, which depends only because of the motion, because this is in when the fluid is at rest this is the only stress. But when the fluid is not in rest fluid in motion even the tangential stresses will be there, and even some remaining normal stresses will be also there, because this is the sum of the average. Each of the normal stress element that is sigma 1 1, sigma 2 2, sigma 3 3 none of them are equal to this, so little amount of or some amount of normal stress is left out. So the stress tensor including these remaining part of the normal stresses, and the entire tangential stresses is not included here, so the complete stress tensor now can be written as say sigma ij as minus p delta ij plus we will call it d ij, this d ij is commonly called as the deviatoric stress tensor.

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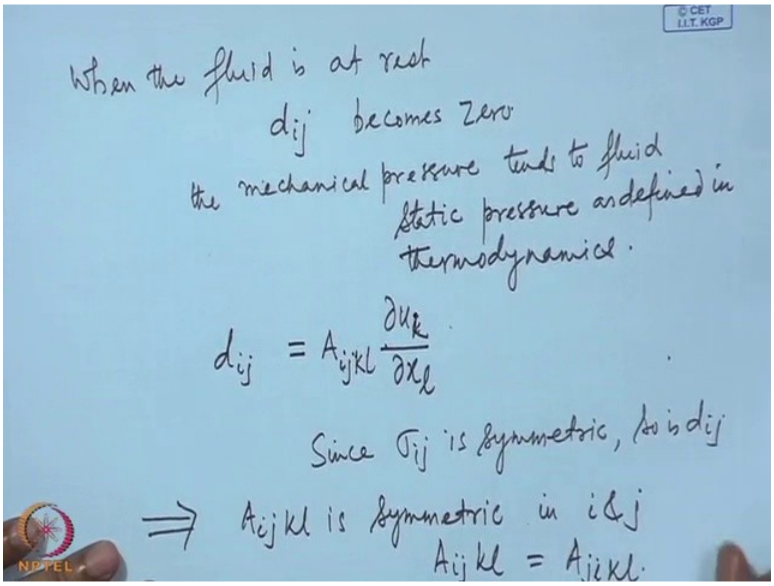

When the fluid is at rest, this d ij vanishes, and this mechanical pressure changes to that thermo dynamical pressure or static pressure, so when the fluid is at rest. Now, if we assume the fluid to be isotropic, the stress tensor will behave in that way it will have no direction preference, irrespective of the orientation it will remain invariant, and that is possible if Aijkl is also a symmetric tensor, sorry isotropic tensor, this will be possible if A ijkl is an isotropic tensor. So this is the meaning of this assumption that is why you have made this assumption, that the coefficient matrix A ijkl is an isotropic tensor. and this delta kl is of course, symmetric, it is only a diagonal matrix diagonal tensor, with diagonal element as 1. This then means that A ijkl is symmetric also in k and l, because this right hand side now symmetric in i and j as well as in k and l, so the left hand side A ijkl also must be symmetric in k and l. Now we can look to this that second term containing the vorticity term containing the vorticity term again mathematically, look this says A ijkl is symmetric in k and l, symmetric in k and l then we say that epsilon kl will also be symmetric in k and l, but we know that by definition that alternating tensor epsilon kl is not symmetric in k and l, but this needs that it must be symmetric. So obviously it cannot be represent, so mathematically also we get the same thing that this second term cannot be present in the stress, or other way that vorticity will have no contribution in the stresses.

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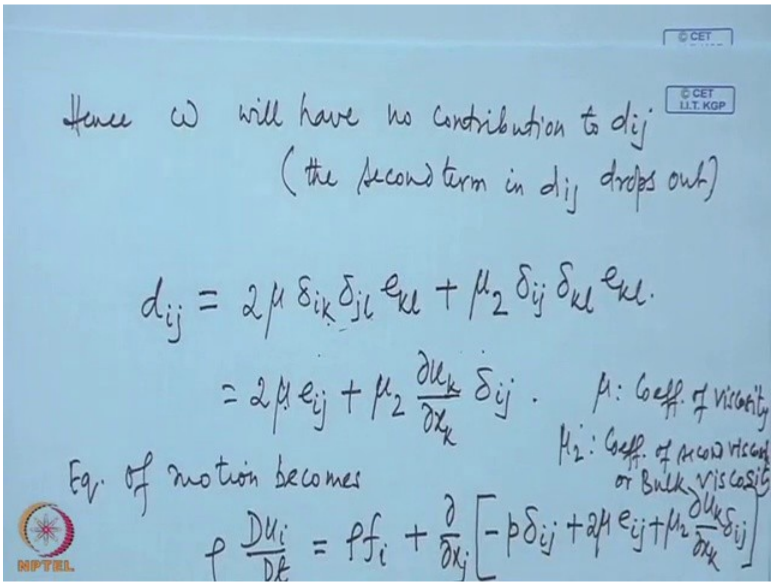

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And

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Delta ij

Student: Delta kl

Delta kl

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e kl, what is this? This is 2 mu e ij plus mu 2 du k dx k delta ij, see in this kl are repeated in this, so if you just make all these, if you go on writing the term for different values of k and l of course, in three dimensional case k and l become 1, 2, 3 both k and l can be 1, 2, 3, so if you carry out that product and the values of delta are either 0 or 1, the result will be this, and same thing this will become this, this is now the deviatoric stress tensor.

Now, we have expressed everything in terms of velocity, e ij is already expressed in terms of velocity, this mu is called the viscosity coefficient of viscosity sometime also called coefficient of dynamic viscosity. Because there is another parameter which is mu by rho, mu divide by rho which is also called viscosity, often we called often called kinematic viscosity to differentiate it this is called dynamic viscosity. But actual name is just viscosity, coefficient of viscosity. And the second one mu 2 is called of the coefficient of bulk viscosity, because you see this term present with this mu 2 is du k dx k, what is that du k dx k? It is divergence of the velocity or rates of expansion this du k dx k, since it is associated with that volume expansion this is called the second coefficient of viscosity or bulk viscosity. So, this is coefficient of viscosity this is coefficient second coefficient of viscosity or bulk viscosity.

And now of course, we can put it in that general form of the equation so the equation of motion becomes equation of motion becomes rho du by dt equal to rho f i plus d dx j of minus p delta ij plus 2 mu e ij plus mu 2 du k dx k delta ij Now, let us once again go back to our deviatoric stress tensor, this is the general case this equation is known as Navier-stokes equation of course, it is a vector equation for each component of momentum, I can take the value of 1, 2, 3 each will give 1 equation for each direction, momentum equation in three direction. So there are three equations n here, but once again let us go back to that d ij, what will be d ii? What will be d ii? That is sum of d 1 plus d 22 plus d 33, what it should be? If you remember that from sigma we have taken the sum one average of the sum of the normal stresses as the mechanical pressure, which we have defined as pressure, then what remains you have taken out that isotropic part, then what remains of that sigma 11, sigma 11 minus p is the d 11 now.

So, what will be the sum of it? Sigma 11 minus minus p that is sigma 11 plus p, sigma 2 2 plus p, sigma 33 plus p that is what are the three diagonal terms now in d 11, so sum of them what it is? Sigma 11 plus sigma 22 plus sigma 33 is three times p minus three times p, so it is 0. We will continue it in the next class.