Introduction to Aerodynamics

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Lecture No. # 17

Equations of Fluid Motion –Navier-Stokes Equations

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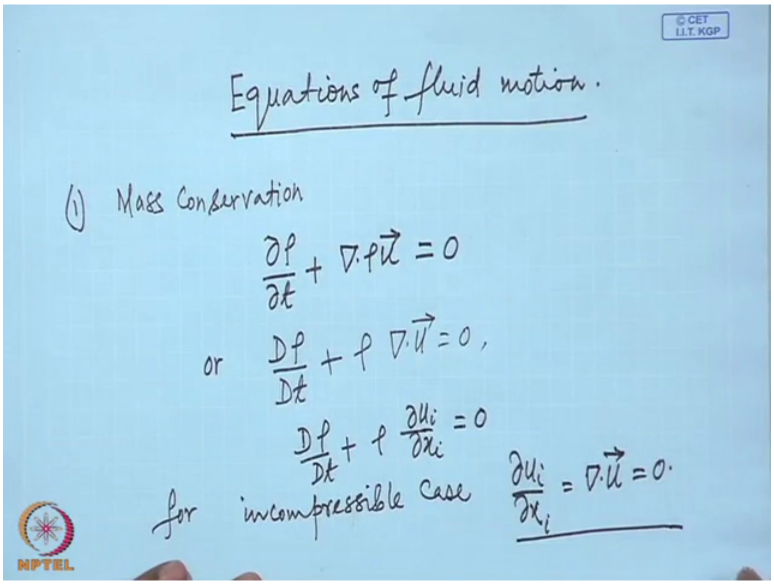

We will now consider what are commonly called as equations of fluid motion. You can call it that the governing equations of fluid motion, eventually the governing laws are nothing but the statement of the physical conservation laws, one of these physical conservation law we have already considered and discussed, which talk about conservation of mass and we have derived the mass conservation equation or equation of continuity.

Just to remind you once again, mass conservation that is the first equation and if you remember that we consider two three approaches to derive this equation, in which in one case we considered a control volume and then found how much mass is entering that control volume and leaving the control volume, and the difference of that is a rate of change of mass within that control volume. For that we consider say a cubic control volume and you consider each phase how much mass is entering, that is the way we did it one case, we also used a straight forward vectorial approach by which we found out the total flux of mass out of that of control volume, which again we equated with the rate of change of volume rate of change of mass within the control volume.

And the third alternative that we considered, that we consider a material volume that is a volume which is not fixed in space, but a volume consisting of or comprising of certain amount of fluid, then you said that the volume of that material element can change because of the movement of the boundary, how much the boundary is moving, and that is the way we found out the rate of expansion or dilatation of course, the equation is same in all the approaches and the equation to remind you once again is d rho d t plus divergence of rho u equal to 0, or can be written as like this d rho d t plus rho divergence of u equal to 0.

In index notation this also we wrote that d rho d t plus rho d u i d x i, so this is a different form of conservation of mass, popularly known as equation of continuity or continuity equation, and from here we said that for an incompressible flow in which the change in temperature is also not large, change in temperature is also small so that density remain constant, the equation of continuity simply becomes divergence of u equal to 0, or divergence of u is the dilatation or rate of expansion so rate of expansion is 0, is the form of continuity equation for incompressible fluid.

So, for incompressible case write… in particular I will ask you to recall that

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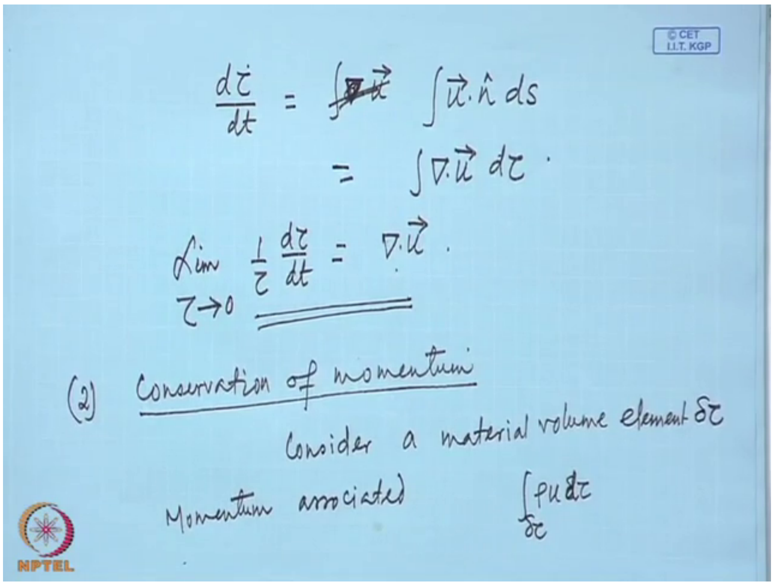

we also defined that rate of change of the material volume, rate of change of material volume tau, tau is the material volume that is volume comprising of certain amount of metals, certain amount of fluid, not just a volume in space, is I am sorry I am sorry not divergence, or in terms of divergence sorry write both, in terms of the movement of the boundary where this s is the boundary of this same material volume. So, this is the amount by which the boundary is moving, which can be written as… And you defined that rate of dilatation, rate of change of material volume per unit volume is divergence of u, rate of change of volume per unit volume is the rate of expansion so divergence of u is the expansion or dilatation which of course, you have done earlier, we need this for our next equation so that is why I just wanted to recall this.

The second conservation or which now we said to derive is the conservation of momentum, which is of course, known to you in the form of Newton's second law, see eventually that is what gives the so called equation of motion or governing equation conservation of momentum, what it says that it relates the rate of change of momentum with the force acting.

Now, when we discuss about rigid body mechanics these are more or less obvious the rate of change of momentum. Rate of change of momentum is what? Obviously of the particle or the body, in rigid but mechanics there is no ambiguity, but when you come to fluid mechanics, because of our nature of description of the fluid motion, we have little ambiguity, rate of change of momentum of what, so you have to be very clear that this is the rate of change of momentum of the material, rate of change of momentum so in case we are using the eulerian description, where everything we fixed in space a volume a fixed volume in space, the material is not fixed the volume is fixed.

So obviously the since the material is not fixed the rate of change of momentum of that volume is really not applicable in this context, what we need is rate of change of momentum of certain material fluid and similarly, the force acting on it, so let us consider a small or infinitesimal material volume, infinitesimal material volume then what will be the momentum associated with it. Let us consider a material volume material volume element we are calling it delta tau, so how much is the momentum associated with this material. Rho u delta tau for all the mass of this material element is rho delta tau, and so momentum is into u of course, integrated over this entire volume oh sorry rho u d t integrated over that volume element delta tau.

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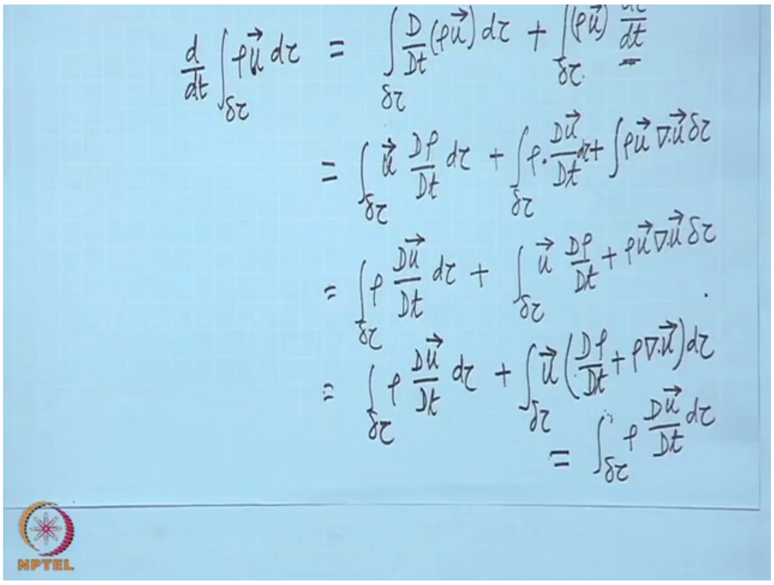

Now, the rate of change of this however we already mentioned that see this description of fluid motion based on material is not very convenient, is not very suitable, the description based on the control volume approach or point approach field approach is more convenient and is more widely used. So, this will now shift to that control volume or volume based approach, and we see how that be obtained, this we can write this u the way we have defined is that eulerian description, the velocity is described by eulerian description, that velocity associated at a point, so this derivative is basically the material derivative, here we have derived it on the basis of the material, so to look for the derivative of the velocity this is basically the material derivative, and we will use that same notation, that this is now become this, is that all, no you see even this the material volume that is also changing, the material volume that is also changing.

So, we have even this integration, and now replace this by using that relation here, using this by the rate of definition of rate function, this we can further write sorry… And now see the second term is 0, this term is 0 by equation of continuity. The second term is 0 from the equation of continuity, so we have only… please look to this particularly carefully, because this is a question of perhaps again we will hear from you many times that in this the density is out of the derivative term, it is perhaps again you will say that this is valid when the density is constant, or say for incompressible flow, what you see it is not so, we have started here where the density is taken within the derivative, only it has become like this, so here even though this density is outside this derivative sign we have never considered density to be constant, it has become 0, because of this continuity equation, and this continuity equation you can see is valid when density is variable, because in our final equation this is what will remain, where you will not see that the density is being differentiated throughout the equation.

So, most of the time you try to believe or you think to or you think that this is an equation in which density is taken as constant, but no see the density is been taken as a variable, only because of this continuity equation this has taken this form where density has come out of the derivative. This rate of change of momentum as you know is balanced by the forces that are acting on the fluid element, we have already discussed a great deal about the forces that acts on a fluid element, and what are these in general in a specific case we will have specific forces, but in general case the general forces, the body forces and surface forces.

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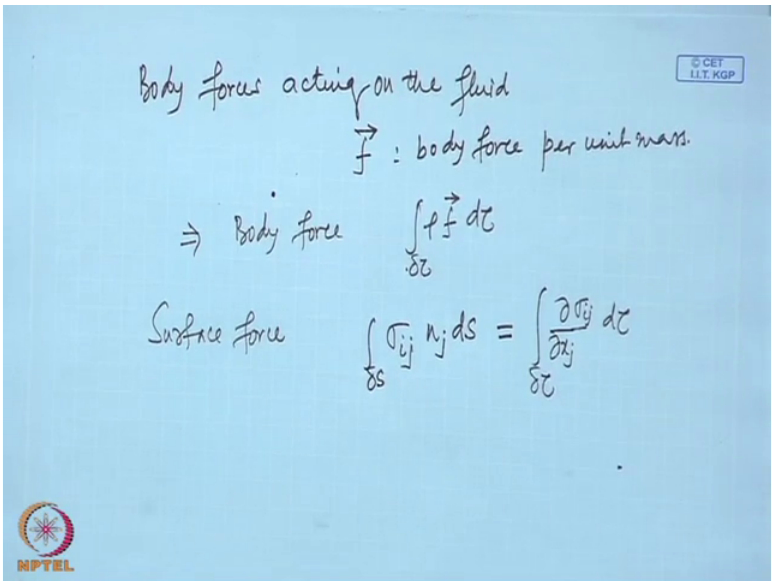

So, you can write what is a body forces acting on the fluid, that can straight away be written body forces acting on the fluid, once again if we follow the same notation that f is the body force per unit mass, f is the body force per unit mass then the total body force then the total body force is… so the most general form of the equation of motion, however you see that the equation in this form is perhaps not that useful unless we know little more about the stress tensors, particularly how the stress tensor is related to the motion, how the stress tensor is related to the motion, unless we can specify that this equation cannot be used this equation cannot be used.

As far as the body force is concerned you see the most common cases the most common case is the body force is the gravitational force where you can write that f i is the g, gravitational acceleration, in some problem if there is different type of body forces usually you will be able to write what is that body forces, if it is electromagnetic body forces it will come with a cross product with the velocity and magnetic field, electric field multiplied by the charge.

So, that sort of force stamps will come if there is any other type of body forces like electromagnetic or if you have some centrifugal type of fictitious body forces that also you can write in a in a specific case, but we must know something about this stress tensor, how it is related with the motion in general, until we can do it there is no question of solving this or doing anything with this equation.

However, before we go to look for something about this stress tensor, we will try to write this equation in another form, we try to write this equation in another form known as an integral form of course, we can say that we have you have already derived in integral form, but we will now do it whether we will write this equation in another integral form. See the first term here this d u i d t as you know that this material derivative can be written in this form think about a jet coming out of a nozzle, let us say a nozzle from this a jet of fluid is coming out, and this jet of fluid is heating a plate, as you can understand that this jet of fluid will try to displace this plate it will act exact certain force on it.

So, if you want to keep it fixed you have to apply the opposite type of force, how much is that force? consider say the diameter is 10 centimeter and the velocity here is just 8 meter per second, the velocity it has only 1 component of velocity the x component we will call it say u 1 8 meter per second, you can consider a control volume like this, and apply this momentum and apply this momentum equation in integral form, and you will get the force or the solution straight away, in this problem of course, we can neglect all these viscous stresses, sorry we will we should not call viscous stage, at this stage that stress there is no stress acting on it, what happen when this jet heats the plate? you know whether jet speeds into up it follows the plate, so there will be flow in this here after coming here, when it heats the plate it can no longer go in the straight direction so it will go up and down following the plate.

(( )) Cannot field particles rebound field particles can rebound also (( )).

No see in this case rebound means with the they will be again carried this flow is continuous know, so they cannot come back this way, this flow is going continuously so a particle cannot come back wait, it will be again taken away by other particles, so in this problem see this i and j cannot take 1, 2 3, we will treat this problem as a 2 dimensional problem, we will treat this problem as a 2 dimensional problem. So, i and j both can take 1 and 2, and eventually that rho into u 1 that gives the in to area of course, gives the mass flow rate, the mass flow rate rho e 1 Into this cross sectional area that will give you the mass flow rate.

Yes see the force acting on the fluid, the force acting on the fluid can be obtained from this just by writing the left hand side term only, the right hand side whatever is there that is the force acting on the fluid. So, that can be obtain by writing only, the left hand side terms only so how much is that rho u 1 a, now this u 2 that is the velocity in this direction there is of course, what in that there are both terms upward as well as downward.

However, both are same, yes both are same yes you take this area of course, here is an assumption that the fluid is split in two half exactly, but of course, there is no reason why it should not be, anything else, yes also in this direction is there rho u 1 a u 1, what will be the sign of this term, if we consider force acting on the fluid then it should be negative, if we consider force acting on the plate then it is positive. So, actually this is what is the term ultimately, this is what is giving us everything, so force on the plate. u 1 and u 2.

Student: (( )).

Sorry.

Student: (( )).

This is 1 is in the upward direction the other is in the downward direction.

Student: Sir why cannot we take a (( )).

I perhaps not following u.

Student: Sir we took all the combinations u 1 u 2.

U 1 u 2 again in the u 1 u 1.

Why did not we take u 2 u 2.

Student: U 2 u 2.

Yeah.

No see this part if you write this equation this i in the direction of x we are writing only in the force in the x direction, force writing in x direction, this i actually gives the direction, as you can see this j is summed, j is repeating so j mean this it will be sum over j so actually this i will remain in the final equation, in each terms you see this i will remain so this is an equation in the i direction, you are writing here the equation in the x direction or the first 1 direction.

Student: I is the potential function (( )).

psi

Student: (( )).

No this is not velocity potential in this case this is the.

Student: (( )).

This is the we said that the force is a potential force so it is the potential force.

Student: (( )).

So, in this case this psi do not treat it as a streamline, please do not give much weight age to the symbol itself, look what that symbol stands for, that whenever there is a psi; the psi is streamline no, in this case the psi is not streamline it is that force potential, so there it is a potential energy per unit mass, the numerical figures are not important that can be obtained.

So, see this is the utility of this integral form, in any situation wherever you will be able to apply integral form, you see that the answer is obtained almost without any effect, but as already mentioned that you will not get the details of the flow picture, you will get the important information without getting the details, this is of course, one simple example but there are many applications, you can consult any standard text book on fluid dynamics, and they will give you the application and momentum principle for various numerical problems, perhaps we will also give you a few more problems as tutorial problems which you may try. Then I think we will stop and discuss about the stress tensor in the next class.