Space Flight Mechanics
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Lecture No. # 45
Propulsion (Contd.)

So, we have been discussing about, the rocket dynamics in that context last time. I told that, I will be taking up the specific energy at the burn out.

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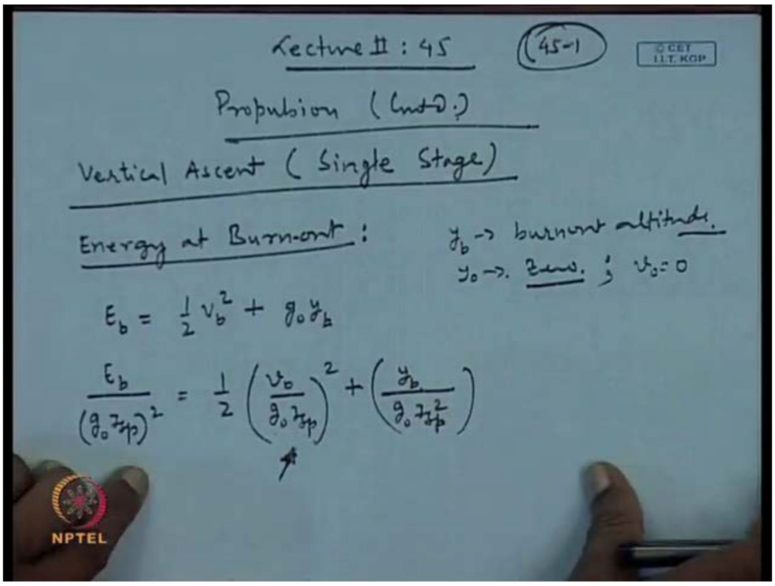

So, a specific energy is nothing, but the energy per unit mass at the burnout, we can write as 1 by 2 v b square plus g 0 times y b, where y b is the burnout altitude and here y 0, we are assuming to be 0. Now, if we divide both side by g 0 I s p square. So, this can be written as v b by g 0 I s p square plus y b by g 0 I s p square.

Now, in this format you know that already, we have found out the expression for the v 0 by g 0 I s p here v 0 also, we can assume to be 0. And similarly, for y b also we have got that expression for this. So, we just need to insert those expressions from the last class.

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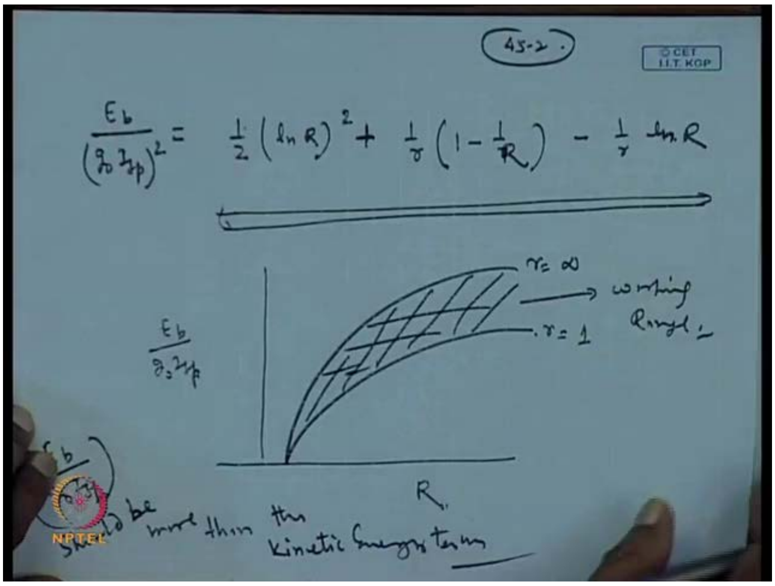

So, this will yield, insert the expression here for v 0 by g 0 I s p and this 1. So, once we insert this and solve it, we get 1 by 2, the simplification leads to this equation and this, you can check yourself by putting the expression for v 0 by g 0 I s p and y b by g 0 I s p square. And plot of the same, you should do the plot here, E b by g 0 I s p and R on this x.

So, you will get this kind of curve r is equal to infinity and r is equal to 1 and this is your working range E b by g 0 I s p, this should be more more than the kinetic energy term that is, this quantity E b by g i this is square here, we are putting the square. We have missed out this square in this place this is a square. So, E b by g 0 I s p square, this quantity should be greater than the kinetic energy term, which is present here why, because if it is greater only then, you will get the y b. You will have certain burnout altitude.

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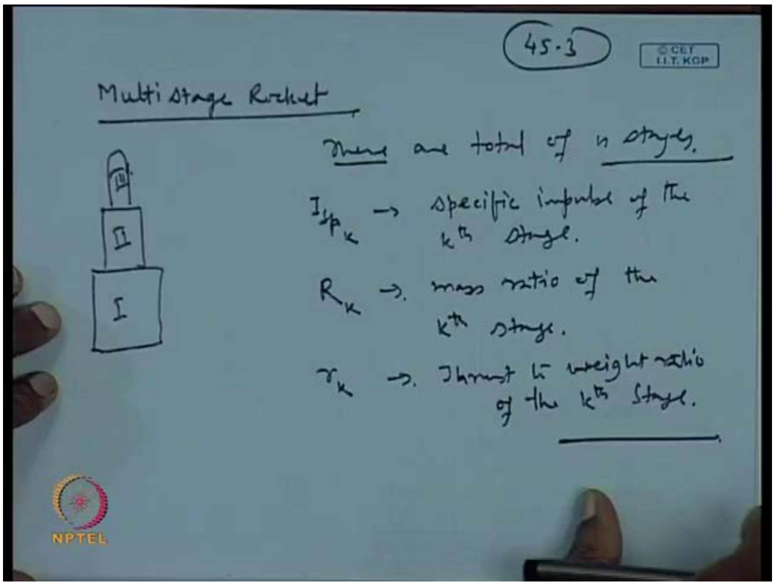

Now, will not go into much details of this, we quickly finish the Multi stage Rocket in the multi stage rocket. We have one stage here and another stage here, the third stage may be present like this. So, we can assume that, there are total of n stages.

And for each of the stage, we can have certain specification like the I s p under a score k, this can represent the specific impulse of the k th stage. Similarly, we can write R underscore k as the mass ratio of the k th stage and along the same line, r k can be written as the Thrust to weight ratio of the k th stage.

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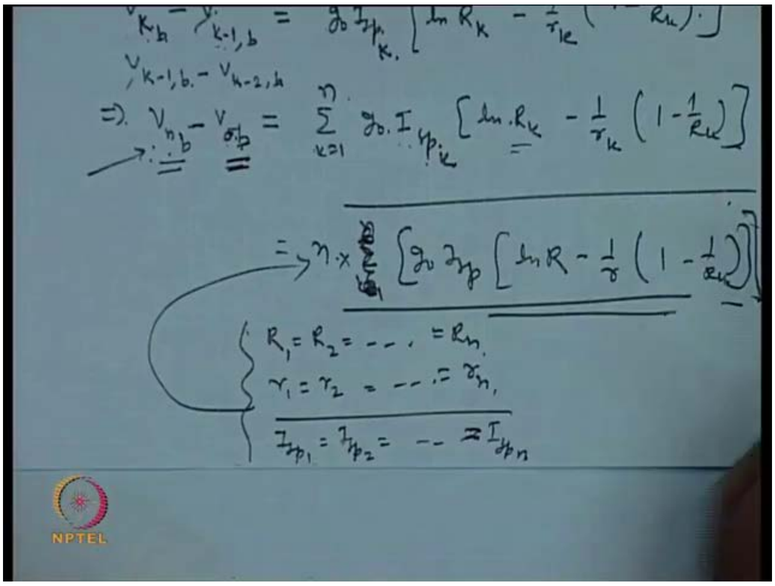

And you know that, we have already developed the equations for y b and v b. So, it becomes easy writing the equation, here in this place for the Multi stage Rocket. Now, let us say, we write v k b the burnout of the k th stage. So, this can be written as v k minus 1 b. So, 1 v k minus 1 b is nothing, but v k 0 v k minus 1 b this is nothing, but v k 0 velocities at the start of the k th stage.

And this from our earlier derivation can be written as l n R k as the usual notation, we are following. So, this way we can keep writing for the all the stages starting from the beginning therefore, if we add all of them together. So, this will imply. So, the terms like for the let us say, the previous stage. We can write it as v k minus 1 b and v k minus 2 b. So, if you add this kind of terms, you can see that this term and this term, they will cancel out and ultimately, leaving you with the n th stage here and this is the starting phase, the starting stage for the first stage.

The starting value of v 0 b, which is we will write here v 0 b is equal to 0. So, add them up and this summation takes place from k is equal to 1 to n and here also, we will put k l n R k minus 1 by r k 1 minus 1 by r k. So, this gives you the burnout velocity, for the last stage now, if all their stages having the same values of specific impulse and r k etc. So, you can just multiply, you can write this as k is equal to 1 to n g 0 I s p times l n R minus 1 by r 1 minus r k.

Where we are writing R 1 is equal to R 2 is equal to R n and similarly, this summation will then drop out this gets multiplied by this quantity r n, and then r 1 equal to r 2 is equal to r n and similarly I s p 1 is equal to I s p 2 is equal to I s p n.

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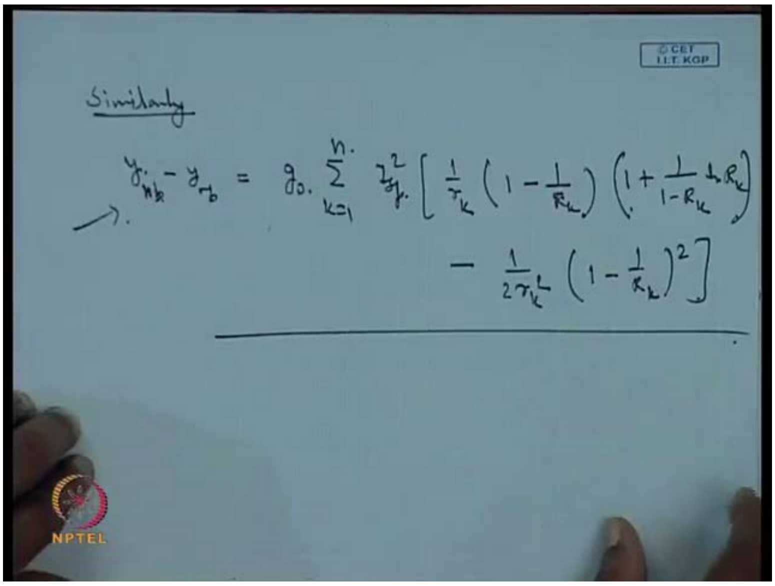

So, under this these assumptions, we can write this statement here, in the same way. Similarly, we can write y n b minus y 0 b is equal to g 0 summation k is equal to 1 to n , where the same symbols as we have used for this is the burn out velocity here, the same stands for also the burn out altitude. So, thus we complete this rocket propulsion part.

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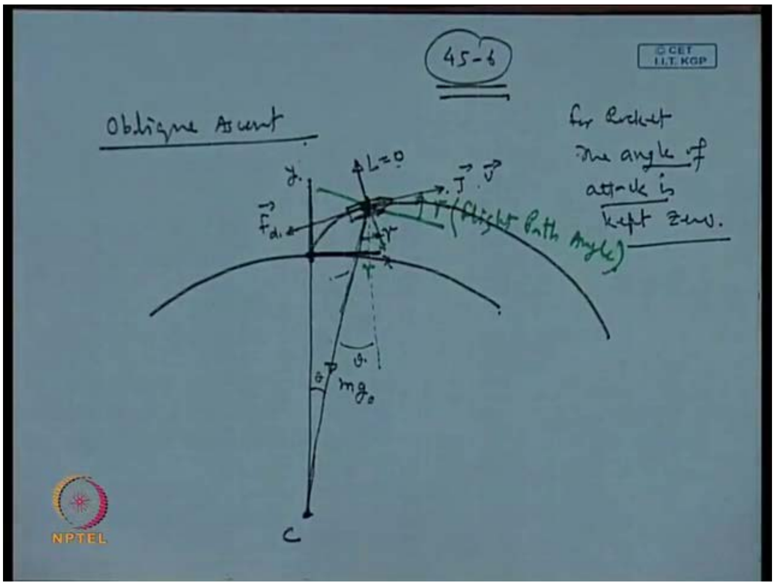

Now, going back to our earlier derivation, where we started writing the equation of motion for a rocket in Oblique Ascent. So, now we are trying to take a more comprehensive view, what we have done earlier say, we fix the x and y axis here.

Now, rocket is launched from this place, the rocket will follow certain trajectory and this has to be injected in an orbit. So, your orbit may be looking like this. So, at some intermediate stage, the thrust vector can be written here, has we have written earlier. This is the drag vector v is in the same direction as the thrust vector.

For rocket the angle of attack attack is kept 0, this is this implies, that we will not be showing any lift. We will just write lift is equal to 0, this is meant for avoiding the avoiding any lateral force. So, the rocket is not designed to take the lateral force like the missiles, which undergoes very high acceleration along that, trajectories perpendicular to the trajectory. So, the rocket is little different from that.

So, one is kept here 0 and that is achieved by keeping, the angle of attack all the time equal to 0. Now, if we assume that, this is equal to the centre of the earth. So, you can see that the gravity vector, the gravitational force will be directed along this direction. This is m 0 and the local horizontal is defined as the line, which is perpendicular to the m g 0 lines. So, this is the force acting here and this angle, we write it as gamma, this is the flight path angle. So, you can see that the lift vector, it will be directed some like this and this will be making an angle of gamma with this line. While, if you try to draw this line. So, parallel to this line, it will appear somewhere like this. So, if this is your angle theta, this will be your angle theta from here to here, this angle theta from here. We have written as gamma this is your gamma angle.

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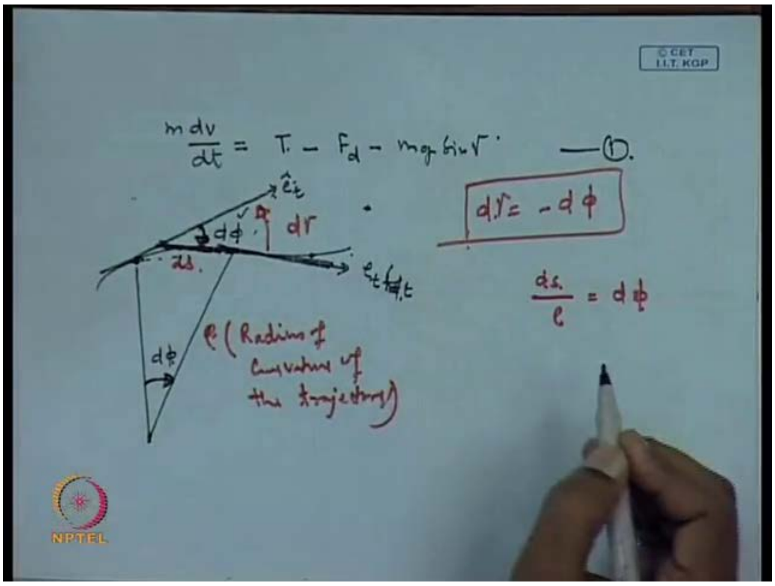

So, for this kind of motion if, we write the equation of motion for this, we can write m d v by d t along the trajectory is equal to thrust minus F Drag minus m g. You can see from this place here, m g is acting along this direction m g 0. So, one perpendicular to this in this direction, you will have m g cos gamma or cos theta whatever, you write. Here, we have written this angle as the theta to differ it from this gamma and this is the usual notation the gamma is often, even for the aircraft the flight path angle is described by this symbol, only flight path angle this is equal to gamma. So, this is the F D and then in this direction, we will have m g sine gamma.

So, we can write here m g sine gamma, this is equation one and then another equation. We can which is perpendicular to this trajectory. So, for deriving the equation perpendicular to the trajectory as, you may recall from our earlier lecture. If, we have a trajectory like this and say, this is the tangent vector e t, here in this place and this is the tangent vector in this place e t plus delta t or d t. So, this angle the angle between this and this two vectors, if it rotates by d phi here.

So, if we extend this from here to here. So, you can see that the angle between, we are just extending it back this tangent from here to here, and meeting it here in this place. So, this angle will be also d phi. So, d phi we are measuring from this direction and from here, the d phi is being measured from this to this direction.

So in our case, we can take a small part of this trajectory and which is near in this place. So, this trajectory is shown here, a very small link is taken here, which is nothing, but here d s. So, d s can be written as let us say, this is rho, which is the Radius of curvature of the trajectory and which will be different from the radius of curvature of the earth, it is a two different things.

So, Radius of curvature we can draw from this place and let us say, that it is lying somewhere here in this place. It is the centre of the curvature you will show it as make it o. So, we will make it c this is the centre of curvature of the trajectory. This is also called osculating centre.

So, this serves as the instantaneous centre for this trajectory, here in this place. So, d s by rho, this will be equal to d phi. Now, d phi is been taken from this side to since d here also, this is from this side. So, this is the positive since, we are taking contrary to this gamma is in the opposite direction. So, gamma will be from here to here and therefore, we will write the gamma equals d phi.

You can see from this place, if the trajectory changes little bit. So, we have the gamma angle here say, this is the local vertical and this is the local horizontal. So, we measure the angle gamma from here to here. This is your angle gamma from this place to this is your tangent; this is your angle gamma.

You go to next point which is along the trajectory by d s. So, you will see that the gamma will also, change by d gamma the flight path angle will change by d gamma. So, d gamma can be written in d gamma is equal to minus d phi. Therefore, writing in the equation of motion perpendicular to that trajectory or this is the perpendicular.

So, this is a perpendicular to the trajectory and this is the local vertical, they are two different things local vertical. We decide it from the gravitational point of view while the perpendicular to the trajectory is decided from the acceleration point of view of the rocket.

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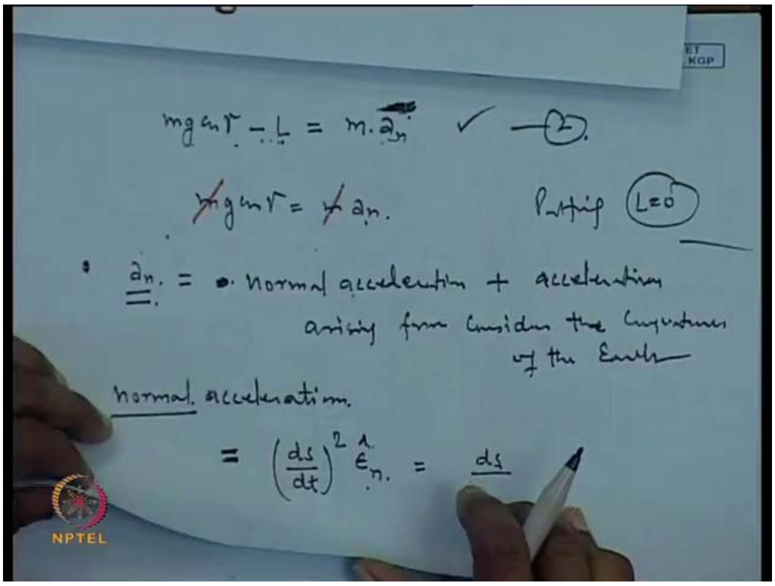

So, thus we can write the equation of motion perpendicular to the trajectory as m g cos gamma. This is the m g cos gamma in this direction, and if we write the lift here in this direction. So, this will be minus L and this we can write as m times the acceleration, the normal acceleration. Which, we are showing here in this direction, this is the normal acceleration. So, we removed this arrow here, which is indicating a vector, because all this quantities are in the same direction. You can say along the same line. So, and we have already taken care of the sign. So, it can be written in this way, this is our equation number two.

So, if we make l is equal to 0 putting l is equal to 0, now we need to find out what will be this acceleration a n. So, the acceleration along this direction, it is composed of two components, one due to the we can assume that, this rocket is moving in a trajectory about the centre of the earth. So, this case we are taking once, we are considering the curvature of the earth is present. So, one acceleration will come, because of this. Because it is a moving, in a trajectory and about the center of the earth another one.

We can see this is locally that, this is also moving about this centre. So, we can add them and represent the acceleration a n as acceleration due to this is called the acceleration, normal acceleration. We will write here, a n is equal to normal acceleration and plus acceleration arising from considering, the curvature of the earth. So, here also we canceled out this m and m.

Now, this normal acceleration we can write as we can recall from our earlier lecture, this is nothing, but d s by d t square times E n where E n is the unit vector in this direction E n cap. So, we can write this as d s by d t, from let us say, from our very experience we know that, if any particle is moving in a path along the trajectory.

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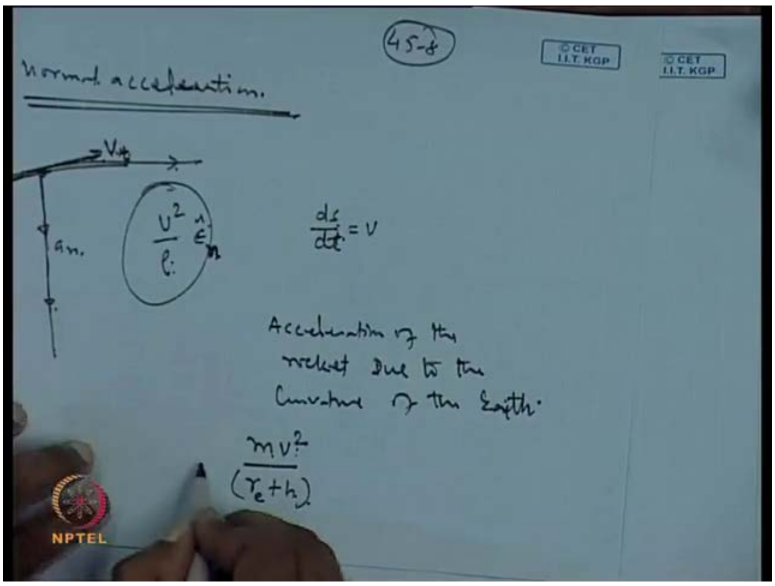

So, this is the velocity which is tangential to this trajectory. So, in the normal acceleration a n is given by v square by rho and here, this is directed along this direction. So, v square by rho and we can put this as n. So, we are cancelling out this only v square is appearing in this place and we have to have the term here present. So, I have canceled this and v square by rho where v is nothing, but d s by d t. So, this is the acceleration along this direction and what we call as the normal acceleration.

Now, this normal acceleration and acceleration, we are having from the gravitational force. So, gravitational force due to that of movement about the earth, it is also rotating about the earth. So, if it is to rotates about the earth the acceleration of the rocket will be given by.

So, the acceleration of the rocket due to the curvature of the earth of the rocket, this can be written as we know that the acceleration is given by m v square, if v is the velocity with respect to the centre of the earth. So, m v square by r. So, r is the distance which is equal to this distance which is r earth and plus this distance which is the altitude h.

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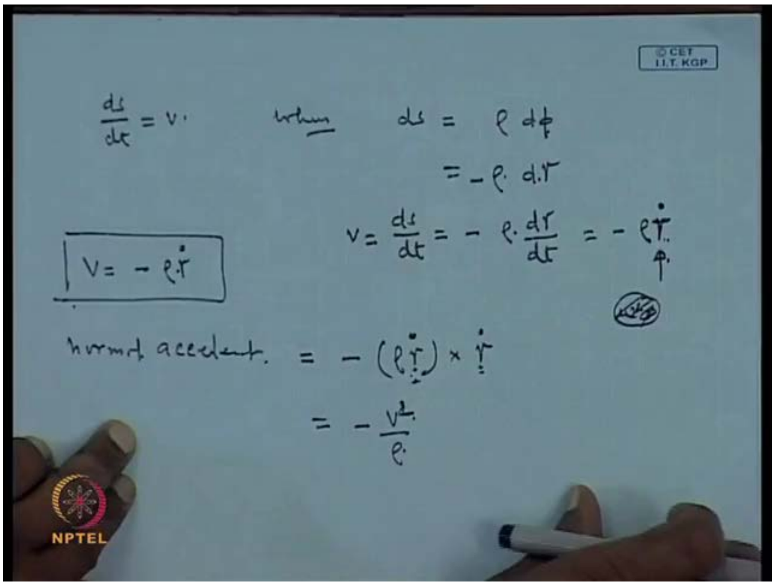

So, this can be written as r earth plus h. So, now, let us write here d s by d t equal to v where d s can be written as following, this figure d s can be written as rho times d phi and we replace in terms of gamma. So, this becomes with a minus sign d gamma and therefore, d s by d t we can write as minus times d gamma by d t and we write as rho gamma dot.

So, we have converted in terms of the flight path angle and this is equal to your v. So, what we get here v is equal to minus rho times gamma dot. So, normal acceleration with proper sign, what we have written here this will be written in terms of gamma. If we write in terms of gamma then, this will be converted into with minus sign and this can be written as rho times gamma dot, which is equal to v n times gamma dot.

So, this is basically omega you can see that, this is in the form of omega s square times R. So, this is giving you the local acceleration, because of this trajectory curvature of this trajectory and the same thing, we can write as minus v square by rho.

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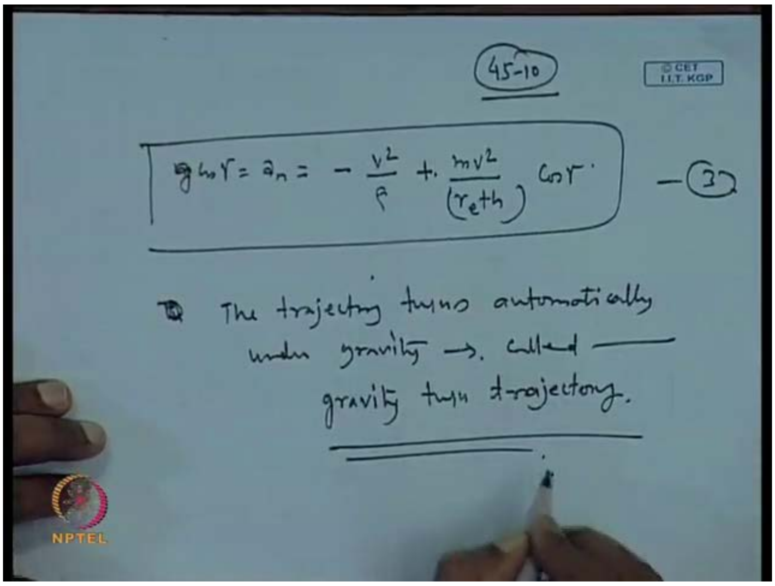

This is with proper sign writing in terms of gamma now, adding both of them. So, this will give us g cos gamma is equal to n, we can write it as v square by and plus. This term m v square by r e plus h, but this quantity, as we can follow from this figure. This quantity is directed towards the centre of the earth as, it is moving in a trajectory about the centre of the earth. So, this quantity is directed about the centre of the earth. Now, we can take a component of along this direction. So, we will put here cos gamma. So, ultimately acceleration due to the acceleration of the rocket; due to the curvature of the earth here, also we can put quantity cos gamma. So, this completes our derivation or this is our equation number three.

So, equation number three combined with equation number one, this way our equation number one. So, this need to be this two need to be solved simultaneously, to get the complete trajectory in Oblique Ascent assuming that the earth is having a curvature. It is a not to here, we are not assuming that to be a flat. And c here obviously, we are writing all the equations in inertial reference frame not in terms of the non inertial reference frame, as if you try to write in the non inertial reference frame. So, what will happen that you need to inject into the orbit. So, orbit the velocity in this orbit whatever, is and velocity in this will be a rotating reference frame the earth is rotating.

So, if you fix in reference frame here in this place, which is rotating along with the earth then, this problem will be their then, you have to take into the corollas acceleration and other things. And if you write the same equation in the inertial reference frame, which is not rotating with the earth along with the earth then, the situation becomes different and you can write the equation in a very simple way.

Thus, we have completed the all the equations and this equation can be integrated. This equation can be integrated numerically, using mat lab you can do it. So, here once the rocket is fired rocket is fired vertically, and there after it is tilted little bit and then, the rocket follows it is own course. So, under the gravitational force, itself the trajectory will keep bending and it is called the gravity turned trajectory.

So, the rocket turns automatically, the trajectory trajectory turns automatically under gravity and this is called gravity turned trajectory.

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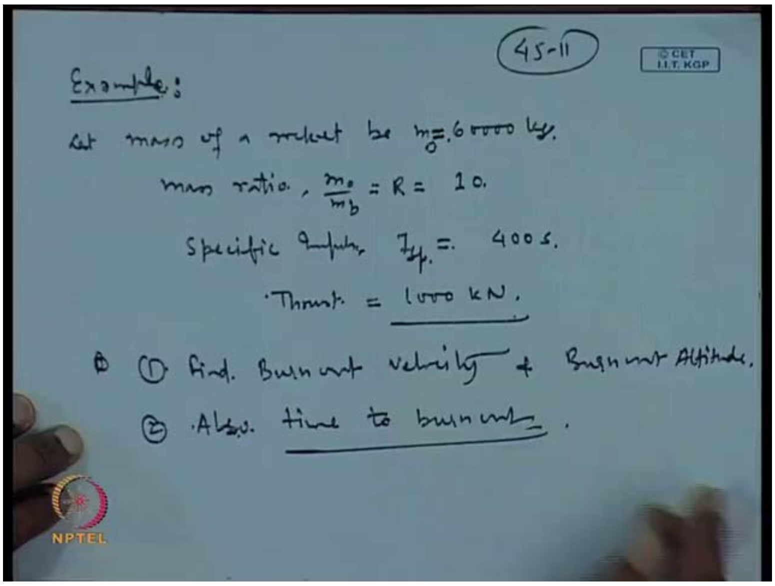

We have finished all these things now, we will take up an example, this is a very simple example, but source whatever, we have done till now, let mass of a rocket be 60000 kilo grams. So, this is your m. Or say this will put as m 0 and the notations are same, as we have done earlier the mass ratio means, m 0 by m b equal to R this is given to be 10 and specific impulse I s p. This is given to be 400 second and thrust of the rocket. This is given to be 1000 kilo Newton. So, of objective is to find burnout velocity and burnout altitude and also, time to burnout.

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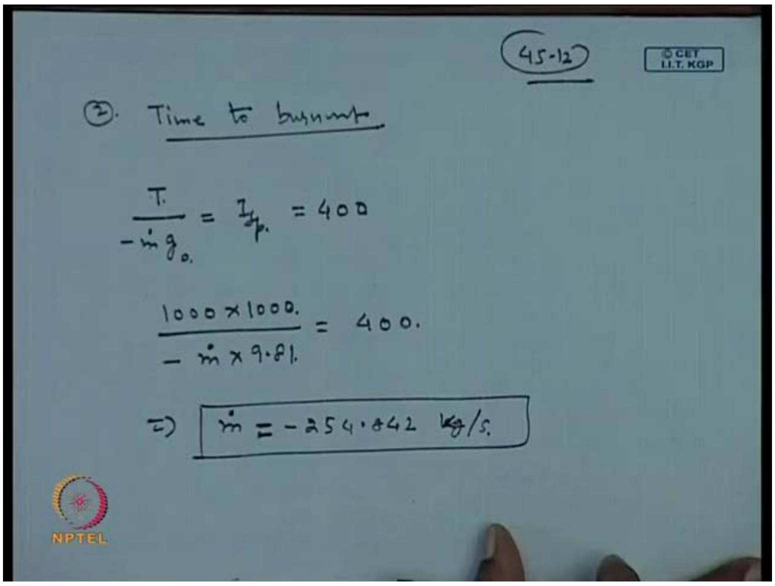

So, we can take the second problem first. So, time to burnout, because time to burnout it is involved in the equation for the burnout velocity and burnout altitude. So, this is the equation for the specific impulse and the quantity I s p is given, this is given to be 400 and thrust is also given to us. So, thrust is given to be 1000 kilo Newton. So, we multiply it by 1000 to get it in terms of Newton and then m dot minus m dot and g 0. We can write as 9 point 8 1 is equal to 400 and this implies m dot equal to, we can solve it and this will give you minus 250 4 point 8 4 2 k g second. So, this is the rate at, which the mass is been burnt and obviously, for as we have stated earlier for all the rockets the same dot is negative.

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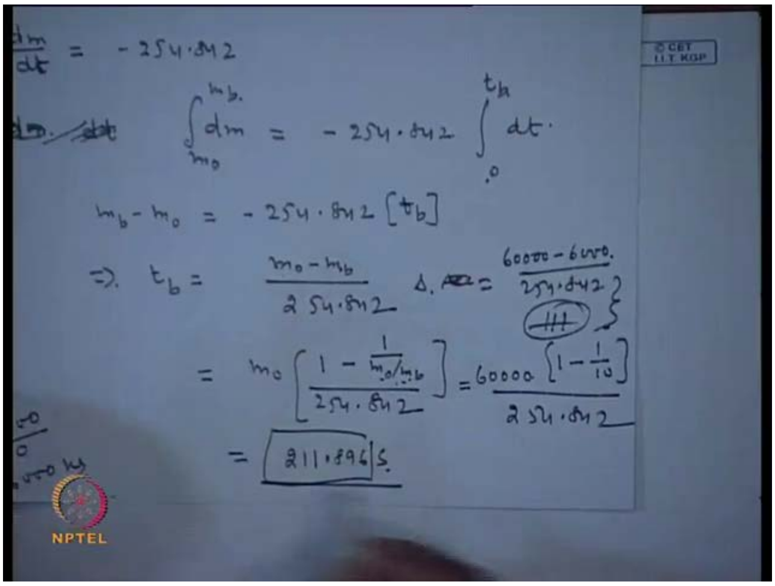

Therefore, d m by d t is given to us we can integrate it to get the time for burnout. So, d m integrating or we can write here as d t fine let us integrate it like this m 0 to m b t 0. So, t 0 we will assume as 0 and here we will write as t b and this is equal to d t time t b and this implies t b will be equal to m 0 minus m b divided by 254 point 8 4 2 and this many seconds. So, inserting the values for m 0 we know already. So, m 0 we can take it outside and we can write here as 1 by m 0 by m b divided by 250 4 point8 4 2. So, m 0 is given to be 60000 kilogram and 1 minus 1 by r is ten. So, m 0 by m b is nothing, but r. So, we insert it for ten and this is 250 4 point 8 4 2.

Or alternatively you could have written as because m 0 by m b is known to be 10. So, this implies m b will be equal to m 0 by ten means 60000 kilograms divided by 10 and this is 6000 kilograms. So, here directly you could have subtracted from 60000 minus 6000 and this could have been written like the 60000 minus 6000 divided by 2 5 4 point 8 4 2. So, both are same this one and this two are equivalent and this will give you 2 1 1 point 8 2 6. So, this is the burnout time.

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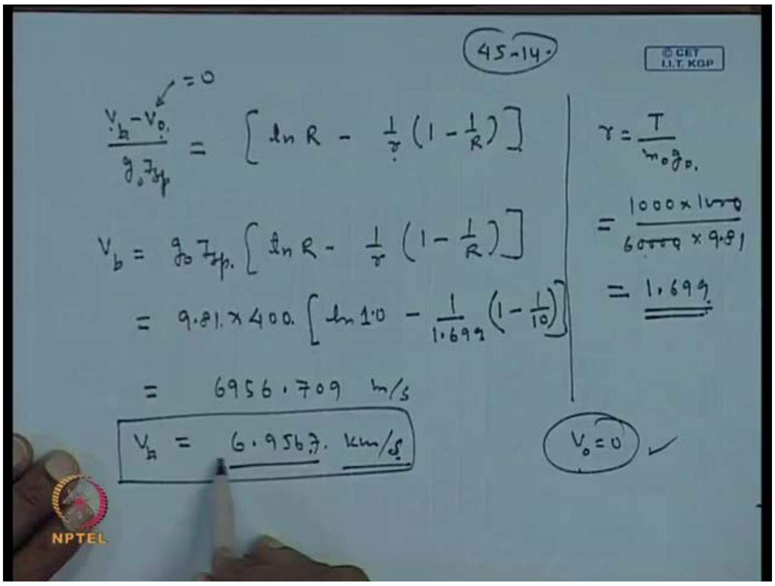

Now, we can calculate the burnout velocity. So, for finding the burnout velocity earlier we have developed this equation where these parameters, you were aware of this is the thrust to weight ratio and this is mass ratio. So, from here we will have v b equal to assuming this to be 0 g 0 times I s p and g 0 is 9 point 8 1 I s p is we have taken to be 400 second l n 10 R equal to ten and this is 1 by R. So, r we have to find it out how much R is. So, R we can write as we can calculate here in this place R is thrust to weight ratio m 0 by g 0. So, this is 1000 time 1000 divided by 60000 time 9 point 8 1.

And this will give you around 1 point 6 9 9. So, this is the thrust to weight ratio. So, here we can put it 1 6 9 9 and 1 minus 1 by 10. So, computing this will give you 6 9 5 6 point 7 0 9 meter per second or 6 point 9 5 6 7 kilometer per second. So, this is the burnout velocity where we have assumed v 0 to v 0 if v 0 is non 0 and this will simply add to the v p.

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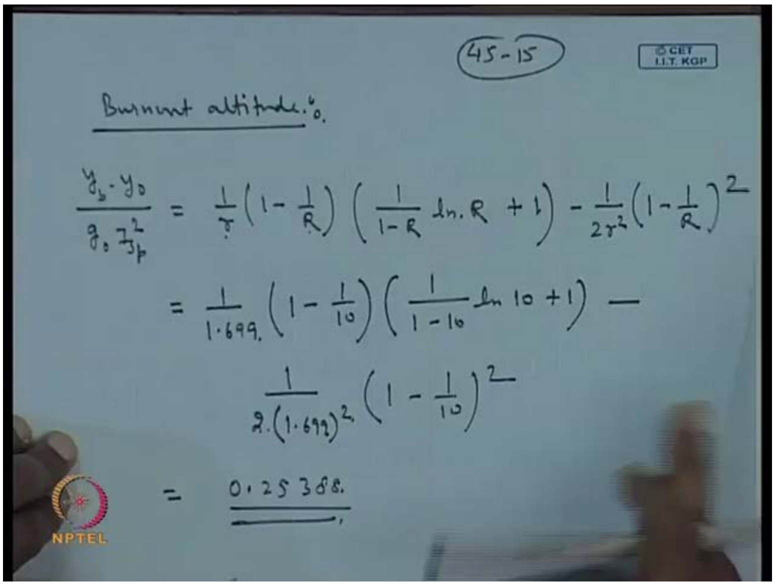

Next, we take the burnout altitude now the for the burnout altitude, we have the equation, because we have written earlier. This equation as y b minus y 0 divided by g 0 I s p square 2 R square.

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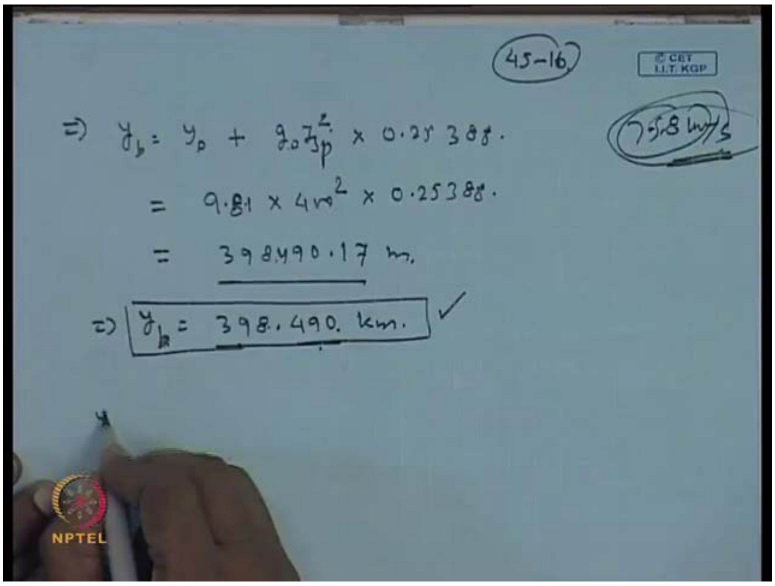

So, using this equation now, we can insert the r value and all these quantities are known on the right hand side and this is 1 point 6 9 9time 1 minus 1 by 10 2 time 1 point 6 9 9 square and if, you compute it this will turn out to be 0 point 2 5 3 8 8.

Assuming y 0 to be 0 g 0 9 point 8 1 times 400 square. So, this is the burnout altitude. So, we can see that our once, we have put the equation in this parametric form how conducive. It is to calculate the final burn burnout altitude not only that, this burnout altitude helps us in this parametric equation. This helps us in doing the parametric study. That is the most important part that is the reason why, we write the equation in terms of parameters it is a it becomes, because there are various parameters involved and with respect to those parameters, how the equation is behaving that remains the main objective of the study.

So, it is not that only is once, we have done the design and that design then becomes the final aspect, we have calculated once. So, if we have a broad aspect of the design like by varying, which parameter what happens or in what range. It is a possible to vary the r the capital, r the small, r how the system will behave varying these parameters. So, if we have a broad prospective of this, then we do a better design for the rocket. So, all these details it is not possible to cover in this course, because only 4 lectures are assigned for this. So, what is still in the brief, we have covered the dynamics of the rocket and we have got a lot of insight into how the rocket motion takes place.

And if, you remember the earlier what we did that in the earlier case, we assumed the earth to be flat, if we assume the earth to be flat and in the vertical ascent obviously, this gamma was taken as 0 for vertical ascent. So, vertical ascent simply implies that, we are just going here in this direction and the sounding rockets generally, it is a fired and it goes vertically and it gives you up to what altitude the rocket is going to get over by the term. It burns out and that becomes a very important parameter.

Now, one thing you should take care of here while, working out we have not considered anything about the horizontal motion of the earth the horizontal motion of the earth. It adds to the motion of the rocket, that is why the rockets are fired sternly means the as in the last class, I have been explaining that if is this is earth and here. This is the equator and if I am firing the rocket in this direction. So, already the earth is rotating and last time wrote the value for this tangential velocity, because of the rotational motion of the earth to be around point 4 kilometers point 4 5 kilometers per second at the equator.

So it is a substantial point 4 5 means 450 meter per second while, we require for to put in the orbit around 7 2 8 kilometers 7 point 5 2 8 kilometers per second, this is the requirement for putting in the orbit. So, even if you are firing the rocket, here like this you are not considering the horizontal velocity the rocket will keep drifting like this, because it has the horizontal velocity, because of the motion of the rotational motion of the earth.

So, irrespective of that, what we have done till now, we have just assumed that rocket is going vertically and we have written the equation for the vertical motion by putting gamma equal to gamma equal to 90 degree not 0 this is gamma equal to 90 degree, this is for the vertical ascent. Remember gamma equal to 90 degrees for the vertical ascent and gamma equal to 0 means, it is a just it is a going tangential and of course, from the beginning you cannot fire the rocket tangentially. So, the rocket is first fired vertically, up it is a fired vertically up it goes at up to certain altitude while, it crosses the dense atmosphere and thereafter it is a tilted.

And under the gravitational force then automatically, it keeps bending what is called the gravity turn trajectory and all this things can be optimized numerically, and we have to if we write the exact equation for that and if we think and we worked out in details using computer. But the main objective was here to get the idea about how, the rocket dynamics looks like how does calculate, the burnout altitude burnout velocity etcetera. So, those those objectives are achieved by the end of this lecture.

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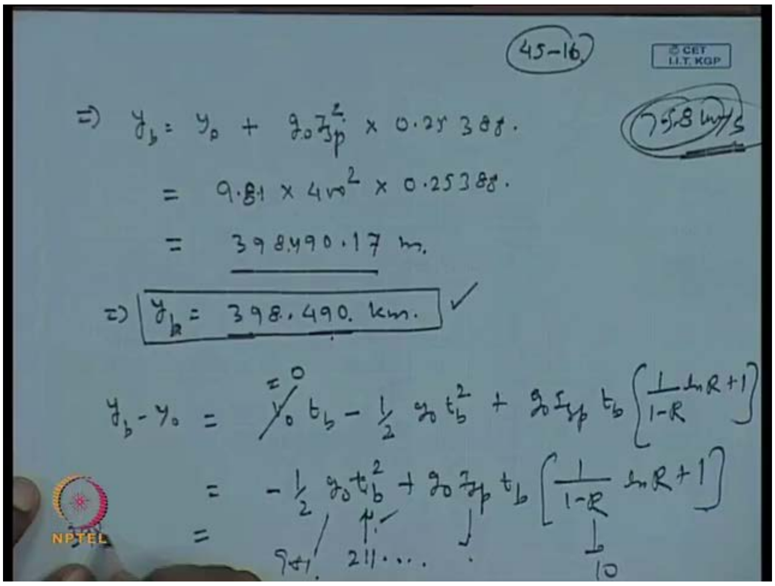

So, for the burnout altitude we have also written the equation in this format, this was the original equation without any parametric representation changing much into this. So, in this b 0 once we are assuming this to be 0. So, this get reduced into this format and this equation and the equation that, we have written here both are same. This is nothing different and you can check by putting the proper values of t b, we have already known to us 211 point something just. Now, we calculated it g 0 is 9 point 8 1 I s p is given and this value is 10.

So, if you put this, and you compute, I got around 398 point 5 1 7. So, it is a little bit difference from this, I am getting and that is a numerical error this kilo meter. So, using either the parametric representation or either use, the original equation results are same. So, we end the lecture here and this whole course comes to an end, after total 45 number of lectures thank you very much and whatever, you want about the numerical. So, numerical will be putting on the wave course. So, look for the wave course, which will be of adulteration thank you very much.