Introduction to Aerodynamics

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Lecture No: # 16

Irrotational Solenoidal Flow in Multiply Connected Region (Contd)

So, we have seen that by simple logic we can say that the flow field in a doubly connected region can determined uniquely, if we know or if we specify the cyclic constant, and as before if we know the normal component of the velocity on all the boundaries or the potential on the all boundaries or a combination of the two. So, one additional constant is required additional constant has come that the cyclic constant has to be specified, this of course, we saw from simple logic, but let us check it with mathematics also mathematical analysis.

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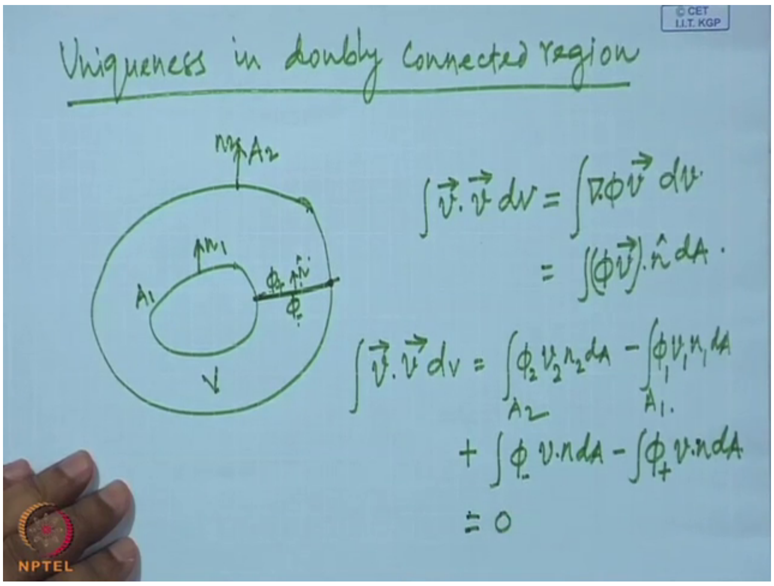

As let us first of all consider the single or doubly connected region, think about this barrier, as before this is A1 with normal N1, A2 with normal N2 and this barrier with normal N as before in this region, now in this domain now we can apply that result which we had earlier, for singly connected domain also we use this result, remember this v does not include this barrier this is the boundary of that volume, and this now we can apply to this applying that gauss theorem by which we get it over the surface, and this area in this case is as before these two are the integration on the two sides of the barrier, this way doubly connected region, so pi is many valued, and the difference between the potential pi at two points on the two sides of this barrier is pi plus pi minus, on one side it is pi plus on the other side the value is pi minus, and how much this will be this integration is (( )).

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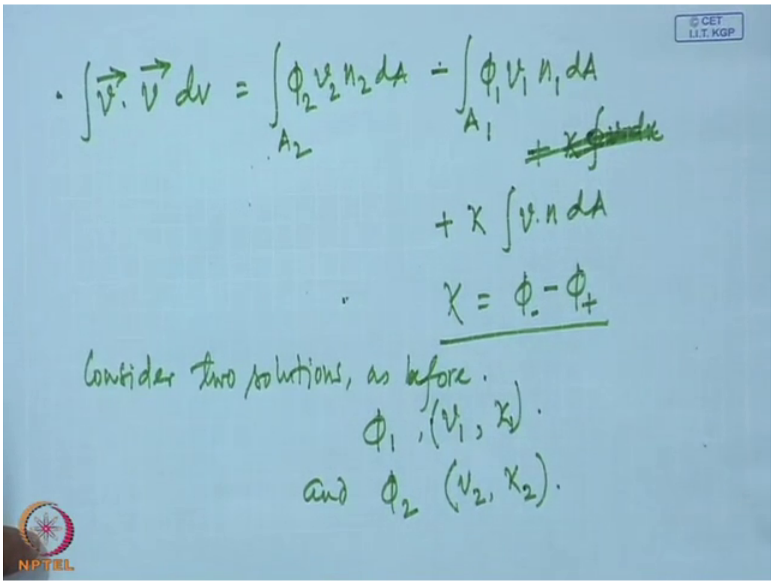

Now, as before let us consider again two solution, before that we can combine these two and we can make it like this, which term we wrote first 2 pi 2 v 2 n 2 d a sorry plus 0.2 where this cyclic constant is the difference in potential at that on the two sides, that is what is the cyclic constant.

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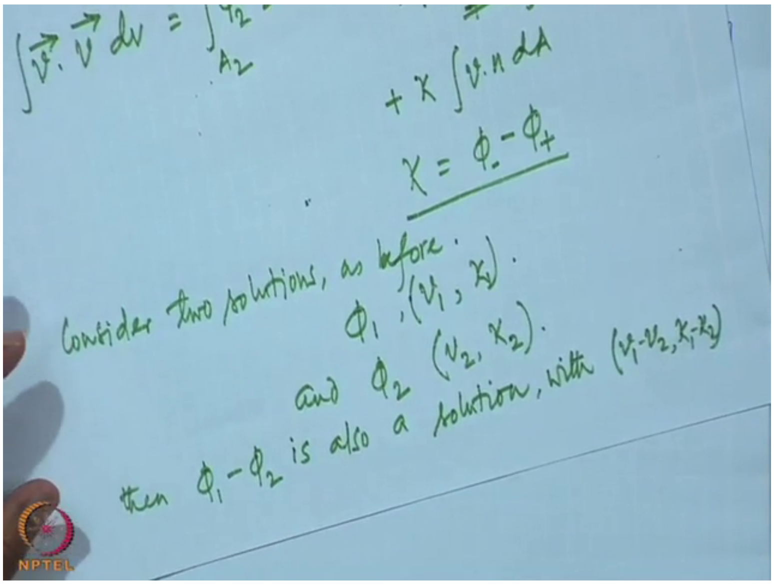

Let us now consider two solutions, let us now consider two solutions pi1 which is associated with velocity field v1 and cyclic constant kappa, and pi 2, which is associated with the velocity field v2 and let us say kappa 2 then which is associated with the velocity field v1 minus v2 and cyclic constant kappa 1 minus kappa 2.

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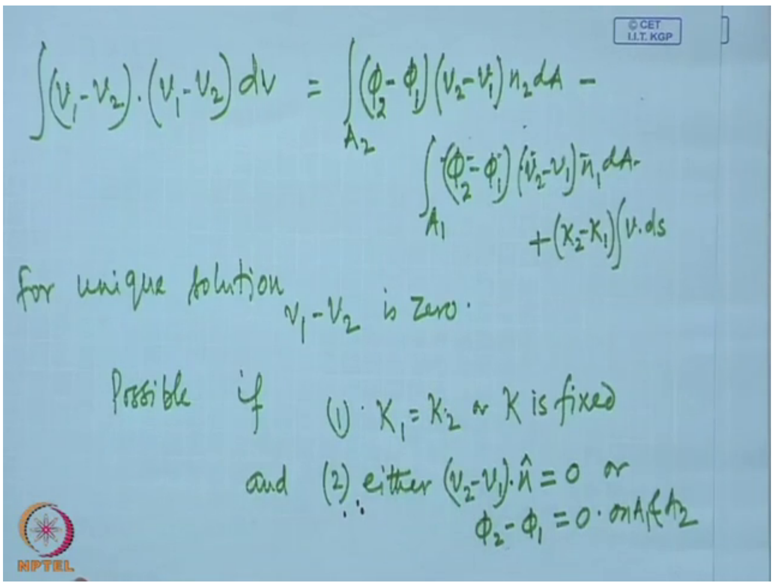

We have little difficulties in the notations, pi 1, pi 2 we have already associated with 2 surfaces, we have associated with 2 surfaces, yes unnecessarily we have made it, it was not necessary to write as a pi 2 v2 this is pi v n on it, should do we make this there has no necessity because integrated over the area A2, integrated over the surface A2, it was not necessary to write pi v n here.

Student: (( ))

Yeah N1 and N2 will be there N1 and N2 will be there, but this we did not associate pi 1 pi 2 there, there simplify pi n v, N1 and N2 should be there they are normal specified to that to that 2 surfaces. So, this will be and similarly, the other terms, and acting this we see that for write it. Now, once again that if the solution is to be unique, is the solution has to be unique then this v1 minus v2 must be 0, that is a left hand side must be 0, for unique solution… And how will that left hand side will become 0, if either v2 minus v1 dot n equal to t 0, on a two and a, A1 or pi 2 minus pi equal to 0, both must be on A1 and A2 on the surfaces.

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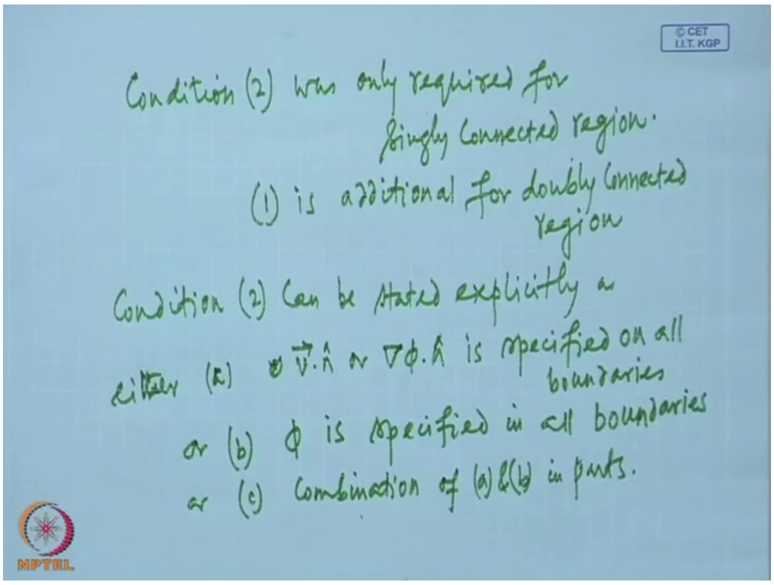

This of course, this is the same condition which you had for singly connected domain, and this is the extra. and condition two we have already explicitly stated, pi is specified in all boundaries, or remember about this third the combination of a and b, it is always that you do not need both at the same part, any 1 on 1 part, and the other on the other part not on the same part, there is on 1 portion of the boundary if you know both that is of hardly in use, knowing 1 of these on 1 portion, and the other on the remaining portion that is what is required, such that you know everything, you know either of the 2 on the all boundaries that that is what is the requirement, either of these 2 must be known at each and everywhere on the boundary surface, and also this in case of a flow in a multiple region multiply connected region that cyclic constant is also an additional factor that you must know.

If the region is (( )) connected then there are n number of cyclic constant what we have mentioned earlier, so for flow or solenoid irrotational flow you need n number of cyclic constant that is if you have a solenoid irrotational flow in (( )) connected region to determine that flow you must know n number of cyclic constant, and these either the normal velocity or the potential or a combination of the 2 on the entire boundary.

So, these are the things must be specified only then you will be able to get the unique solution is mathematical possible, mathematical it is possible to get the unique solution for a given solenoid irrotational flow field.

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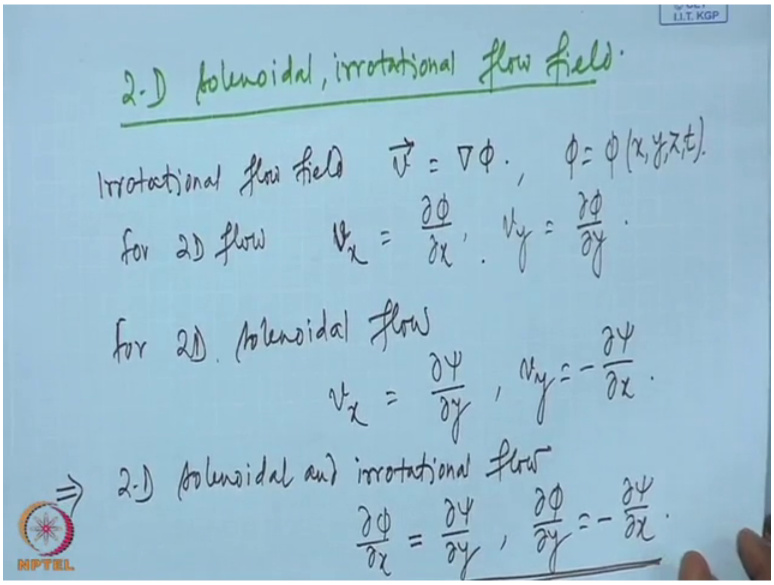

Let us consider this type of a 2 dimensional a special property of 2 dimensional solenoid irrotational flow field, a 2 dimensional a general case solenoid irrotational flow field, for a 2 dimensional irrotational flow field, we have seen that the velocity is gradient of potential, for irrotational flow field… So, for 2 dimensional flow then what becomes we have the… let us consider the straight forward cartesian coordinate system, then we have v x the x component of the velocity is d pi dx, and the remember that pi is in general function of x, y, and t in this case let us say x, y, z, t and v y is… we have seen that in general for any solenoidal flow field is not necessarily irrotational, for any solenoidal flow field we can define a stream function, any 2 dimensional solenoidal flow field we can define a stream function it is in not necessarily to be irrotational, the potential function can be defined only if the flow field is irrotational, otherwise not, but the stream function can be defined if the flow field is 2 dimensional and solenoid, but not for 3 dimension.

This potential function can be defined for 3 dimensional flow, that stream function is not valid or the scalar stream function is not valid for 3 dimensional flow, you have go for then a vector function so for 2d solenoidal flow, we have see the x component of velocity again let us say this x component of velocity is... If we combine the 2 if we combine the 2 then we get the result which is applicable for 2 dimensional solenoid irrotational flow, these results are for 2 dimensional solenoidal flow, this is the result for irrotational flow, so the combined result is for 2 dimensional solenoidal flow, 2 dimensional solenoidal and irrotational flow, what are these solenoidal and irrotational flow? That is the flow field in which there is rate no rate of expansion rate of expansion is 0, and the velocity is 0, we have what d pi dx is equal to d psi d y, and d pi dy equal to…

So, these 2 functions the 2 dimensional potential function and the stream function they satisfies these relation, in a solenoidal irrotational flow field, however come across this relation or this type of relation, what is that what do not think in terms of this velocity potential or stream function flow stream function, just think in terms of mathematical function, pi is a function and psi is also a function. Yes why did you come across this relation, or you may think in terms of pi and pai you can replace by them f and g or anything you like or f 1, f 2.

Student: Linear equation.

Linear equations.

Student: (( )).

Ah.

Student: (( ))

Exact differential, anything else, you have come across this relation in your complex analysis, where you have called this relation to be quasi remarkation, a quasi remark conditions, if z is a complex number defined by x plus iy, then a complex function pi plus i psi, and if the real and imaginary part of the functions satisfies these relations they are called the quasi remark conditions, and the function is called analytic function, the function is called analytic function.

So, you see that for the potential function and stream function in 2 dimensional solenoidal irrotational flow field satisfies a relation such that 5 plus i psi which we may call a complex potential, is an analytic function of the argument z which is x plus iy.

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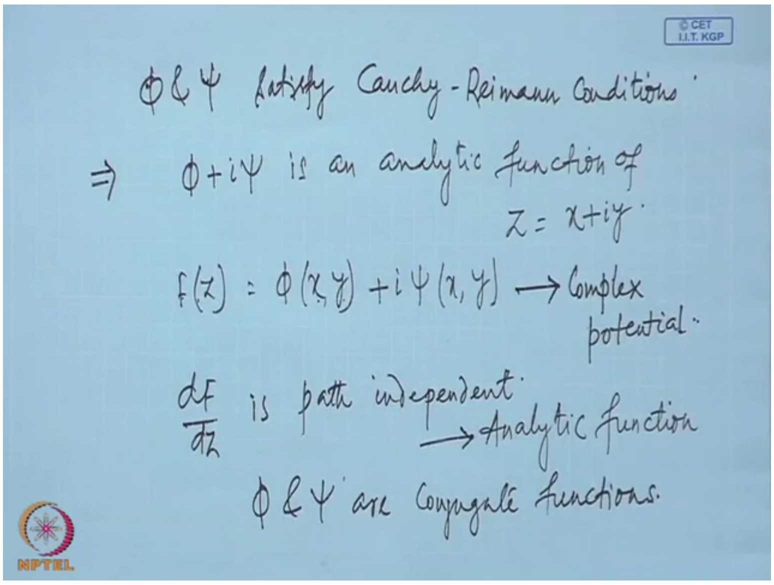

So, that is a that condition is that pi and psi we will call this as complex potential. This is a property of the analytic functions that the derivative of this function is path independent, the derivative of the function is path independent. This I hope you remember, that is delta z you can take in any direction to find the derivative of this f z with respect to z, you can take delta z in any direction the result will be the same.

So, your z direction or the change in z may be only along the x or only along the y or any other direction, the derivative will be same, that is what is an analytic function, that is df dz is path independent the real and imaginary part of an analytic function are also called conjugate, conjugate to each other, meaning they are orthogonal to each other, that is pi and psi are orthogonal to each other. You will be able to appreciate the idea better in this way, instead of the 2 function being orthogonal, the functions are also called orthogonal, in but you will appreciate it this way, think about psi equal to constant, what is psi equal to constant? We have already seen psi equal to constant is a stream line, in a 2 dimensional flow field psi equal to constant represent a stream line.

Similarly, pi equal to constant will represent an equipotential line, line on which potential is same, and when you say that these 2 functions are conjugate the stream function and potential function in 2 dimensional flow field is conjugate or orthogonal, means that this equipotential line and stream potential streamline they will intersect each other at right angles, whenever they will intersect they intersect at right angle. in this relation this pi xy, psi xy what we have written, we have not written any t but in general they may be function of time, in general these both pi and psi may be function of time.

And the real and imaginary part the real and imaginary part of this complex potential or real and imaginary part of any analytic function in general, real and imaginary part of any analytic function satisfies laplace equation, both the real part and imaginary part of any analytic function satisfies laplace equation, and you see that as a result what happened, if you have an any an analytic function if you have an analytic function then it real part will represent a potential and the imaginary part will represent a stream function, in any general 2 dimensional irrotational solenoidal flow field, any analytic function think of. If you just an analytic function it is real and imaginary part will be a potential function on a stream function, whether it matches to a particular flow or not that is a different matter, but it will satisfy both of these the real part and imaginary part will satisfy the laplace equation independently.

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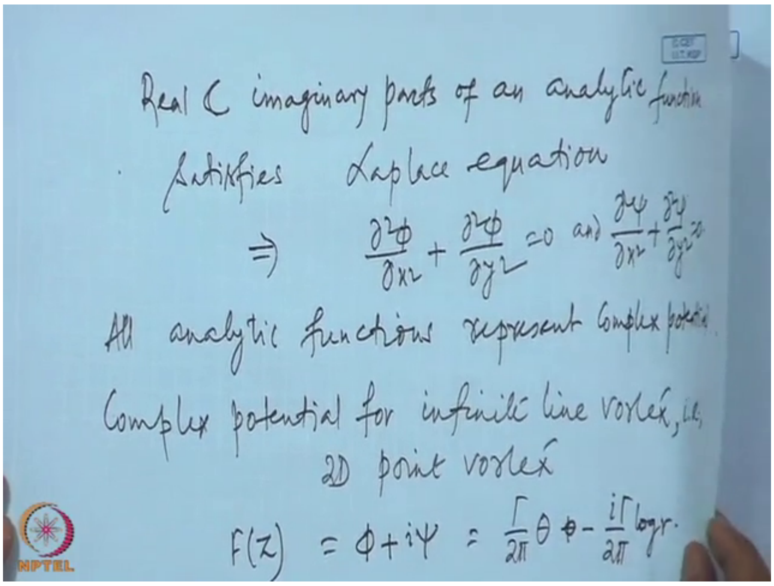

So, and if f represents a potential so does I f, if represents the complex potential so does i f, we have already obtained the velocity potential and stream function for 1 particular flow, some other flows also you can write them now, but at least for 1 we have already written, the potential function as well as the stream function that is flow due to an infinite line vortex, or a 2 dimensional point infinite line is 2 dimensional point. So, for we have already written in the potential function and stream for a 2 dimensional point vortex, now we can write the complex potential for that, let us write the complex potential for infinite line vortex, that is a 2 dimensional point vortex.

See how much it will be, we have already written pi and psi for this, so find what is this complex potential f the potential function was gamma by 2 pi theta, and the stream function was minus gamma by 2 pi log that that distance, if we consider 2 dimensional then it is straight away r, then add it pi plus i psi make it how much it is pi plus i psi.

Student: (( ))

Pi is gamma by 2 pi theta and sorry minus.

Student: (( ))

I gamma by.

Student: (( )).

Two pi.

Student: Log (( )).

Log r, then what it becomes, take out i gamma by 2 pi, or say minus i gamma by 2 pi, from both the terms take out minus i gamma by 2 pi, then what happens.

Student: (( )).

Log r.

Student: Log psi pi.

Minus or plus.

Student: Minus.

Plus.

Student: (( ))

Plus.

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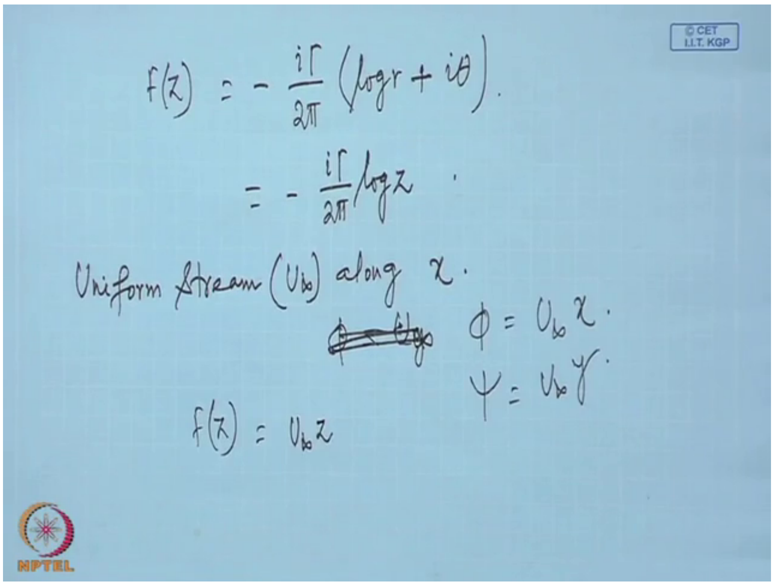

F z is minus i gamma by two pi.

Student: Minus (( )) minus i theta.

Plus.

Student: (( ))

I square is minus 1, and what is this log r plus i theta.

Student: Log of (( )) r.

R e to the power i theta log of r e to the power i theta, what is r e to the power i theta is z.

Student: (( ))

X plus iy, so this becomes… 1 more very simple flow which is both irrotational and solenoidal, I will tell you and tell me what would be the complex potential, a uniform strain along x, a uniform stream along x a uniform stream with speed u infinity, or say capital u whatever you like, a uniform stream with speed u infinity along x, what would be the complex potential, a uniform velocity will call it a uniform stream, first of all what will be pi?, what will be pi? d pi dx is u infinity d pi dy equal to 0, what is pi?

Student: (( ))

A just pure constant will drop, because of that just a pure constant does not mean anything as for as pi is concerned, if there is a if you go on adding any constant it hardly matters, because see our main interest is the velocity field, so if there is some unknown constant associated with the pi it hardly matters, because when we take the gradient the constant will vanish, so just a pure constant has no meaning, we will always drop it, when we said the solution is unique say that is again there might be a difference in constant additive constant that hardly matters, so pi is infinity x. What is psi? sorry.

Student: To the infinity was.

plus or minus.

Student: Minus

plus or minus psi

Student: Plus

Plus sure d psi d x equal to 0, d psi dy equal to u infinity.

Student: (( )) infinity

And the complex potential is pi plus i psi u infinity z, now let us see what happen if we differentiate this complex potential pi f.

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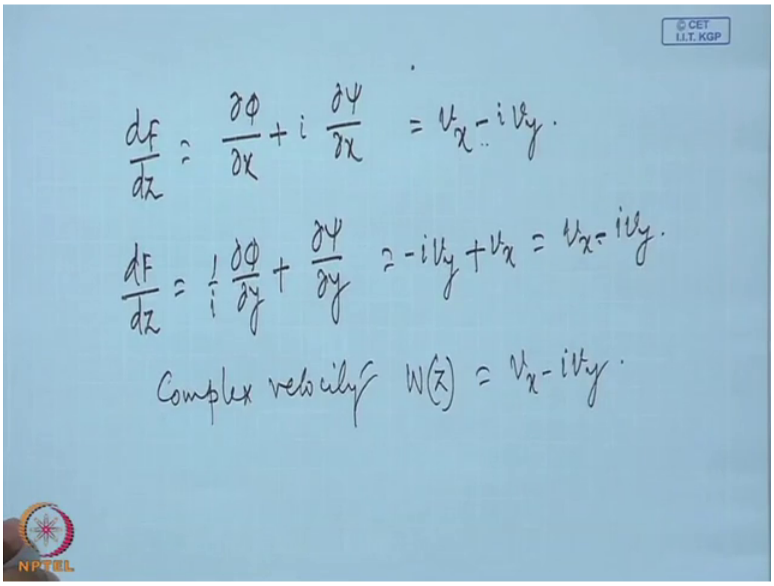

Now as before we have mentioned that a since f is analytic the differentiation is path independent, so when we differentiate with respect to z we take the consider change only along x, it hardly matters, then what it will be d pi d f d z can be written as simply d pi dx plus i d psi dx, because it is path independent, and what it is d pi dx is v x and d psi dx is minus vy.

So, this becomes vx minus vy, as a check we can also think that instead of taking the change along x reaction let us take the change along y direction, that also can be done, if we consider change along y direction then z is i delta y, the z is i delta y, see if we consider change along y direction then the derivatives will be 1 by I, d pi d y plus i i get canceled, what is this d pi d y is dy y component 1 by i is minus i.

So, you have got the same result, so this is also another verification, now look following the same notation that the gradient of the potential is taken as the velocity similarly, if we consider now the derivative of the complex potential as the complex velocity, then you see this complex velocity is not plus, if you have the velocity component v x and v y you would say the complex velocity is v x plus i v y, that is the way we generally define any complex number, if we have the real and the imaginary part known we say this plus i into the imaginary, but here you see this is not plus is not v x plus i v y it is b x minus i v y.

So, if we call d pi d f d z as complex velocity which we may call and we will call it the complex velocity, then you remember this complex velocity is having minus here, not plus, if we call this as a complex velocity, we will not go further into this complex or in the analysis of these a irrotational solenoidal flow field, because up to now we have seen this irrotational solenoidal flow as just a part of the flow, we have seen that the flow contained consist of flow which is associated with a specified rate of extend expansion plus or specified rate of vorticity plus this irrotational solenoidal flow, though we have mentioned in the beginning while started discussion on this irrotational solenoidal flow, that there are many situation there are many situation where the complete flow is irrotational solenoidal, there is no expansion there is no rotation, but we have said that there are, we have not seen how is possible, when do the occur.

So, before continuing this further we will now try to see what are the condition, or when a flow can really be solenoidal or irrotational, at least approximately without going on just discussing irrotational and solenoidal flow would like say that the flow the in which we are interested, as a pure mathematics we could have going on discussing on this, but would like to see the talk what are those flows, where these solutions are applicable, where we can have this type of situation.

So, that we can use this, this complex analysis are very well developed mathematics and if we can use it at least for 2dimensional flow we got a very good result, all ready known results which we can state way use, but before that we must know that the real flow field is solenoidal irrotational otherwise there is no point in going on doing it.

So, now hence forth we will for sometime we will try to see that what are those conditions, or when a flow can be solenoidal or irrotational, or what they are mathematically solenoidal irrotational all those vector properties we are talking about, but in reality what are they, in terms of say engineering sense what type of flow can be called it, that is what we will try to see hence forth and of course, to see that we have to consider some more things which we have not yet consider, that is how do the forces effect the fluid motion, so subsequently for next few class we will be considering the dynamics of flow, next few class means next all classes.