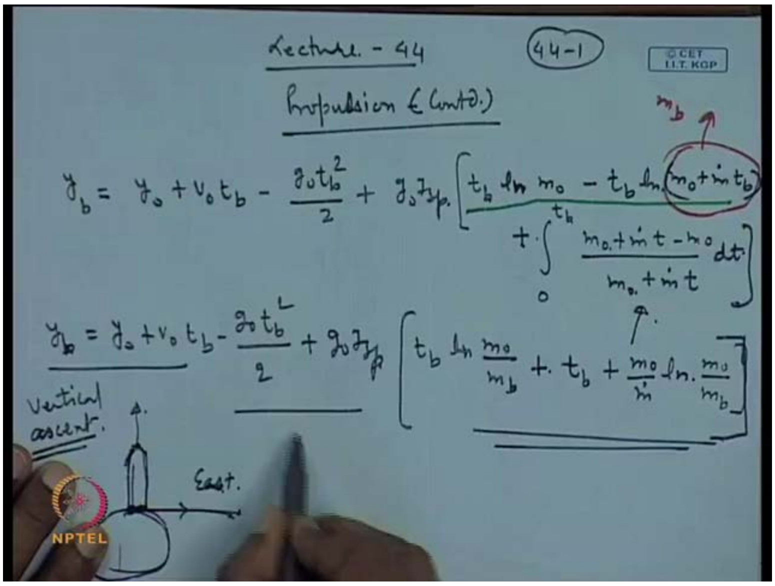
Space Flight Mechanics

Prof. M. Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 44
Propulsion (contd.)

In the last lecture, we have been discussing about the vertical ascent of the rocket and we calculated the burnout velocity in the vertical ascent and then, we were working out for the burnout altitude.

(Refer Slide Time: 00:32)



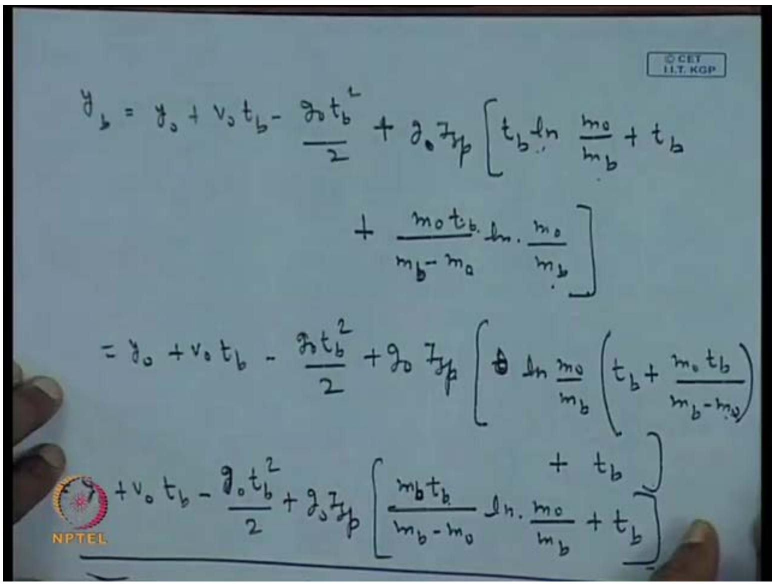
So, in that context we wrote the equation y b equal to y 0 plus v 0 t b minus g 0 t b square divided by 2 plus g 0 I s p times t b l n m 0 minus t b l n m 0 plus m dot t b and then, plus the quantity the, if you look into the last lecture. So, this was the quantity the time copying here in this place. So, this is the t b l n m 0 minus. This quantity and this is inside the curly bracket, this quantity and it here, it is minus sign. So, this minus and this becomes plus and therefore, here we get a plus sign 0 2 t b and this is m 0 plus m dot t minus m 0 divided by m 0 plus m dot t d t and this quantity, we were computing also last time. So, first of all let us, simplify this term simplifying this term gives you. This is y 0 v 0 t b minus g 0 t b square divided by 2 plus g 0 I s p and this becomes t b times l n m 0 by m b t. This quantity is your m b the quantity, which is present here this is nothing, but m b.

So, this two terms whether we can write as m 0 by m b and then, this becomes plus and if you remember last time. We derived this term to be t b minus m 0 by m dot times l n and this quantity. Now, this minus sign can be absorbed inside this by taking m 0 from denominator to numerator and numerator can be passed to the denominator. So, this will become then, t b plus m 0 by m dot and l n. Now, this m 0 will go into the numerator and this term which is nothing, but equal to m b this, we can push to shift to denominator.

So, this is the simple equation, we have got for the burnout altitude in vertical ascent. So, irrespective of if you, assume that the rocket is been launched from this place. So, rocket will in the vertical ascent, it will keep ascending vertically up while, once gets a living the surface of the earth and if it is on the equator. So, you know that there will be a velocity towards the east. So, it will have an also horizontal towards there is a horizontal velocity towards the east.

So, in the inertial reference frame, this will tend to this will appear as a velocity. So, rocket will appear drift laterally also, but if we look from only for the vertical part. So, we will looked at the rocket is ascending vertically and whose equation is given by the expression. We had derived here in this place g 0 I s p times.

(Refer Slide Time: 04:34)

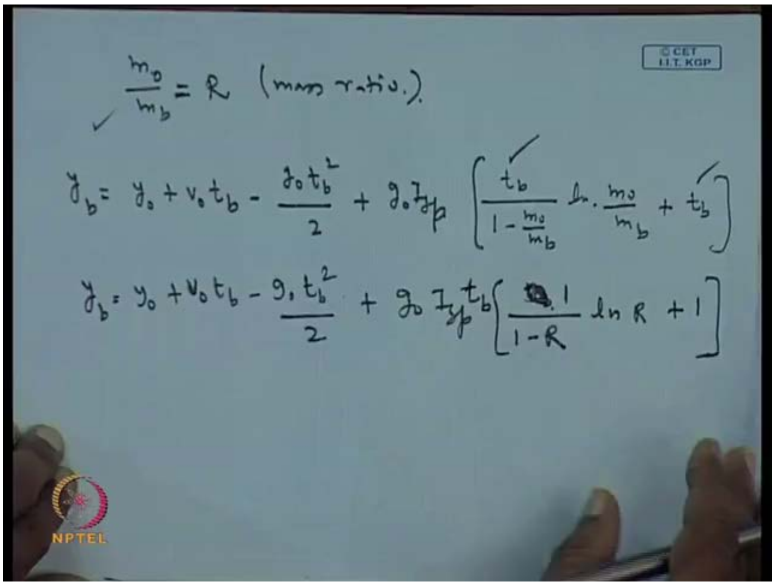


Now, this quantity m dot which is present here, this can be written in terms of m and m 0. So, this m dot basically, if you remember we have written m as m dot t plus m 0. So, m dot can be written as m minus m 0 divided by t. So, this we what we are trying to do that, we are trying to eliminate this m dot and writing whole thing in terms of the burnout mass and the initial mass and the burnout time rather in terms of m dot.

So, this becomes m 0 divided by m burnout minus m 0 times t l n m 0 divided by m b. Now, once we have got in this format, some more simplification is possible like, we can see that the term here l n m 0 by m b. and here the term is l n m 0 by m b. So, we can take this common and add this term and let us look, what the expression results in.

So, we can take out the term l n m 0 by m b outside and then, the quantity inside the bracket will be t b plus m 0 t b divided by m b minus m 0 and plus t b this simplification is little long, but not difficult. So, if we multiply this, we can see that in 0 t b terms will cancel out living, out with living a with the term, m 0 times m b times t b divided by m b minus m 0 l n m 0 by m b plus t b.

(Refer Slide Time: 10:19)

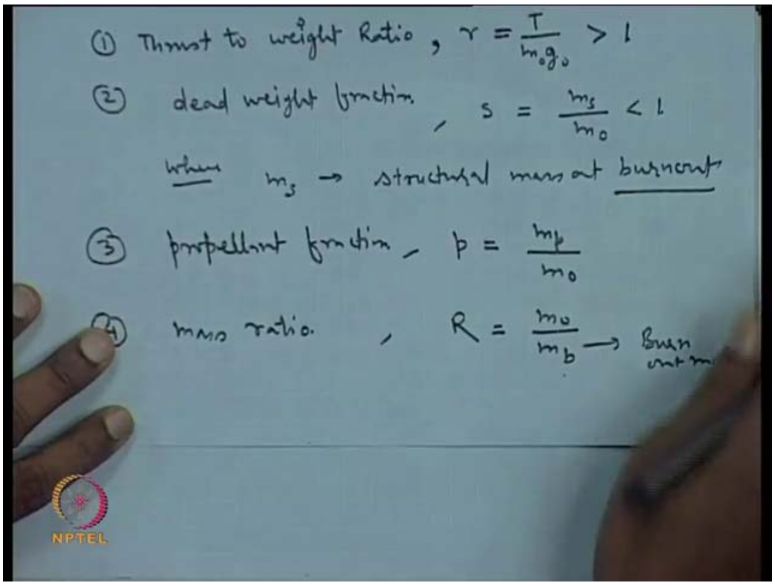


This is what we get in the now, to do further simplification, we just put m 0 by m b equal to r, which is the mass ratio. So, this gives us y b equal to y 0 plus v 0 t b minus g 0 t b square by 2 plus g 0 I s p. I am dividing this term, m b in the numerator and the denominator dividing by m b. We get here, as the t b and in the denominator 1 minus m 0 by m b and similarly, m 0 by m b plus t b.

So, in this format now, insert this value m 0 by m b equal to R v 0 t b minus g 0 t b square divided by 2 t b divided by 1 minus R and this t b also, we can take it outside and write it here in this place, because t b is present here. t b is present here. So, we can take it out of the bracket and write 1 by 1 minus R l n and this becomes R plus 1.

So, this expression gives out, gives us the burnout altitude. Now, little bit more parametric representation of this equation is possible and that puts by the parametric representation. We will be able to draw some graphs and have little more differ look into the nature of these equations.

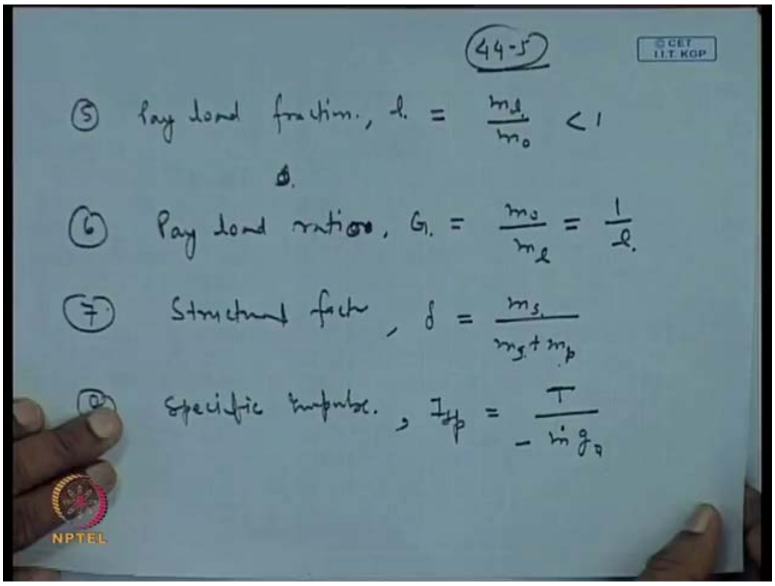
(Refer Slide Time: 10:48)



So, we define some design parameters such as, Thrust to weight ratio and this quantity is greater than 1, dead weight fraction s and we define as m s by m 0 and this quantity is less than 1. Where, m s is the structural mass a structural mass at burnout.

Then, we define propellant fraction fraction p as m p by m 0 m 0 is the initial mass is includes the total thing as the structural mass plus, the propellants plus, the pay load mass ratio already we have defined this, we write as R equal to m 0 by m b. Where, m b is the burnout mass, this is the burnout mass then, we define pay load fraction pay load fraction as the l and this can be written as m l by m 0 and this quantity is also less than 1.

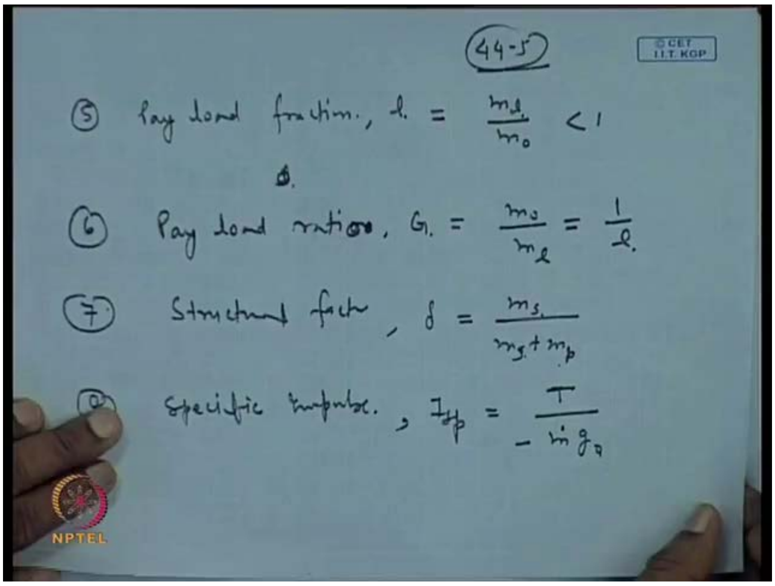
(Refer Slide Time: 13:00)



Then, pay load ratio as the fractions are less than 1 and the ratio will be greater than1. So, pay load ratio this we write as G and this we define as m 0 by m l. So, the inverse of this, we can write this as 1 by l.

So, a structural factor delta, this is m s by m s plus m p. So, this is the structural mass and this is the propellant mass. This is the structural mass. So, pay load mass, we have removed from this and then we have a specific impulse I s p, this is T divided by minus m dot g 0.

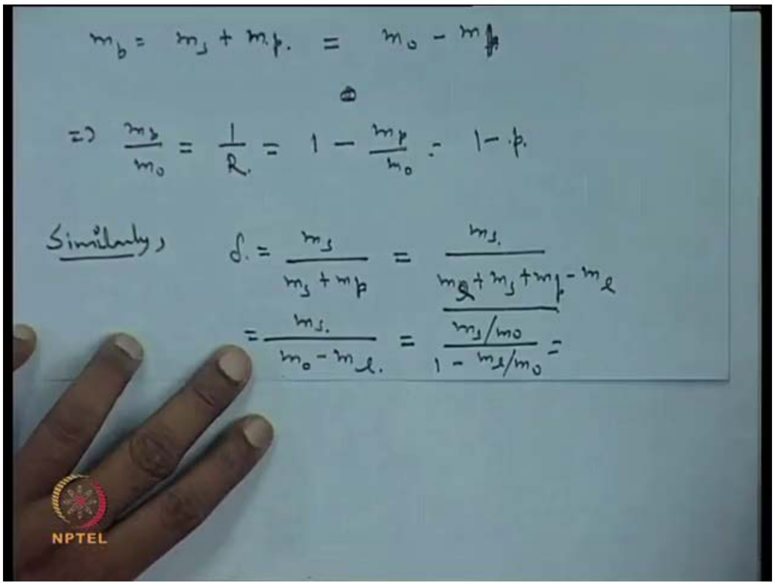
(Refer Slide Time: 14:42)



So, these are the design parameters of a rocket. Now, m 0 can be written as m pay load plus, m propellant plus, m structure mass. Now, at the burnout all the propellants will be burnt. So, we can write, this as m p plus m l plus m s and this is nothing, but mass of the rocket at the burnout m b and this is the mass of the propellant. So, m b is nothing, but m p plus m s. So, what we have written here m b is m s plus m propellant.

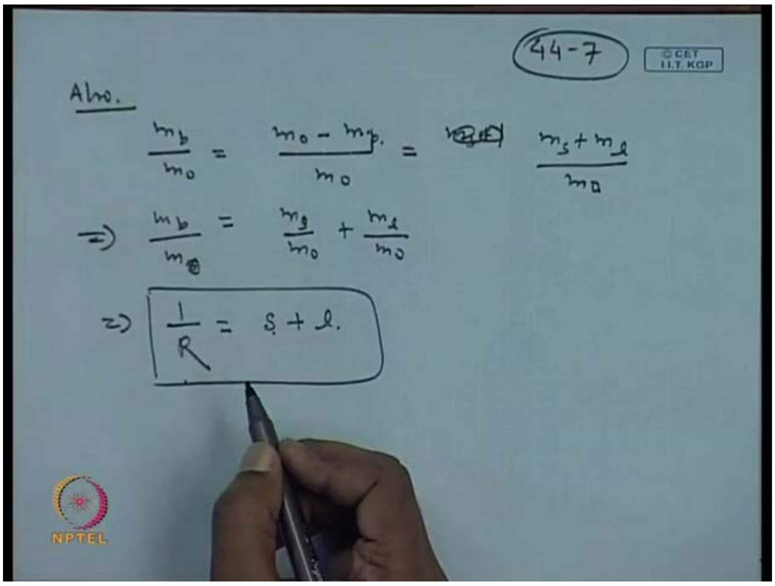
And therefore, this we can also write as you can see from this place, this is your m 0. So, m m b can be written as m 0 minus m p. So, this implies m b by m 0 m b by m 0 is nothing, but 1 by R. So, if we divided on the right hand side, this becomes 1 minus m p by m 0 and this is the propellant fraction and we have used the notation for propellant fraction as p, this is 1 minus p.

(Refer Slide Time: 14:42)



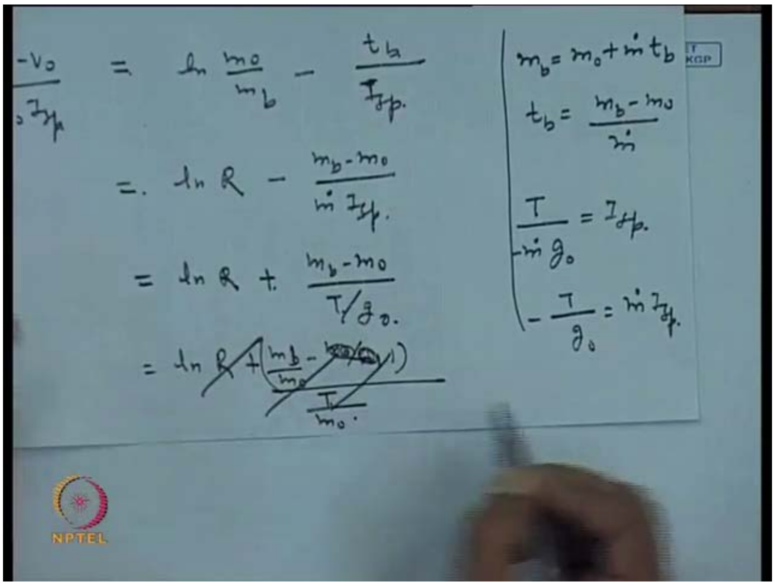
Similarly, you can get the other relationship delta is m s by m s plus m p m pay load plus m s plus m p minus m pay load. And this we can write as m s divided by m 0 minus. This quantity is m 0, divided the numerator denominator by m 0. So, this is m s divided by m 0 1 minus m l divided by m 0 and this quantity, we have written as the quantity s which is the dead weight fraction m s by m 0. So, m s by m 0 is our dead weight fraction s and this become 1 m l by m 0 is nothing, but pay load fraction m l by m 0, this is we have written as l. So, delta becomes equal to s by 1 minus l.

(Refer Slide Time: 17:35)



Also, we can write m b by m 0 is equal to m s plus m l divided by m 0 and this becomes m s by m 0 plus m l by m 0. So, this will be left hand side is 1 by R and m s by m 0. This is the dead weight fraction s, we have written as an m l by m 0 as l. So, this is another relationship we are getting.

(Refer Slide Time: 18:41)

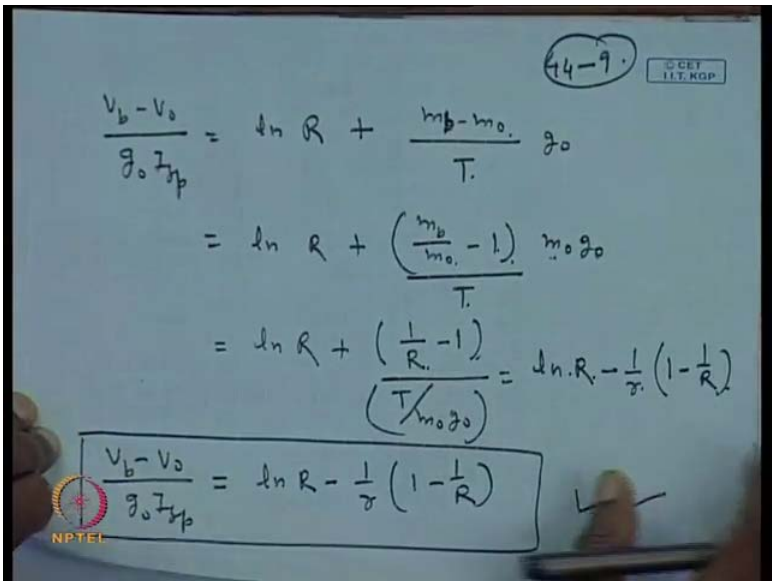


So, this way you can combine the different parameters and write them and these are very useful, while reducing the rocket equation. So, if we remember from our last lecture. So, v b minus v 0, this was written as g 0 I s p l n m 0 by m b minus g 0 t b, and this we often write it deleting the drag term and considering the rocket is in vertical ascent.

So, divide both side by g 0 I s p and this gets reduced to l n m 0 by m b minus t b by I s p g 0 cancels out. So, m 0 by m b, we have written as R and this we need to work out little bit. So, we can write m burnout is equal to m 0 plus m dot t b and therefore, t b becomes m b minus m 0 divided by m dot.

So, this we can write as m b minus m 0 divided by m dot I s p. Now, if you remember that thrust by m dot 0 with s g 0 with negative sign, this has been written as I s p. So, this I s p, we can eliminate from here. So, m dot I s p is here, this can be written as t by minus g 0 this is equal to m dot I s p. So, this gets reduced to l n R plus m b minus m 0 and this quantity we can write as t by s 0.

(Refer Slide Time: 21:42)

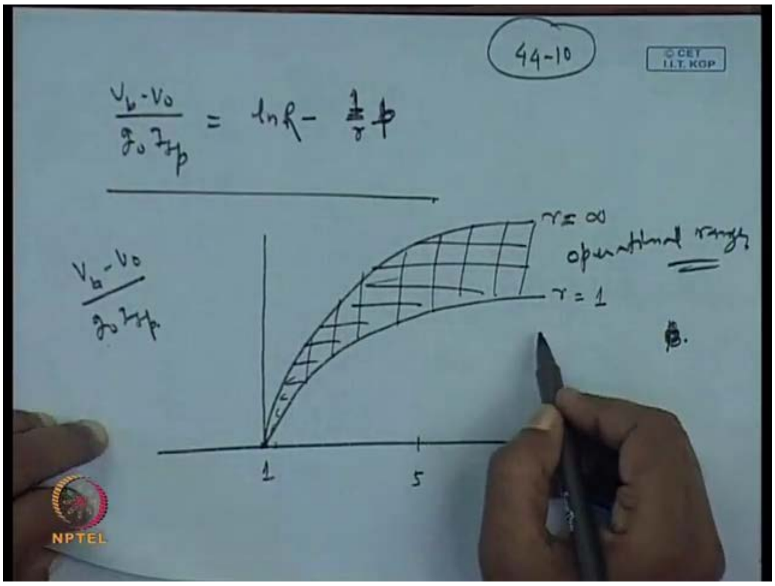


Now, divide the numerator, denominator by m 0 in this term. So, this is m 0 by m b divided by m 0 minus 1 m divided by t divided by m 0 let us, written on a separate page g 0. We can put here, and t in this place. So, this is m b not m 0 now, dividing this by m 0. So, l n R plus m b divided by m 0 minus 1, this becomes m 0 times g 0 divided by t. So, l n R plus m b by m 0, we know that this quantity is equal to 1 by R, this is minus 1 and t by m 0 g 0, we have written as the thrust ratio.

So, in the beginning itself, we wrote it thrust to weight ratio t by m 0 g 0. So, here we can write it as t by m 0 g 0. So, this is your t by m 0 g 0 and the next step is you can reduce this as l n R minus. This quantity is nothing, but R. So, this becomes minus 1 by R and here, we are changing the sign here, because this quantity is less than 1, 1 by R. So, we will put it in this.

So, what ultimately, we see that v b minus v 0 by g 0 I s p. This can be represented as l n R minus 1 by r and 1 minus 1 by R, this we have also written as if, you looks from this place. This equation m b by m 0, this is equal to 1 by r and this has been written as 1 minus p. So, this implies that 1 minus 1 by R, this is equal to p.

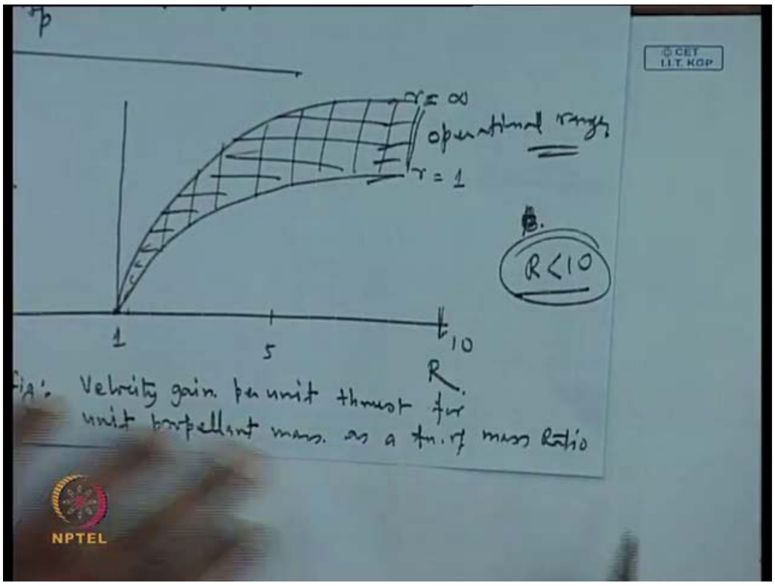
(Refer Slide Time: 24:20)



So, this equation for v b minus v 0 by g 0 I s p, this can be also written as l n times l n R minus 1 by r 1 by r minus times p 1 by r times p. Now, we can if we draw a plot for this by plotting r on x axis v b minus v 0 and divided by g 0 I s p on the y axis. So, this we take as y and this as the x and this as the parameter. So, in this you look in this equation basically, we are trying to plot this equation. So, here minus 1 by r is appearing multiplied by 1 by r.

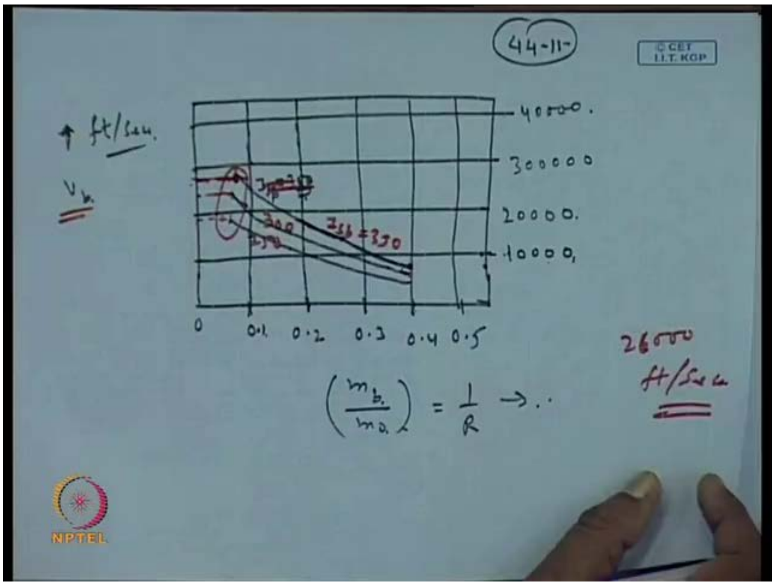
So, basically we are trying to plot, this equation and this is a short representation of given here, if you do the plot. So, your v b minus v 0 divided by g 0 I s p will be limited to this range. So, this shows that even, if your increase the thrust infinitely R becomes infinitely. It does not mean that the quantity v b minus v 0 by g 0 I s p will become infinite.

(Refer Slide Time: 24:20)



So, your velocity remains finite. So, this is your operational range basically, operational range we cannot go beyond this range and we have to work with in this limit where, as I already told that R is generally less than10. So, this figure, we will term as the velocity gain means v b minus v 0 is your velocity gain per unit thrust for unit propellant mass as a function of mass ratio.

(Refer Slide Time: 27:52)



So, now one more figure, we will draw if, we plot the burnout velocity on this axis in feet per second and m b by m 0, which is nothing, but 1 by R. On the axis. So, you will get, some this kind of plot that on this axis let us say, this quantity is 10000; this is 20000; this is 30000 and here, this is 40000.

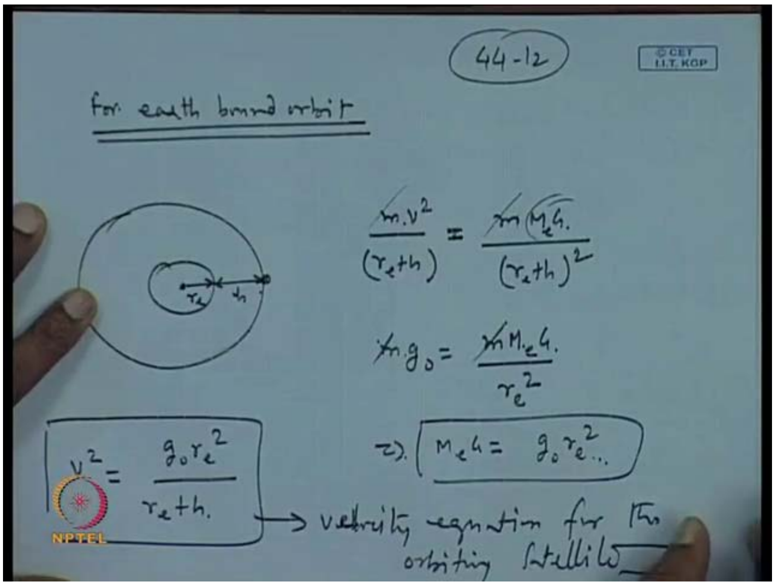
This kind of curve can be plotted now, here if you look into this curve is for I s p for 350, this curve is for, here I s p is equal to 350, this is for 300 and this is for 250.

So, on the top we can see from this place, that velocity that the burnout velocity that, you can achieve with the I s p of 350. This is limited similarly, for with I s p of 300, this is limited. So, while on the other hand for injecting the satellite into the orbit some we require, the velocity in the range of some, 26000 feet per second.

So, this is not up to the 26000 marks, but it is a rather less whatever the quantity, we are getting this is for the ideal case. We will do a derivation of how much velocity is required for the injection into the orbit. So, this figure we refer as, burnout velocity as a function of mass ratio specific, impulse and thrust ratio thrust to weight ratio thrust to weigh ratio, which is R this is I s p mass ratio. We have written as R and in this case the R has been present as R is equal to2.

So, using the equation that, we developed earlier this equation, you put R equal to 2 and g 0 I s p you choose as 300, 250 and 350. So, you can do (( )) plot and you can assume v 0 to be equal to be 0, because in the vertical ascent, we are taking. So, in the vertical direction v 0 is equal to 0. So, we can a start with this and plot this v b. So, this kind of curve you will be obtained.

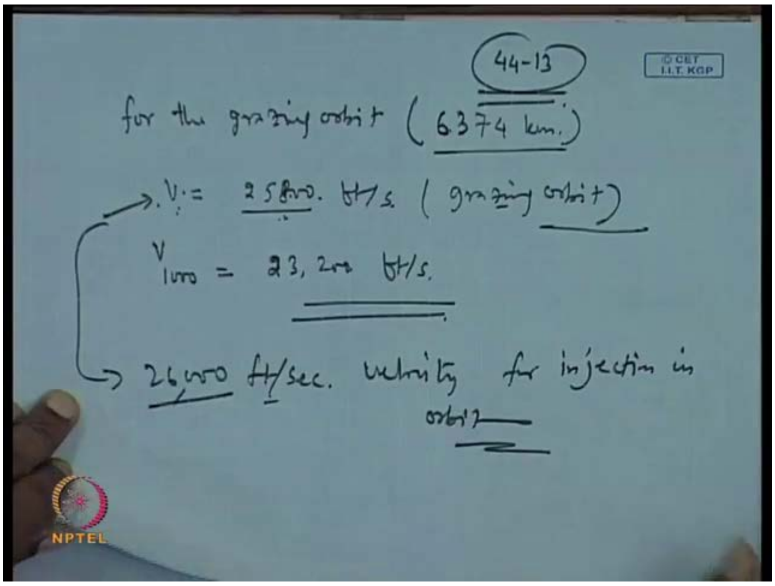
(Refer Slide Time: 34:01)



Now, for earth bound orbit let us calculate the required velocity, we can write a very simple equation this is the earth. So, here this is your radius of earth r e and this, we will write from here to here this is the altitude. So, m v square by r e plus h the necessary, centripetal acceleration is provided by the gravitational force. This we cancel out and here also this term will cancel.

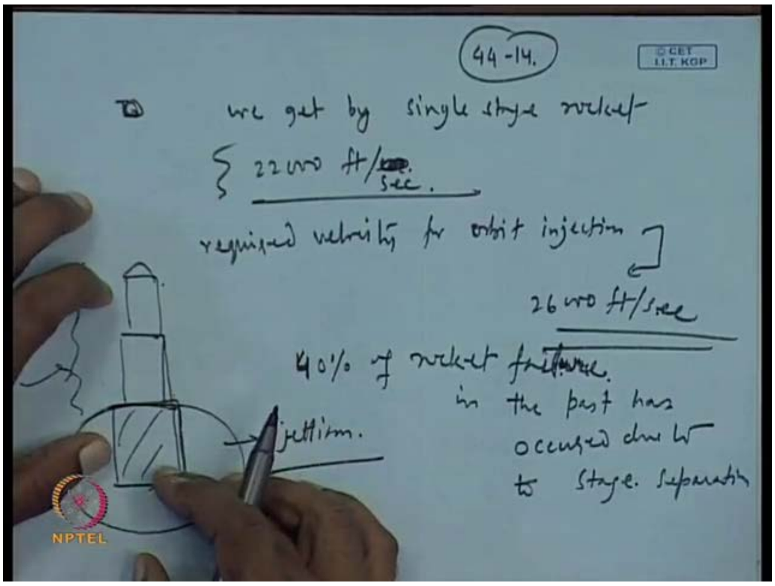
Out now, we know that on the surface of the earth m times g 0. This is nothing, but equal to m times m earth times g by r earth square. This is what we get on the surface of the earth. So, from here what this implies this cancel out. So, M e times g this is nothing, but g 0 times r e square. So, this we can these terms can be replaced by from here. So, we will have v e square by v e square equal to g 0 r e square divided by r e plus h. So, this is the velocity equation equation for the orbiting satellite.

(Refer Slide Time: 36:12)



So, for the grazing orbit means, the orbit of the satellite is same as the radius of the earth. So, this calculation 6374 kilometer is the radius of the earth. So, based on this calculation the following results can be obtained. So, we can write v equal to 2 5 8 0 0 feet per second, this is for grazing orbit if we go to 100 miles 1000 miles of v 1 1000, this can be written as 23 2 0 0 feet per second. So, for injecting the satellite in the orbit, you require around 26000 feet per second velocity for injection, in injection in orbit and for lunar orbit or lunar injection or lunar mission, the required velocity is 35000 feet per second.

(Refer Slide Time: 38:12)



While the burnout velocity, which is available to us that will be confined to around 22000 feet per second. So, the figure has been not very exact. So, here in this case, this will be around 20000 feet per second that we get with the I s p of 350 say. So, we see that by using a single stage rocket the required velocity the just, we have done a simple calculation in the vertical ascent. So, required velocity is 26000 feet per second, while we are getting around some 22000 feet per second. So, this is not solving our problem.

And therefore, multi staging of the rocket becomes in this case necessary. So, to sort out this problem, we get by single stage by single stage rocket 22000 feet per second velocity, while required velocity required velocity for orbit injection. It is a around 26000 feet per second.

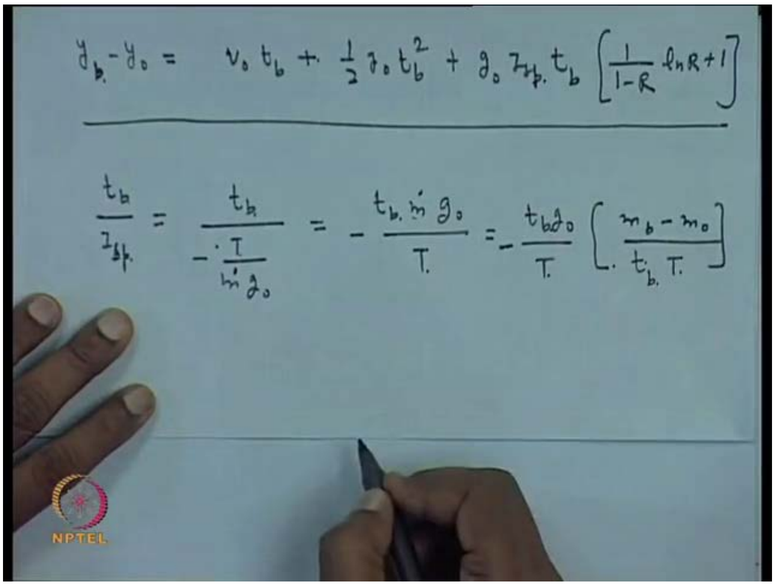
So, using a single stage rocket therefore, it is a not possible to inject the satellite into the orbit and the kind of the specific impulse. We have used here with r equal to 2 therefore, a multi staging concept. It becomes a necessity and multi staging concept also, it is of efficient in the since, that say if you have a single rocket and you are taking, it to the final orbit at the time of the burnout. So, you have to accelerate and accelerate a big mass from beginning to the end.

And for accelerating a big mass from the beginning to the end it requires a lot of energy and that energy at the end will go as a waste these are the structural masses. So, it is a possible that the structural masses whatever if, we do the stretching like the first stage, second stage and third stage. So, we can keep ejecting those masses, the stages 1 by 1 as the burnout takes place and therefore, this kind of a design will be much efficient.

So, what I imply here that, this is the first stage of the rocket; this is the second stage and then, we have the third stage of the rocket. So, after the first stage propellants are burnt-out, we can jettison it, if we jettison. It if we throw it out in the air. So, we are left only with this mass and only this mass needs to be carried out. So, this becomes energy wise or propellant wise more efficient, but the problem involved with this, jettisoning, this is more complicated a complicated mechanism and 40 percent of the rocket failure in the past has occurred due to this stage separation.

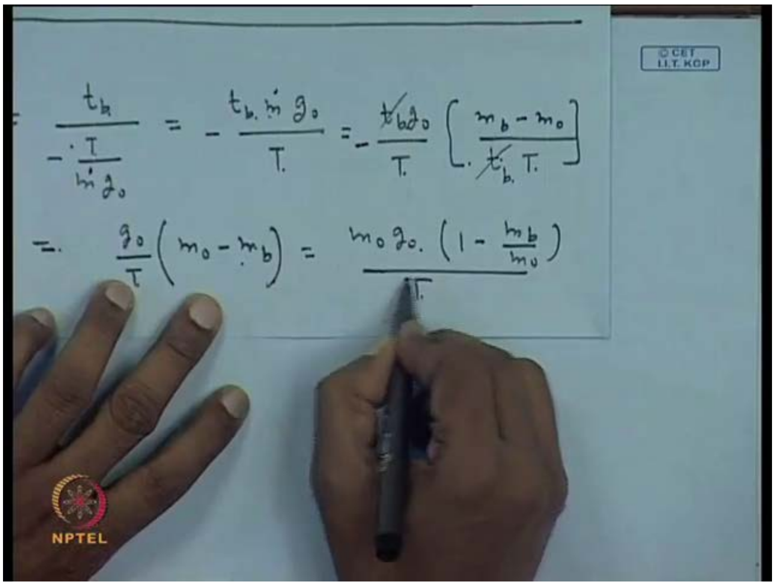
So, we have to shift it out jettison, this we have to separate of this stage and this is a complex this involves a complex mechanism and in that course the basically, the failure occurs, if you are not able to eject this. So, whatever the previous design, you have done this before the launching of the rocket, the whole design comes as a failure cannot go to the final orbit with, much of mass attached to the secondary stage.

(Refer Slide Time: 43:54)



So, we will discuss about the staging of the rockets, also as we have done the parametric representation of the burnout altitudes a burnout velocity in the same way. We can also represent, the burnout altitude in parametric terms and that gives us more comfortable way of describing or presenting the equation, and doing the parametric studies by representing the themes in terms of a the graphs. This is the equation that, we have written for the burnout altitude here, y b indicates the burnout altitude.

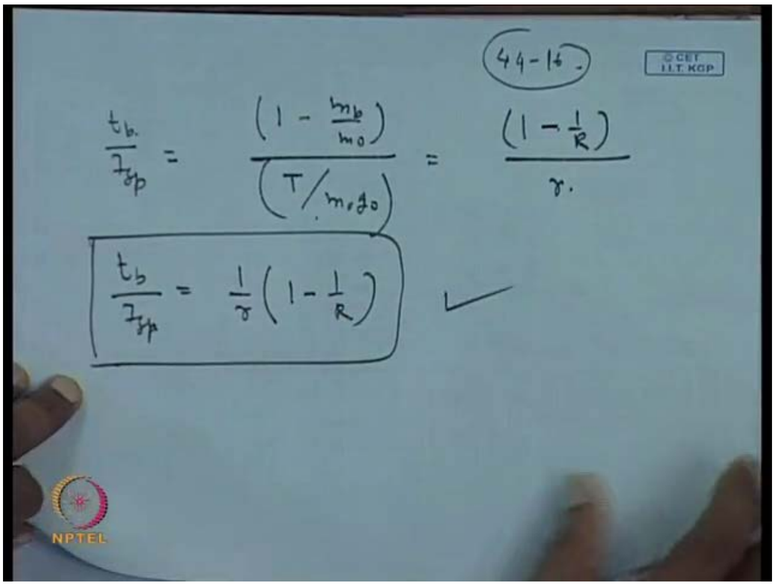
(Refer Slide Time: 43:54)



Also, we have written t b by t b by I s p, this can be written as t b by minus t by m dot g 0 I s p is nothing, but quantity. Now, m dot can be written as m b minus m 0 minus sign here, divided by t burnout. So, this t b and this t b this will cancel out living us with g 0 by T. This we can write as m 0 minus m b, minus sign we have observed inside.

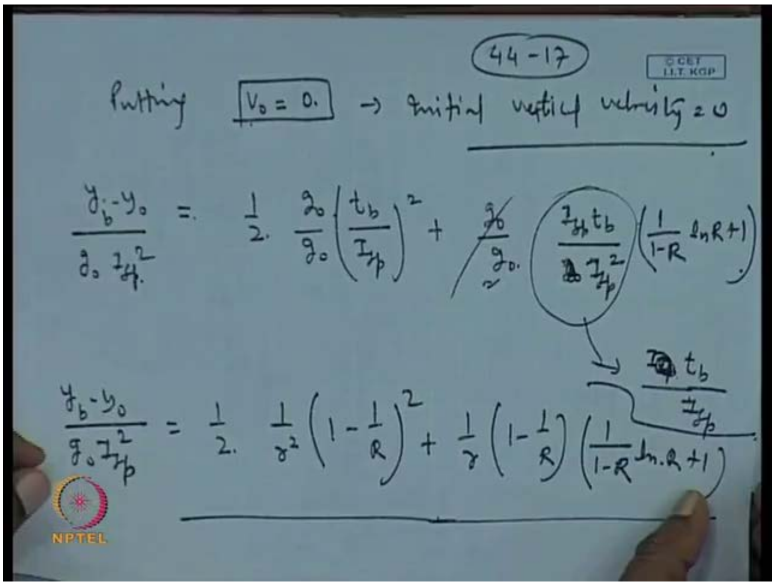
Now, if we divided it by if we take m 0 outside. So, this is m 0 times g 0 1 minus m b by m 0 divided by T, T by m 0 by T. We have written as the thrust ratio. So, this can be written as let us get to the next page.

(Refer Slide Time: 46:49)



So, we have t b by I s p equal to 1 minus T by m 0 g 0 and m b by m 0. This is nothing, but 1 by r, this quantity and this we have written as small r. So, ultimately this reduces to this expression t b by I s p. Now, we can utilize this expression in reducing the expression for the y b minus y 0.

(Refer Slide Time: 47:54)

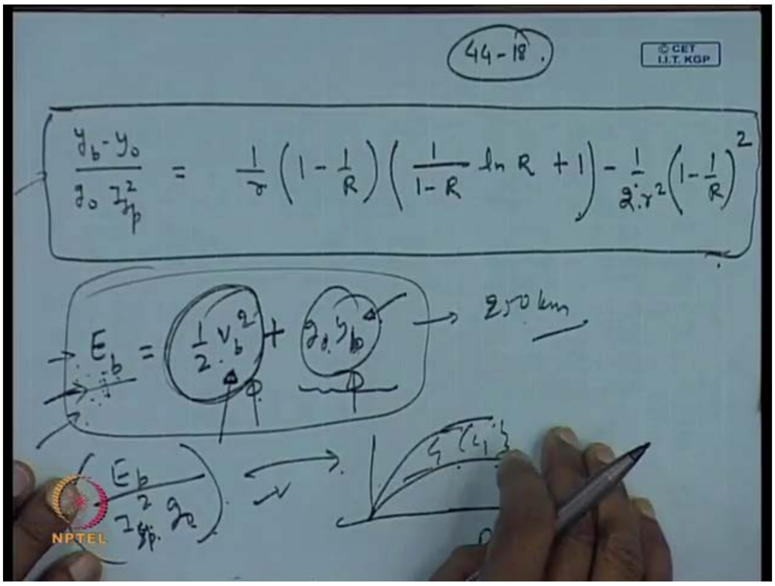


By putting v 0 equal to 0 that is, initial vertical velocity is 0. So, this becomes y b minus y 0 is equal to 1 by 2, we divide both the sides by g 0 I s p square. So, this will give us g 0 t b by I s p square plus g 0 by g 0 and I s p time t b, we have, we can insert from here. So, this is I s p times t b divided by g 0 g 0, we have already taken here into account this I s p square times 1 by 1 minus R l n R plus 1.

So, the quantity which is present here this is nothing, but I s p by t b by I s p and t b by I s p just now, we have derived here in this place. So, just we need insert in this place y b minus y 0 by g 0 1 by 2 times t b by I s p square. This is 1 by r square times, this cancels out and here, we have again 1 by e times 1 minus 1 by r times 1 by 1 minus R l n R plus 1.

So, this is the expression we have got now, this was our with a minus sign will be minus see here, we had this minus sign, here in this place minus g 0 t b square divided by 2. So, wrongly we have put here the plus sign. So, we will make a minus sign, here in this place this is the burnout altitude and we utilize this here in this place. So, here we will have this is fine; this is plus sign here, we will have a minus sign. We will have a minus sign here in this place.

(Refer Slide Time: 52:01)



So, finally, what we have got y b minus y 0 divided by g 0 I s p square equal to 1 by r times minus 1 by 1 by 2 term is also present. So, this is 1 by 2 times r square times 1 minus 1 by R whole square and for this also, we can do a parametric representation in the same way, as we have done earlier.

So, all the parametric representation can be done at what our objective is ultimately to a study, that with the various factors of the parameters, how the rocket will be behaving. If we change some of the parameters then, what are the limits it. So, basically we would like to optimize the parameter in such a way that, our burnout altitude becomes maximum.

And in the multi staging again the altitude the maximum altitude that can be raised, that can be optimized with respect to the parameters. Similarly in the case of the staging also the maximum altitude, that can be raised; that can be optimized with while is doing the parametric study. So, the situation given at our hand, what are those parameters, what are our specific impulse given, what kind of rocket we are using what is the launch altitude, depending on that, we will have some set of already prepared rockets. Now, it is I serve as it is using, it has already the p s l v or the g s l v it is already there.

So, those rockets will be used for launching. So, every time modifying and designing the rockets, it is not possible. So, use the already the built rocket and what is your mass. So, at what altitude, you will be able to put it, and then you need to do the ejections. So, all those things, you can plan in the beginning itself.

So, the next step we will take it for a specific energy required or the specific energy at the burnout, how much it will be. In that case, we will represent the burnout energy as the E b and this can be written as 1 by 2 per unit mass. So, this can be written as 1 by 2 v b square plus g 0 time y b. So, this is indicating the potential energy gain and this is the burnout kinetic energy and this is indicating the total energy per unit mass.

So, at the time of burnout how much, this specific energy will be that need to be calculated. So, you can see that, if you need to put a satellite into certain orbit, So, obviously, at the time of burnout some of the you need this quantity E b to be larger than the 1 by 2 v b terms 1 by 2 v b square term, because this is the energy. Which is going along with the rocket and this is the potential energy to be will be that will begin.

So, if this quantity is equal to this means, this simply the g 0 y b becomes 0. So, this means simply the g 0 y b becomes 0. So, that is we are not looking for exactly, this kind of situation what we are looking for that y b is equal to here, around 250 kilo meters. So, that our problem of putting the satellite in the orbit is solved.

So, we will do instead of doing the plot for this, we will make a plot for E b the specific energy in the same way like E b divided by I s p times square g 0. So, we will make a plot for this in the next lecture, and then see the how the variation is take place and we will see that, we get exactly the same kind of curves for this also, while we plot this on the y axis and r on x axis and this gives you the operating range.

So, these are to be discussed in the next lecture. So, next lecture we will try to make it the last lecture, because we have already given three lectures on the rocket dynamics. So, in the next lecturer, we will do the specific energy study and thereafter the multi staging off the rocket, but in the multi staging of the rocket again, what I was telling about that we can do the optimization. But that process is complex and it requires more number of lectures and obviously, theoretical theoretical theoretically finding out the optimized value it is a difficult. So, we have to make many more compromises means, making more approximation and by making the approximation. We go away from the actual situation.

So, that part we will omit and we will just confine our self multi staging of the rocket and how much will be the altitude achieved or the velocity at the time of the burnout. So, thank you very much, we continue in the next lecture, and that will be the last lecture for this series I hope. So, thank you.