Introduction to Aerodynamics

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Module No. # 01

Lecture No. # 10

Kinematics of Fluid Motion (Contd.)

Continuing our discussion on velocity near a point we have seen that, the difference in velocity between two points are given by two terms; one depending on the symmetric part of the velocity gradient tensor and the other is the anti symmetric part of the velocity gradient tensor.

About the symmetric part of the velocity gradient tensor, we have discussed that this actually represents a straining motion, which can be decomposed into two parts, one is an isotropic expansion and the other part is deformation without change in volume. One part is change with change in volume, the other part without change in volume. That is deformation.

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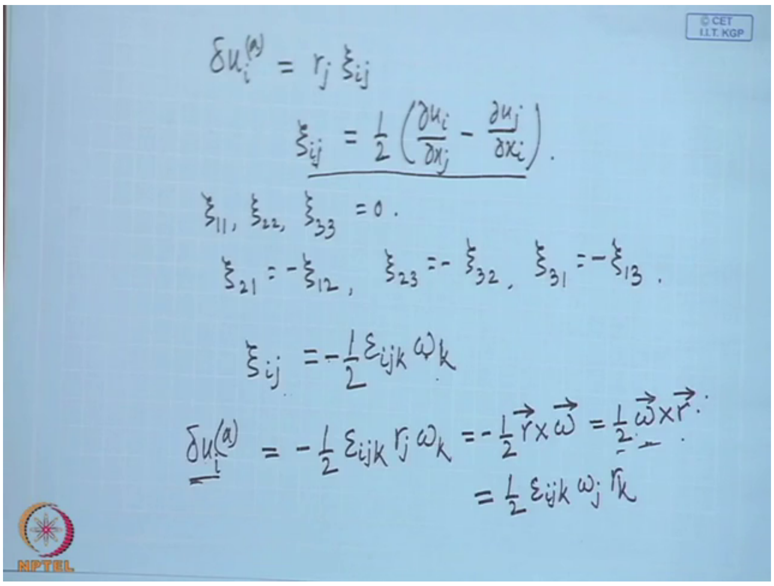

The second part of the velocity second contribution that is the anti symmetric contribution. If you remember that, we had the anti symmetric contribution, delta u i a is r j xi i j. And by definition, xi i j we wrote the anti symmetric part of the velocity gradient tensor half of d u i d x j minus d u j d x i. We have already stated that, this clearly shows that the diagonal terms are all 0, when i is j i and j are same, this term gives 0 that is xi 1 1, xi 2 2, xi 3 3 they are all 0.

So, xi 1 1, xi 2 2, xi 3 3 they are all 0; and also xi 1 2 and xi 2 1 are just equal and opposite. So, we can write xi 2 1 equal to minus xi 1 2 and similarly, xi 2 3 is minus xi 3 2; xi 3 1 is minus xi 1 3. So, which clearly shows that, we have basically three independent term three independent terms out of these six non-zero, three are independent, the other three are just opposite of that. And hence, we said that, xi i j can be written as that alternating tensor epsilon i j k and omega k. We can complete our, this definition here it self, but we will do little more.

See in the definition of xi i j, the initial definition of xi i j there is a half. This, if we write just this, this half can be contained within omega, but we do not want to keep half within omega. So, we keep this half separately again and also we introduce a negative sign. So that, in a later definition of omega we will not have a negative sign, we take a negative sign here; this minus half just to avoid another minus half in the definition of omega, which will be coming across now.

Omega k which has now three components can obviously thought of as a conventional vector having three components. And this velocity then can be written as delta u i the anti symmetric part as minus half epsilon i j k r j omega k, which is which is clearly this epsilon i j k r j omega k is the cross product of vector r and vector omega ok.

So, this is basically minus of half r cross omega in your conventional vector notation or we can write it as half omega cross r; or coming back to tensorial notation, half epsilon i j k omega j r k. Now, this is a very familiar velocity, omega cross r or rather half omega cross r; what does it mean? Velocity of this fluid element which is at point x plus r that is r distance a, from point x has this value omega cross r or half omega cross r. What does it mean? Why do we come across this velocity omega cross r? In a rigid body rotation about distance r or about an axis which is at distance r with an angular velocity of half omega.

So, in this case the particle at point x plus r is rotating about that point x with an angular velocity of half omega, if it is show then this will be the velocity. So, this part of this relative velocity represents a rigid body rotation about point x or about an axis passing through point x, the angular velocity is half omega.

However, see in if you look to the flow sorry I next class I will try to bring you some streamlines and others. So that, you can see, eventually you will not see that, the fluid is really rotating. So, this rotational part of velocity will be needing a little more interpretation, we will come to it. But, let at this stage say that, the velocity has a contribution which is due to a rigid body rotation or the fluid velocity has in general a rigid body type of rotational contribution with angular velocity half omega.

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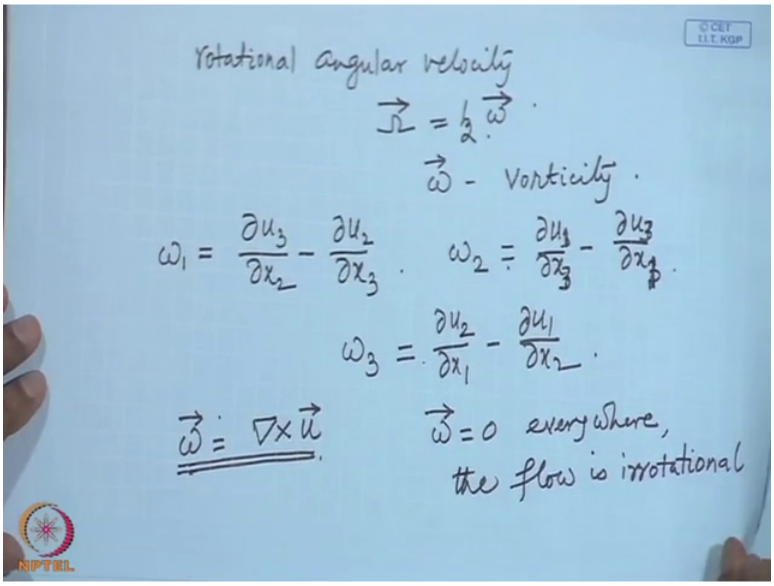

This vector omega you can call it that rotational angular velocity of half, we can give another name to that rotational angular velocity; let us call it this omega is half omega ok you can give that vectorial notation also. This vector omega is a very important fluid dynamical quantity and called as vorticity; omega is called vorticity.

Can you say, what are the three components of this omega omega 1, omega 2, and omega 3? That you can obtain from our earlier definition. What is omega 1? What will be omega 1? Look to that definition xi i j is minus half epsilon i j k omega k (Refer Slide Time: 09:14), we want omega 1 that is k is 1, if k is 1 then we know this epsilon i j k has non-zero value, when all i j k are all different; since k is 1 now, this i and j has to be either 2 and 3, one is 2 the other is 3 3. Let us take it 2 3 1 that is epsilon i j k is epsilon 2 3 1, epsilon 2 3 1 is a cyclic order in cyclic order. So, epsilon 2 3 1 is plus 1. So, this term becomes then xi 2 3, i is 2, j is 3. So, xi 2 3 is minus half omega 1.

Now, you can write what is xi 2 3 from the here, half is already there of course, a minus will be there. So, what it will be, now tell me, what is omega 1? Omega 1 is d u 3 d x 2 minus d u 2 d x 3 yes; similarly, you can write the other component of omega also. What will be omega 2? Omega 2 again let us come here (Refer Slide Time: 11:02), then since k is 2, i and j has to be 1 and 3; let us take it 1 3 2, 1 3 2 is not cyclic order 1 3 2 is not cyclic order. So, your i epsilon 1 3 2 is minus 1. So, xi 1 3 is half omega 2. Now, find what is xi 1 3 half is already there, dou u 3 dou x 1 minus d u 1 d x 3 yes it is correct we should check.

Student: (( ))

It is 1 3 know it was 1 3, so it will be 3 and this is 1. Then, what will be omega 3 that also you can compete, it will be 1 2 or 2 1 which one we will give the, i j can be 1 2 or 2 1? 2 1 will give what? 2 1 3 is negative. So, 2 if you take 2 1 you will get it directly. So, it is d u 2, 2 1, i is 2 j is 1, d u 2 d x 1 minus d u 1 d x 2. So, these are the three components of the vorticity vector. You can see that, I hope you are familiar with the curl operator, this implies that omega is curl of u.

So, that is the final definition of this vorticity vector omega, which is the curl of the velocity vector. If the vorticity is 0, if you find the vorticity is 0 then the flow is called irrotational. If the vorticity everywhere in a flow is 0, then the flow is called irrotational, omega equal to 0 everywhere, the flow is irrotational.

Now, think about say any closed surface surrounding point that x, any closed surface enclosing point x; let us think about for for simplicity are two dimensional picture, any closed curve surrounding that point x and again without loosing any generality we can consider that close surface to be just a circle. So, think about a small circle say of radius a, surrounding the point x, where the vorticity is given by omega.

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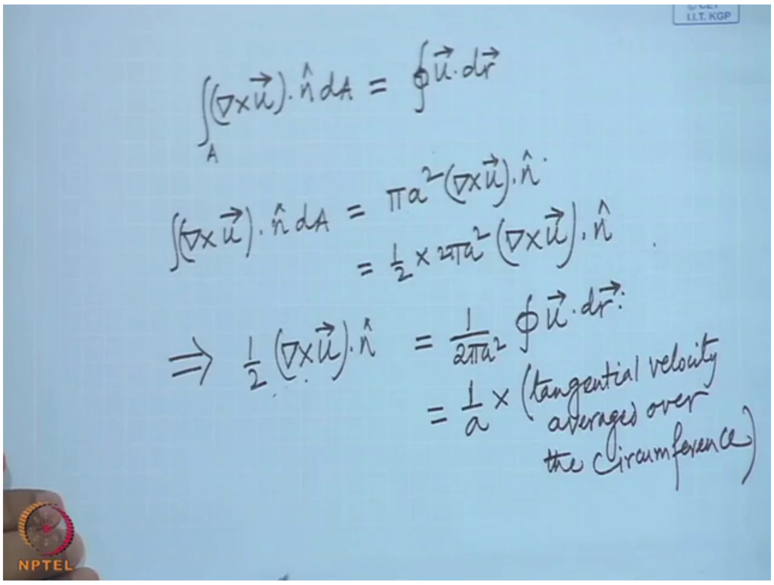

Now, what is this quantity, instead of omega we are writing curl of u again, curl of u dot n d A about that homogeneous, what is this? You are familiar with this divergence, curl of a vector surface integral of a curl of a vector is what? Stokes theorem; this is called stokes theorem is, the line integral on the curve that encloses that area, whether the boundary of that area. That is in this case, since the area we are considering a circle of radius a, this curve is the circumference of that circle. Since it is a closed curve that is what the symbol is.

Now, let us say that this area is small we are considering a small area, then within that small area we can consider approximately that, this curl of u or the vorticity is nearly uniform within a small over a small region, the vorticity is more or less uniform. Then what is the value of this left hand side integral? The left hand side, yes curl of u is nearly uniform over that small area of the circle of radius a.

Then how much is this integration, this curl of u dot n into the area. So, we can write that, for a circle of radius a, this becomes pi a square into And then we write out of this a square leave 1 a, what is the rest? No, that is 2 pi a is the circumference including this integration, the integration divided by 2 pi a, what it is? So, what is u this integration? This integration is the total velocity over the path that we are dividing by the circumference. So, that is velocity averaged over the circumference.

So, this becomes tangential velocity u dot d r only the tangential part will remain; since it is u u dot d r, the path component of u which is normal to that path is vanished. So, it is basically only the tangential part, the total tangential velocity we can call it, if there is something like that of course it is not a not any physical quantity. So, if this is average of the tangential velocity averaged over the circumference.

So, this is 1 by a into tangential velocity averaged over the circumference I am not not giving those vector notations, each u i x plus r j e i j plus r j xi i j. This happens to be u i x plus you can write it as d d r i of half r j r k e j k plus, what it was, half epsilon i j k omega j r k.

So, velocity at any point is given by sum of three contribution sum of three contribution. The first contribution u i x is velocity at any point is sum of a uniform velocity a uniform velocity plus a straining motion a straining motion, which itself is again a sum of an isotropic expansion plus a deformation without change in volume. So, a straining motion this is equal to a an isotropic expansion plus a straining motion without change in volume plus a rigid body rotation Now, before proceeding further that is how to find the velocity distribution in these cases in a discussion, we will take up 1 or 2 say numerical examples to illustrate some of the things that we have done. Consider a two dimensional flow consider a two dimensional flow and say the Eulerian velocity at a point Eulerian velocity at a point (x, y) at time t is given by or the simply the Eulerian velocity is given by the Eulerian velocity or the Eulerian velocity components let us say we are giving in component form.

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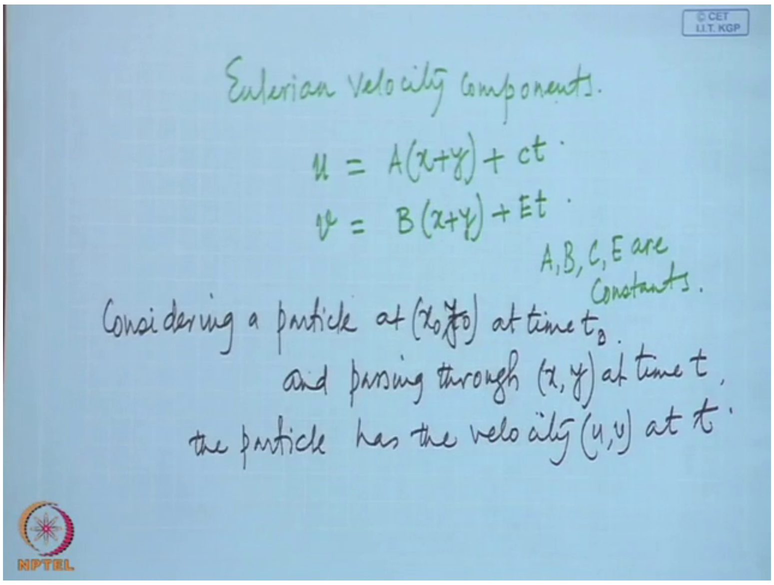

Let us say the Eulerian velocity components are Eulerian velocity components are given by u equal to A into (x plus y) plus C t. It is an unsteady flow as you can see u is function of time, so u is this. And the other component v component let us say is B into (x plus y) plus E t; A, B, C, E are constant A, B, C, E are constant.

Find the displacement of a fluid particle in the Lagrangian system find the displacement of a fluid particle in the Lagrangian system? How will you do it how will you do it? Remembering that, the Eulerian velocity are velocity at a point, any particle passing through that point will have that velocity at that instant, when it is just passing it; once it passes of course it we will not have that velocity, another particle which will come to that position we will have that velocity then. How to do it? Any any suggestion, how to do it?

Student: (( ))

Hmm

Student: (( ))

Which formula?

Student: (( ))

But, where is a two point are coming in this context? In this case is two points required, do we need to consider velocity at another point? Think see, this is the velocity at the point x y at time t. So, any fluid particle that is passing to that point at time t is having this velocity? A fluid particle that is passing thorough the point (x, y) at time t will have the velocity at that instant? Velocity is of course always instantaneous.

So, let us say that this particle was initially at time t equal to t 0 was at (x 0, y 0) and that particle is passing through the point (x, y) at time t. Then for that particle, considering x as a variable, x is the position vector or say (x, y) are the component of the position vector; getting the idea think about a particle, which was at say (x 0, y 0) at time t 0. The same particle is passing thought this point (x, y) at time t, then at that instant these are the velocity component of that particle.

Now, moving the velocity component at a particular instant, can do get it displacement, that is what you need to find the displacement. So, at a particular instant, you know the velocity of the particle, which was at (x 0, y 0) during time t 0. Considering a particle let us write it for a particle at (x 0, t 0) at time t 0 at time t 0 and passing through x sorry (x 0, y 0) actually (Refer Slide Time: 37:09), this is what you have to telling passing thought (x, y) at time t; the particle has has the velocity (u, v) at t at time t that is the this is the velocity of the particle also.

Now, instead of looking to the point think about the particle, treat this x as the position of the particle and let x change; in the Eulerian description x do not change, in Lagrangian description x change; x now represents the position vector.

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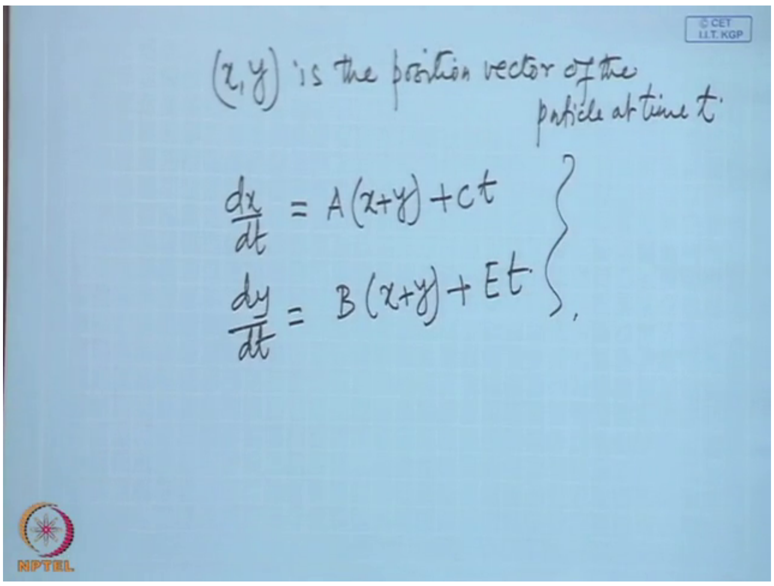

And then (x, y) is the position vector then of the particle at time t. And you can clearly write that d x d t equal to A into (x plus y) plus C t; and d y d t is B into (x plus y) plus E t. This I think you can solve simultaneously to do I will I am not going to solve these equations here.

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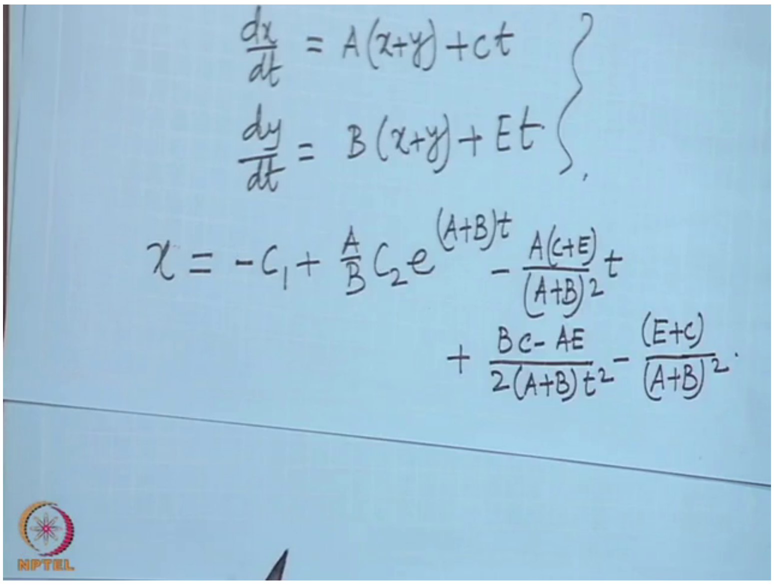

So, I will, but I will give you the answers for this problem; x is minus C 1 these of course are constant of integration this C 1, C 2 whatever I will write they are constant of integration minus C 1 plus A by B C 2 e to be power (A plus B) t minus A into (C plus E) by A plus B square into t plus B C minus A E by 2 into (A plus B) t square minus (E plus C) by (A plus B) whole square. C 1 and C 2 are constant of integration can be evaluated using the initial condition that at time t equal to t 0, the particle was at x is x 0, y is y 0.

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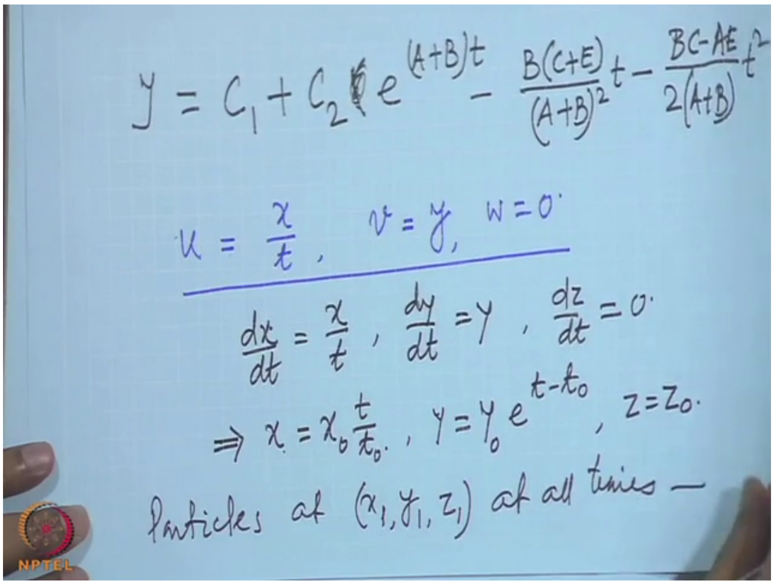

C 1, C 2 and the y is given as C 1 plus C 2 into sorry C 2 into e to be power (A plus B) t minus B into (C plus E) by (A plus B) square into t minus B C minus A E by 2 into (A plus B) into t square at t equal to t 0 which you can take even 0, t 0 can be taken as 0 also; at t equal to 0, x equal to x 0 and y equal to y 0, and C 1 and C 2 can be obtained from there.

Once again let us say that, the Eulerian velocity components are given by u equal to x by t Eulerian and velocity components are given by u equal to x by t; v equal to y, w equal to 0. Find the streak lines find the streak lines, these are the Eulerian velocity components you know how to find the streamlines? Streamlines have given by the equations d x by u equal to d y by v equal to d z by v w.

See in this case of course, basically flow is two dimensional, since w is given as 0 at all time First, find the trajectory of the particle, exactly in the same manner what we did in the last case. Again consider a particle, which was at some location at time t equal to 0 is crossing this point (x, y, z) of course, we need not to a consider that is the a constant, z 0 you can keep.

And then just like the earlier case, here also we have d x by d t equal to x by t, d y by d t equal to y, and d z by d t equal to 0 So, this is the trajectory of the particles, which which has pass through the point (x 0, x 0 y 0, z 0) we can call it this is the trajectory of all those particles, which has passed through point (x 0, y 0, z 0).

Now, instead of (x 0, y 0, z 0) we can take any other point and then find, what is that path just from this relation itself, by taking the inverse relation. If you take the inverse relation, the particle which are say let us say are (x 1, y 1, z 1) the particle at (x 1, y 1, z 1) at all time can be obtained by the inverse relation.

Only (x 0, y 0, z 0) will be replacing by (x 1, y 1, z 1) sorry this x will replaced by (x, y, z) will be replaced by (x 1, y 1, z z 1); all those particle which will pass through a fixed point (x 1, y 1, z 1) at all time will again we obtained from this equation, x 1 equal to these y 1, z 1; (x, y, z) can be replaced by (x 1, y 1, z 1). And then we can inverse the relations say particle at (x 1, y 1, z 1) at all times.

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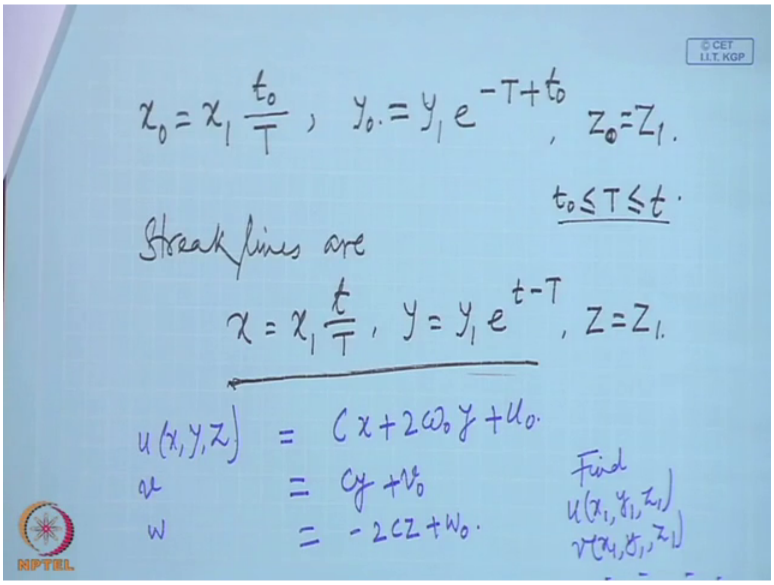

To make it all time let us say that any time between what we should write some other let us say capital T, what will be this for y 0, what will be y 0? y 1 to the power minus T plus t naught. And for z, there is no change sorry z 0 equal to z 1 for this t is when you say all time that is what actually mean. That is what the meaning of all time here is.

And then, since you are looking for a line this (x 0, y 0, z 0 we can now make it a variable. The streak lines will be are will be found on these surface.

So that, velocity at a point, the velocity at a point are given by u equal to C x plus 2 omega 0 y. Velocity at a point (x, y, z) is given by that is u (x, y, z) we are considering as in this case are steady flow is C x plus 2 omega 0 y plus u 0; v is C y plus v 0, and w;

find the velocity at the at the at a neighboring point (x 1, y 1, z 1) So, you can complete this problem there is really nothing in it to explain.

As I was mentioning that, this stream lines that stream lines gives a very useful information and really say picture of the flow, a really picture of the flow; of course, in your laboratory classes later on we will be seeing stream lines, which will show you the flow. But, please if you can remind me, before I come to the next class to bring some stream lines, either in say static picture or a movie picture to show you in the class here.

So, I am giving that responsibility to one of you please remind me, before I come for the next class to bring those with me. Some static view of the streamlines that is streamline at one instant only; if the flow is steady of course that that is the only view is not changing. But, for unsteady cases, the streamlines will change, so you going to movie view will be perhaps where is full. So, to remind me perhaps I will be bringing them.