Introduction to Aerodynamics
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Lecture No. # 13
Kinematics of Fluid Motion-Vorticity Distribution

So, we will consider profiled with the vorticity distribution. We have already defined vorticity as curl of the velocity vector. And in case or this vorticity is just part of the motion we have considered that part of the velocity to be u v so of course, the other parts have no rotation. The other parts that are associated with that rate of expansion or the uniform flow or with deformation without change in volume, none of them have any vorticity associated with it. So, what is curl of u v eventually? It is curl of the total velocity also; curl of u itself is same as the curl of u v and which we have defined as omega.

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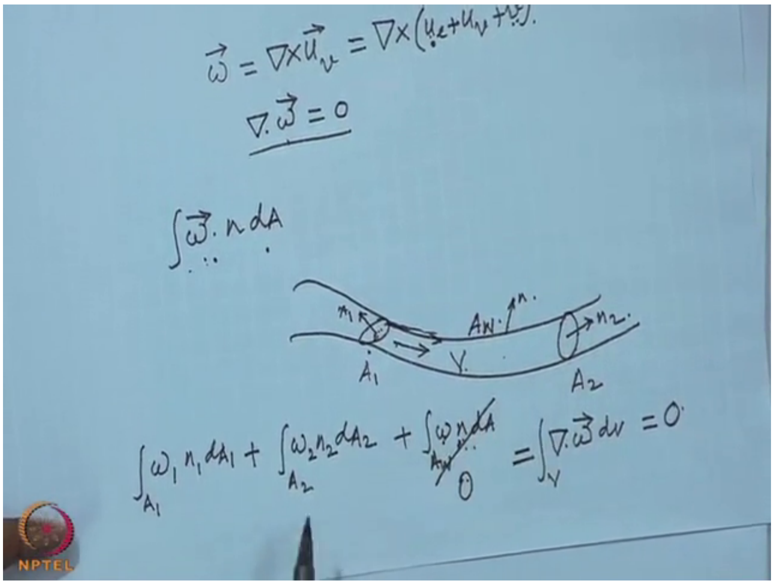

And since the other parts do not have any curl with it. Because this u e and v u e is associated with rate of expansion which is rotational. So, that curls of that is zero. And similarly, v which is associated with the uniform motion plus straining motion without change in volume that also has no rotation. So, the rotation of curl of the total velocity itself is curl of u v and that its means omega. And since vorticity is curl of a vector its divergence is zero this is an identity.

Then we defined what is a vortex line? There is a line which is tangent to the vorticity vector at all points. That is what is a vortex line? And we defined what is a vortex tube? If we imagine a reducible curve within the fluid, then all the vortex line passing through that closed curve forms a tube and that tube is called a vortex tube. So, the vorticity vector is always tangential to the vortex tube also, and the vortex lines can be mathematically expressed again by a similar type of expression equation which we earlier obtained for streamlines. If you remember for streamline we had the velocity vector is tangential to a streamline so we made u cross d l to be zero, something we can make again omega cross d l equal to zero and that will give the equation of the vortex lines.

Now consider a closed curve surrounding the vortex tube once. That is it is looking the vortex tube just once. Then the surface that is found by this closed curve basically an open surface because it does not include you knows any volume. And the integral of this vortex over that area which we can define as omega dot n d A, and we will try to show that this quantity is constant. It will not change, along the length of the vortex tube it will not change. This can be shown very easily like this will be to show this we will be using this equation divergence of omega equal to zero. Now how to get divergence of omega from this relation? See this is an area integral if we convert it to a volume integral then that integral will be will be of divergence of omega.

But you there is a restriction or requirement to apply that condition, which is divergence theorem or gauss theorem? That you can apply the relationship between a volume and an area if that area encloses that volume. So, now we have to think about an area which encloses a certain volume. Now let us consider this vortex tube think about this is a vortex tube, and consider two such area here one here and say another here along with it. Consider this part of the tube that makes a volume, now we can apply this divergence theorem for the surface of these you can call it a cylinder of this cylinder and the volume of this cylinder. See this whenever you try to apply divergence theorem you must be very clear that the relationship between that area integral and volume integral is true provided that area encloses their volume, not just any area and any volume we cannot apply that condition.

So, now let us call these area this one let us take it A1, this as A2 and this area area of the tube surface area of the tube we call it is Aw. Now the total area is sum of these three and the corresponding normal we can call it here as n1, n2. And let us say normal to this n. Now apply this area integral here, the total surface integral is omega1 we have taking the value say here omega1, and omega2 here n1 d A1 integrated over this area A1 plus. This is by application of the divergence theorem is divergence of omega d v where this entire volume is v. Now since divergence of omega is zero for all choice of volume element it is zero everywhere.

So, this total integration is zero. Now we see this third area integral omega dot n d a integrated over these surface. Now since this is a vortex tube all the vortex lines are tangential to this so there is no vorticity which is normal to this area by definition all vortex lines are tangential to the vorticity vector. Consequently there is no normal component of vorticity to this surface. Hence this integration this integral does not contribute anything to this so this is also zero. Because omega is tangential to this surface and since these two normal what we have taken both the normal taken as outward. But they have different sense the two normal have two outward normal have different sense.

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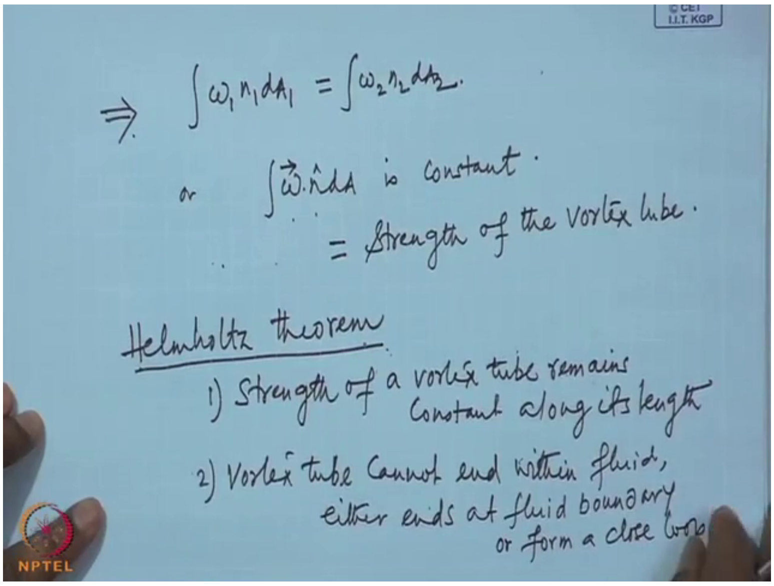

So, this integration now become… We can choose any area any type of area, any area means any type of area. Whether this area is a cross-sectional area or in some inclined area does not make any difference. Because ultimately that end dot d A will take care of it. So, this remains constant along the vortex tube this is called as the strength of the vortex tube. So, you can summarize this result that the strength of the vortex tube remain constant along its length of course, it has the consequences then that vortex tube cannot end within the fluid. Because if it end means its strength is changing its strength is becoming zero. But that it cannot happen, the strength will remain constant. So, a vortex tube will not end within the fluid it can either end at the boundary of the fluid if the condition allows it or it can form a closed loop.

A vortex tube can either end in the boundary of the fluid or it can end and it can form a closed loop. And also that all the material elements which passes through or which makes up the vortex tube at certain instance will continue to make the vortex tube at all subsequent times. All these are result of this simple and these results together are known as Helmholtz theorems on vorticity, that the strength of a vortex tube remain constant along its length. A vortex tube cannot end within the fluid, it may end either at the boundary of the fluid or it may form a closed loop. And the material elements that makes up a vortex tube will continue to make the vortex tube.

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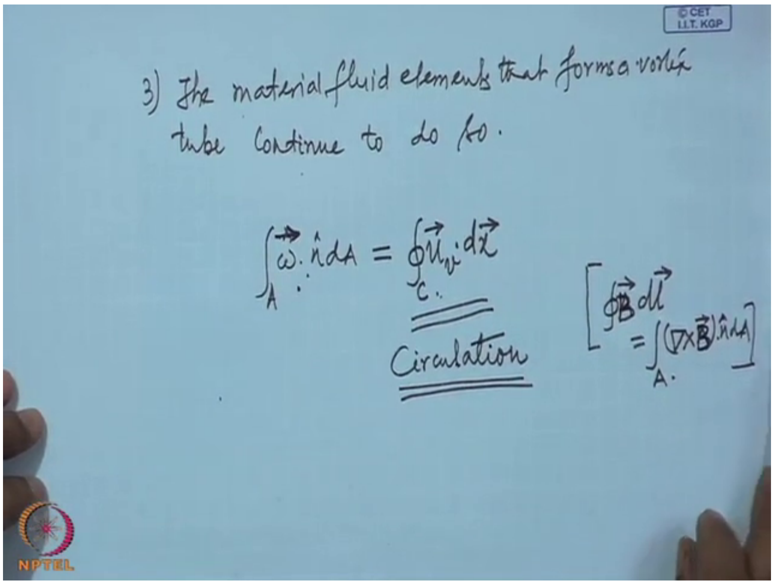

So, remains the material fluid element. Next we will consider the velocity field due to some special type of singularities in vorticity distribution, the one and very common singularity is where we have a very large concentration of vorticity near a line. And everywhere else the vorticity is say zero. Of course, this line has to be parallel to the direction of vorticity, otherwise the divergence of vorticity will not be satisfied. Once again we can make that mathematical idealization that if we have a very high consideration of vorticity near a line. We can now assume that the that region near the line is being shrink to a sink to the line and all the vorticity is placed on the line itself. This is then a singularity in the vorticity distribution nowhere else there is vorticity but around a line there is a very large concentration of vorticity.

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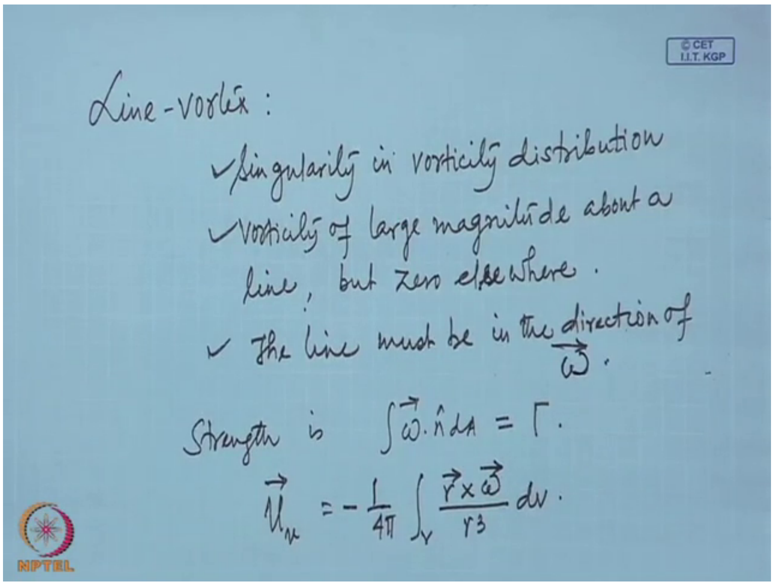

And let us say that the strength of which we have called line vortex. This is of course, a singularity in a vorticity distribution later on we will see that in many practical flow. Flow problem this type of situation occur where there is a very strong concentration of vorticity. Just about narrow region which can be thought of (( )) line that sort of situation comes in many practical problem so in that respect this is quite important. Line so considering straight line vortex.

So, what will be the direction of this velocity? You see the direction of this velocity is always azimuthal about this line, this will be the direction of the velocity and the magnitude of this velocity can be obtained as not a

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Now let us see what would be the velocity field associated with this vortex distribution. Once again that u v can be written as all these are function of position as well as time which we are not writing all the time explicitly. But remember that all these parameters are function of position and time x y z t which we are not writing explicitly most often this will be how much minus 1by 4 pi into

R cross (( ))

Hm r cross omega

Student: (( ))

But we can now change it because that omega d v can be written as

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Now first of all consider that this strength is uniform

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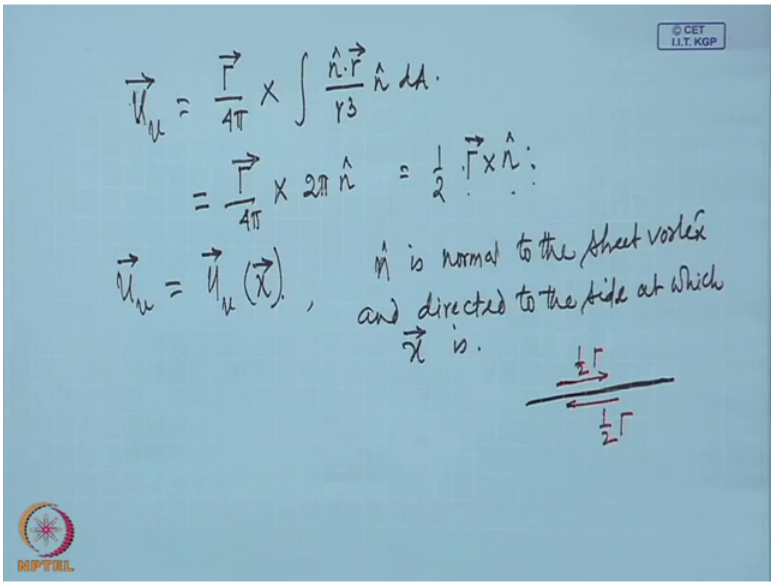

This can be written as just a mathematical manipulation. Just a mathematical manipulation and the result is, n is a new unit normal to the sheet vortex and it is directed to the direction at which the point lies. This u v the velocity we are finding this is of course, at a point we could have written it this u v is at a point x so n is normal to the sheet vortex. And directed to the side at which x is see the magnitude of this velocity simply half gamma and for uniform gamma that is constant gamma half gamma is constant everywhere. So, if we consider one side of the sheet vortex on that side the velocity is half gamma it is of course, from here it is parallel to the sheet the direction of this velocity is parallel to the sheet. As well as perpendicular to gamma.

This velocity direction is perpendicular to the gamma and parallel to the sheet now if you consider this sheet vortex on the two side of the sheet vortex you see the on the both side the magnitude of the velocity same half gamma. But the direction is different or sense is different in both cases it is parallel to the sheet but since the n is different direction. So, considering say the two-dimensional representation of this sheet it is, if it is half gamma like this on this side it is this same half gamma. So, what do we have now that across this sheet vortex there is a jump in this velocity? The velocity component which is parallel to the sheet suppose say jump by a magnitude of gamma which is just strength of the sheet vortex. So, if you know some physical situation where there is this type of possibility that there will be a jump or change or discontinuities in this tangential velocity.

You can model that situation by a sheet vortex. If it is say from physical consideration we can think that here it is expected that there will be discontinuity or jump in this component of the velocity. We can model it by a sheet vortex, but that as if there is a sheet vortex there; and eventually see this is what an aircraft wing does. It creates difference in velocity on the upper surface of it and the lower surface of it. So, it can be modeled by a sheet vortex, if the strength of the sheet vortex is not uniform that is perhaps more general case, the strength of the sheet vortex is not uniform then you will find that this relation still applies but locally. Because that gamma is changing so at each and every point the difference will still remain gamma, but since the gamma value is different the u values are also different, but again that locally the jump of gamma still holds.

Another very useful case perhaps you could have been obtained or you may try yourself assuming this sheet vortex to be a cylindrical sheet vortex the result is known to you. You have done it. Think about the velocity to be the magnetic field and the vorticity to be electric current, you have done that no a cylindrical tube or solenoid what happened there is electric field inside. But not outside you will find the same thing here also if you consider a cylindrical vortex tube we will see that the velocity in the axial direction within the tube zero outside you may try that problem.

One thing I should mention in this case though we have not done it perhaps we will be doing it later a sheet source distribution or surface distribution of source I have mentioned, but I have not found the velocity if you evaluate it again you will see that that creates another jump but, not in this tangential velocity normal velocity. If a source strength of m is placed over a sheet in per unit area, strength distribution of m placed over the surface area of m then it will creates a jump in normal velocity by an amount of m, that will be required in some later time we will do it at that stage.