Space Flight Mechanics

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Lecture No. # 43

Propulsion (contd.)

So, we were discussing about the rocket propulsion. So, in the last lecture we are started writing the equation for the oblique ascent. So, we will continue with that in the oblique ascent.

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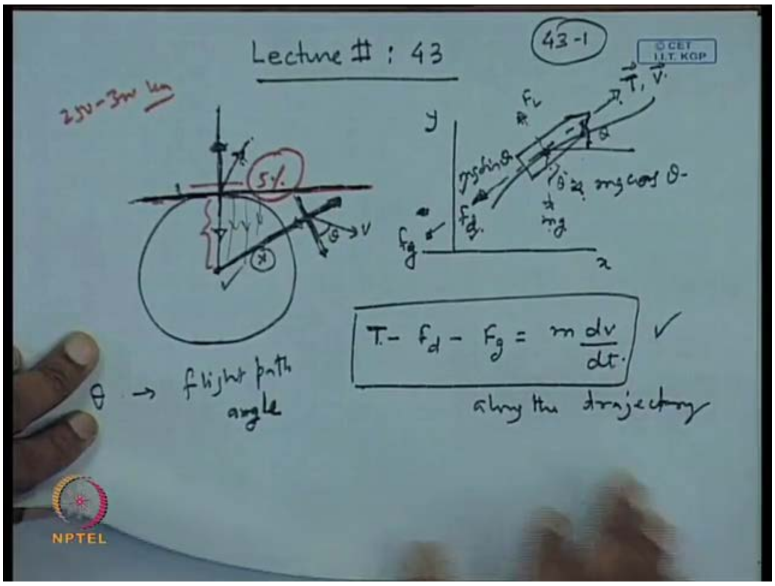

What does it imply that, I have earth here? and if I fire the rocket along the inclined direction from the vertical. So, this will imply that the rocket is under oblique ascent. If, it is going vertically in this direction then, this be called vertical ascent. So, we will write the equation for the oblique ascent, but we will continue with the finally, will continue only with the vertical ascent equation. So, in that context we had x and y reference choosen, and anywhere we depicted the rocket motion along certain trajectory.

So, tangent to the trajectory this along this, we have the thrust and the v vector both are acting here, and in this direction we showed the drag. So, F drag and F gravity also, we showed here in this direction. Gravity will have two components and one will be the drag component, and which is the drag will have two components: The one along the opposite to the velocity vector which we call the drag component, and other one perpendicular to this is called lift component.

And another cause for the gravity, we have if this is m g the weight of the rocket at any time T then, this can be broken into two parts m g cos theta. So, here theta is this angle this angle is theta m g cos theta, and this becomes along this direction we have m g sin theta. So, these are the forces acting on the rocket. So, under this force under this forces the motion of the rocket along the trajectory, we can write as thrust minus F drag minus F gravity is equal to m times d v by d t.

This is along the trajectory and theta here, this act as a flight path angle. So, in our case because as I have told you that, as compared to the radius of the earth. The space craft will be confined into a very narrow region. In which, it is being launched later on it the height may be increased, as in the case of the geo synchronous satellite or the geo stationary satellite. The initially they are launched at the perigee position of say 250 to 300 kilometer, and later on the perigee is boosted. So, perigee will be here, but the apogee may go high.

So, the rocket is really not moving much through the atmospheric (( )) rather, it is confined to a very narrow distance from the surface of the earth, and we showed that this is around 5 percent of the radius of the earth. So, for this case we can assume the flight path angle to be measured, from the horizontal that m showing here. Otherwise in if, you remember for our earlier trajectory transfer or the orbit determination problem we discussed. So, in that case we measured the theta angle, this flight path angle from the local horizontal. Local horizontal implies if I have this line here. So, local horizontal will be perpendicular to this is the radius vector here, going radically outward. So, local horizontal will be this one and the flight if, the velocity vector is this. And, this will be our flight path angle theta.

At here, for a short (( )) we are measuring the flight path angle with respect to this rather. So, we are not discussing about, this rather we are taking this horizontal here, at that place of the launch and from here. We are measuring the theta angle, because it is a confined to a very narrow distance from here So, in that context the theta angle will be measured with respect to this line not, with respect to the different gravitational line moreover. This we are assuming, because now we are assuming to be the gravity vector to be the gravitational field to be parallel.

So, if the satellite is if, the rocket is here in this place. So, we are showing this the gravitational force field, it is directed like this while if, we go in this place says in this place. So, gravitational force field is directed like this, but in a parallel one all of the lines will be parallel directed like this. So, for the short length for short distance, this assumption is also valid. So, we proceed with this and write the equation of motion, but later on we work only further vertical ascent. So, this is the equation for oblique ascent.

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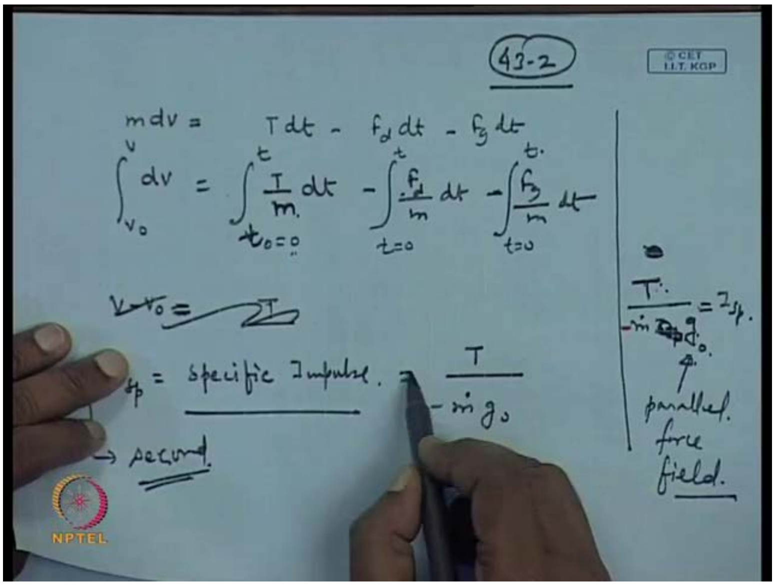

So, we can write d v equal to m d v this will be T d t minus F d d t minus F g d t and d v, becomes T by m and, if we integrate it between v 0 and v. So, T 0 we can assume to start at T equal to 0. So, from here we can write v minus v zero, now this part we have to deal with. So, now we know that T by m dot I s p or you can write it like this, thrust by m dot g and for that we had put a minus sign before this, because m dot this is a negative quantity.

So, minus sign was there. So, thrust divided by m dot g this, we define as a quantity what is called the specific impulse g 0. So, we will write this as let us say, I s p has those specific this is T by minus m dot g 0.

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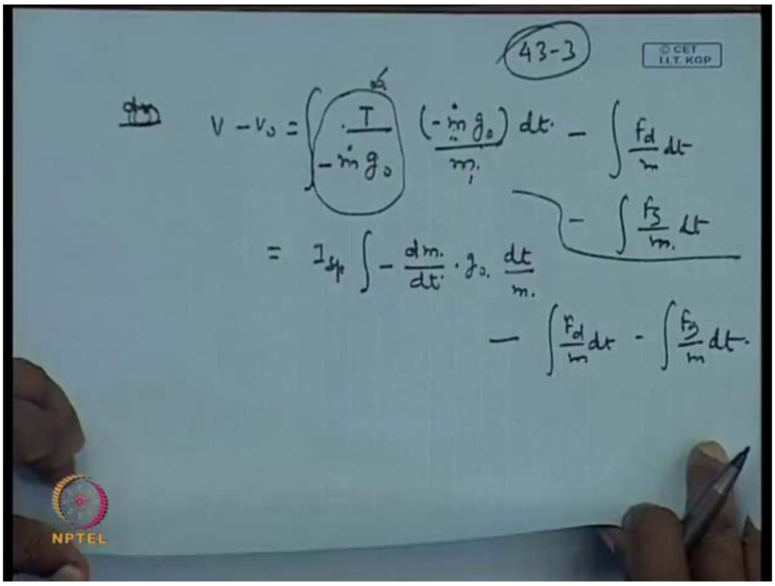

g 0 we have assumed according to the assumption of parallel force field parallel. So, this is the specific impulse, and it is the unit is as you can check this is in second. So, using the specific impulse we can write this equation of the d v, which v minus v 0 this is equal to T divided by m dot g 0. We have put a minus sign here, and we multiply by m dot g 0 and then, d t here in this place and this is to be integrated.

Similarly, other terms here we have F d by m d t and minus F g by m d t. So, the quantity which is present here, this is we can write as the specific impulse as per earlier. We can take it outside and here, we have the quantity m dot g 0 and another quantity. We have this is the m which is missing here, so we put it here in this place. So, this gives us this we can write as d m by d t and this g 0, this is d t by m and we are putting a minus sign here. So, write the other terms.

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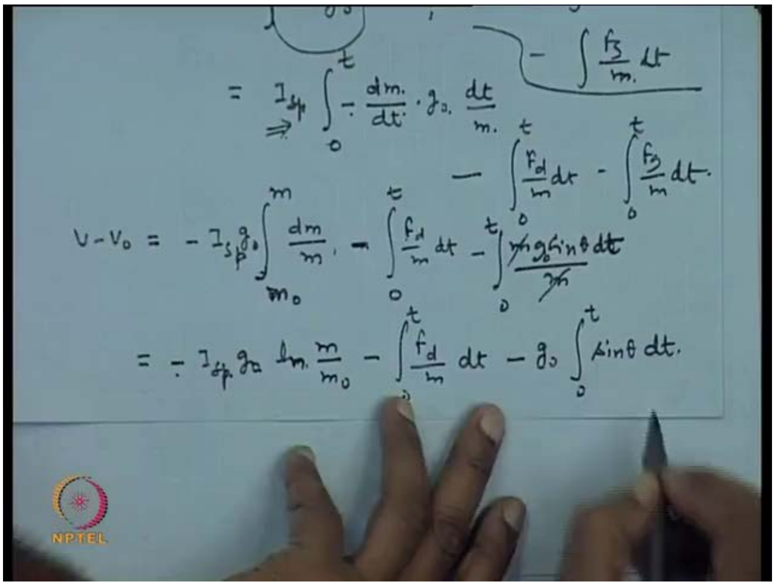

So, integrating between time 0 to T. Here minus sign, we can take it outside and this becomes I s p times, this can be written as then and g 0 also, we can take outside because this is a constant I s p is also a constant for a particular rocket, this is a specific impulse. So, the quantity inside the integrant this becomes d m by m, and now we can integrate it between m 0 to m.

And, F g obviously, we have written as m times g sin theta g d t, and this is divided by m this is between 0 to T I s p g zero integrating, this gives us l n m by m 0 here, minus sign is there F d by m. So similarly, we can write for F d also, and here m m cancels out and g g we are assuming as this value as g 0. So, g 0 we can take it outside and write this as sin theta d t.

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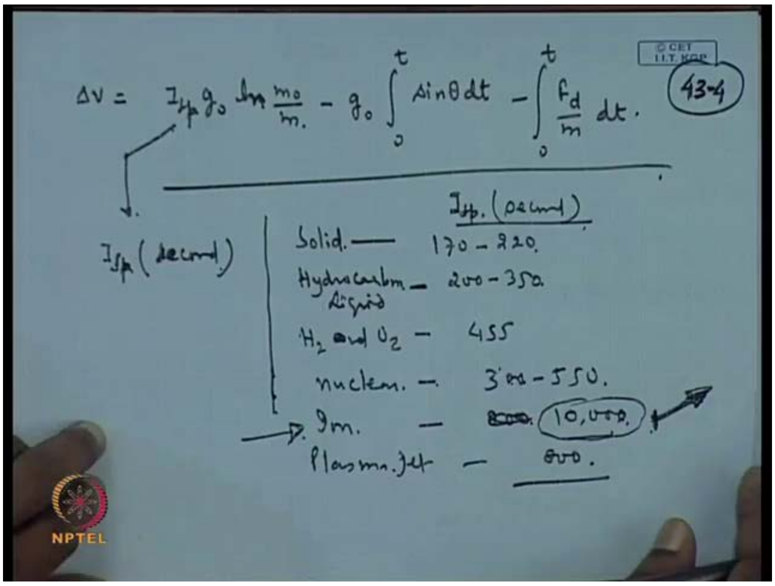

So, delta v we can write as I s p g 0 l n, now minus sign is observed here. So, we can write m zero divided by m, now in this equation. The We will discuss first about this I s p, which is the specific impulse its unit is in second. So, for different kind of propellants it is a different value or different ranges are there.

So, for solid for solid rockets, we can have this range from 170 seconds to 220 seconds for hydrocarbon liquid based rocket. We can have this value from 200 to 350 for h 2 and O two, this value can be this is in seconds this value can be somewhere around 455 then, we have the nuclear (( )) somewhere from 300 to 550 then, ion rockets are here. So, these are based on the ejections of the ion and let us say, some eight hundred seconds and then we have for ion. We have ten ten thousand seconds ten thousand seconds, and we can have the plasma jet plasma jet this is around eight hundred seconds.

So, for ion we have ten thousand seconds, but later on we will see that. So, it appears from here, that using the ion rockets is a very advantageous writing the equation as, we have written in this place and we further work on this. So, in that context we will see that, this there are limits just having the very high specific impulse does not solve our whole problem. The problem is little more complicated. So, details of this we are not going to discuss, solid one I have already told. You that the solid rockets are based on some solid propellants are used and there mixed, with some glue and fill into the rocket and fired by firing in charge other ones are little more complicated.

And, they have their own mechanism here. We are mostly concerned with the dynamics of this rocket not with the chemistry of the rocket. So, we avoid that.

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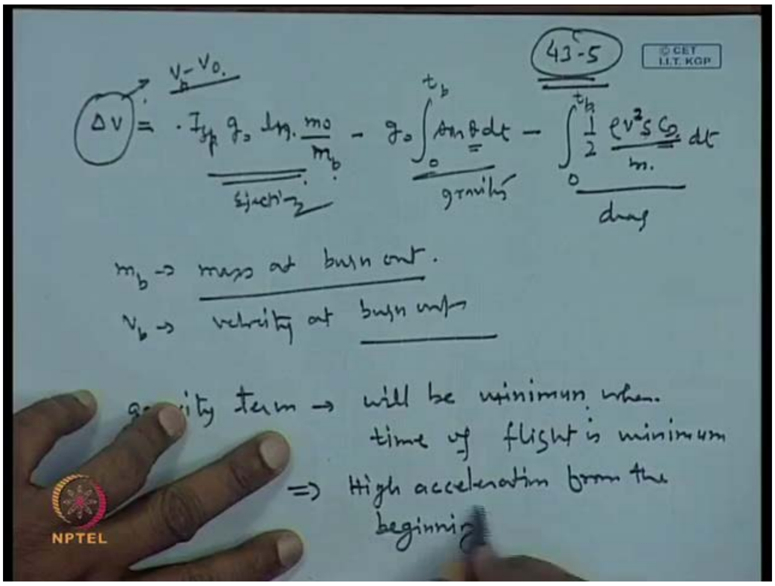

And, the drag force you can write as one by two v square s times C D. So, this is the drag force basically this, you know from the aerodynamics one by two rho v square. This is the dynamic phase; this is the cross section area. So, that makes it force from what this multiplied by C D of coefficient, and then we divide it by m here. And, this is our drag term. So, this is your drag term gravity term and this is due to the ejection of the mass.

Now in this format, you know that, what is our objective our objective is to get, this delta v sorry delta v maximum. So, we can consider these 2 v at the burn out. So, we can write here, v v minus v 0 indicating, the time of the burn out to t b and accordingly mass of the rocket at the time of burn out, we can show it as m b. So, m b is the mass at burn out when all the propellants are burnt, and v b is the velocity at burn out.

Now, this v v to be are delta, v v to be maximum, what is recorded that these two quantities should be minimum, because they are subtracting from a positive quantity which is present here. So, the minimum value of this, the larger will be change in velocity. This quantity will be minimum as we can see from here, if theta is equal to 0, or either the time of flight zero to t b this becomes small.

So, the gravity term gravity term will be minimum, when time of flight is minimum. And, this implies high acceleration from the beginning give high acceleration from the beginning. So, your time of flight will be less or either theta zero to be theta to be zero. So, if theta is zero then, this part vanishes, but it is not possible that theta will all the time zero. On the other hand this is the drag term. So, drag term will be depending on the velocity, the higher the velocity. You can see that, the higher the velocity this is the square of the velocity term. So, higher the velocity larger will be this drag term.

So, if we accelerate very fast. So, the velocity is going very high at that time you will find that this term is becoming high. So, the two terms are contradictory this demands that the initial velocity should be very high. So, that it passes through this time of flight becomes small and therefore, this integrant will become small while on the other hand this requires that v remains the small. So, we have to somewhere optimize this, and this can be done numerically, but not theoretically. So, numerically this can be optimized. So, this is out of the scope of this course. And, theta equal to zero what it relates to like, if we are doing the vertical ascent.

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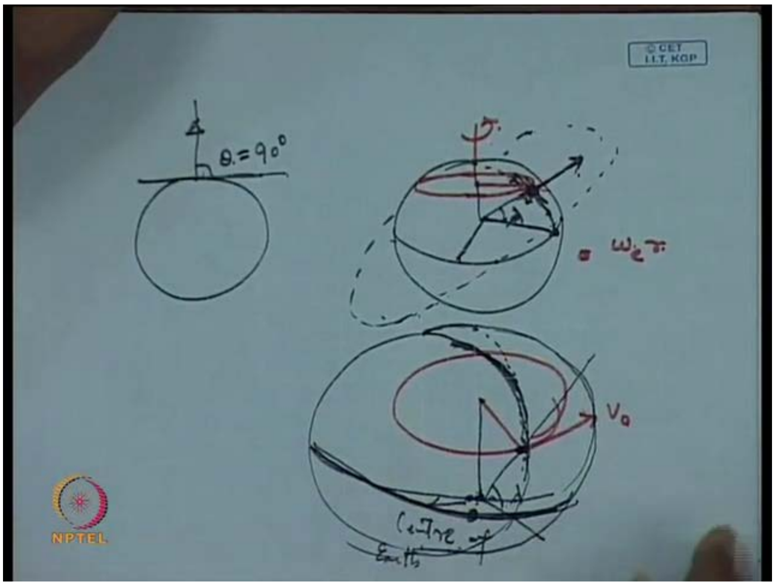

So, here we are measuring the angle from here, the local horizontal this is theta. So, this in the vertical ascent theta equal to ninety degree, besides this you can see that. In the case of the earth, I can have the launcher station at the equator or if our location is away from the equator.

So, our launch equation station will be somewhere away from this, and local vertical will be indicated by this line, and your orbit of injection; obviously, once you are going to inject into the orbit. So, orbit of injection should be inclined, and this inclination is governed by this latitude at, which you are firing the rocket. Now, in this rocket firing, so you can see that, because the earth is rotating about it is own axis. So, here you have your station is also rotating in a circle, and therefore, it imparts certain initial velocity to the satellite in the inertial reference frame.

Just because of the spin or angular motion of the earth, and this you can write as omega, which is the angular velocity of the earth, and times this distance. So, smaller and this will give you the velocity, which is now you can see that if you are here in this place. So, this velocity will be tangential to the circle. So, if you were looking from the top, and your launcher station is here. So, this velocity is basically tangential to the surface, and it is why appearing like, this is v zero while the rocket launch. You are going to do, this is a lambda angle as shown here.

And, this is the centre of the earth this is the centre of earth, and the from the centre of earth, this is the radius vector going outside. So, this is acting as the local vertical, and we can say somewhere the pole is located here. So and this is constituting the surface of the earth. This is the surface of the earth which is (()). So, this is constituting your surface of the earth, and this forms your this is the centre of the earth, and so your equator is somewhere away from this figure is not very good.

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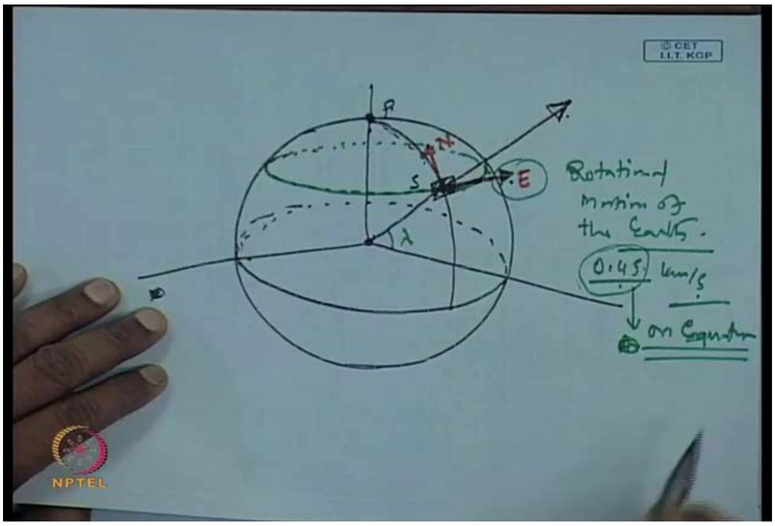

So, we can make another figure here, this is the pole here. So, here locate the station can be located somewhere here in this place. So, this is your station and this is your local vertical assuming, that to be a spherical or otherwise local vertical. It differs little bit, because of the earth. So, we are just ignoring that, now all those things and this is the north south line, and here your east line will appear here in this place. This is your east line; this is the north line; this line and this is the north line.

Because the launching of the rocket, you do along this. This is the local vertical while the initial velocity is also present, because of the rotation of the earth on it is own axis. You can see that, in the east direction, which is ta your velocity initial velocity may be present will be present and that comes because of the rotational motion of the. And if, you compute it it will come around 0 point 4 5 kilometer per second.

0 point 4 5 this will be further around the equator on the equator on equator. So, if you have the latitude present here. So, this value will differ as you go towards the pole. So, at this place you can see that, there is no velocity present here. So, as you are quite close as you become close to the equator, this will keep going down and at exactly at the pole. This will be equal to 0. So, this value I have written here, on the equator this is not the exact value, but an approximately this is correct.

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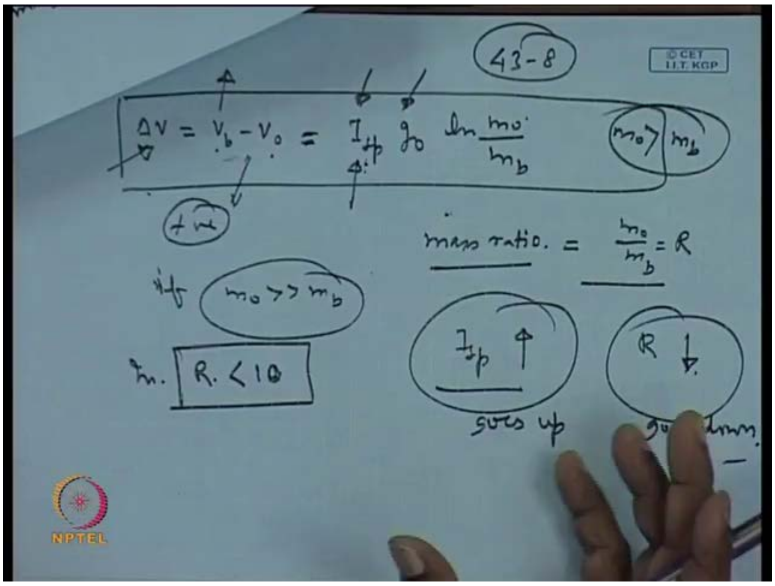

So, we are talking about optimizing these terms. So, this can be they are contradicted each other and they must be numerically sorted out.

So, will we are not going into the numerical sorting of this rather, we can assume that as the rocket vertically ascents. So, for the atmosphere it becomes rear and rear, because the atmosphere is close to the earth. It is a dense and thereafter the density of the atmosphere it exponentially decays. So, this term can be taken care of ad hockly. So, this term we can delete it, and we can concentrate with only with this term and this term we obviously, see that as we ascent. So, for shorter distance the gravity difference is not much, but still for the vertical ascent at least. We can assume this to be theta to be ninety degree and they can work.

So, taking only the first term which is due to the mass, this is v 0 minus v b or v b minus v zero. We have written, because on the and we had a negative sign present with this. So, which we have already observed inside this, so it does not matter v b minus v 0 this quantity is I s p g 0 l n m 0 by m b. For, the time being just forget the other terms and look into this m zero by m b, this we write as r and this is called the mass ration. So, v b is larger at the time of burn out v b will be larger, and v 0 is the initial velocity.

So, once we have start with this value, and we can see from this place that I s p. Which is a positive quantity, g 0 is a positive quantity, m 0 is greater than m b. So, this also becomes a positive quantity. So therefore, v b minus v 0 this is going to be positive. Now, this is that if m 0 is much larger than m b. So, your dealt wave will be larger. So, in general this r, this will be confined to ten means, this mass ratio will be less than ten.

Now, the another thing that, you can control this is the I s p if, the I s p is larger. So, you can see that again delta v b will be large. So, as in the case of the ion rockets, we wrote that the value of the specific impulse is ten thousand seconds. So, that does not mean that your if you use the ion rockets, your delta v b will be very large, because in this case what happens this one increases.

So, as the specific impulse increases I s p goes up r goes down. This goes up this goes down reason is very simple that, the difference between m 0 and m b is small in the case of the ion rockets. The small mass ejection only takes place. So therefore, this value goes down while this value is going up. So, in some way or the other both hot nodes satisfy simultaneously.

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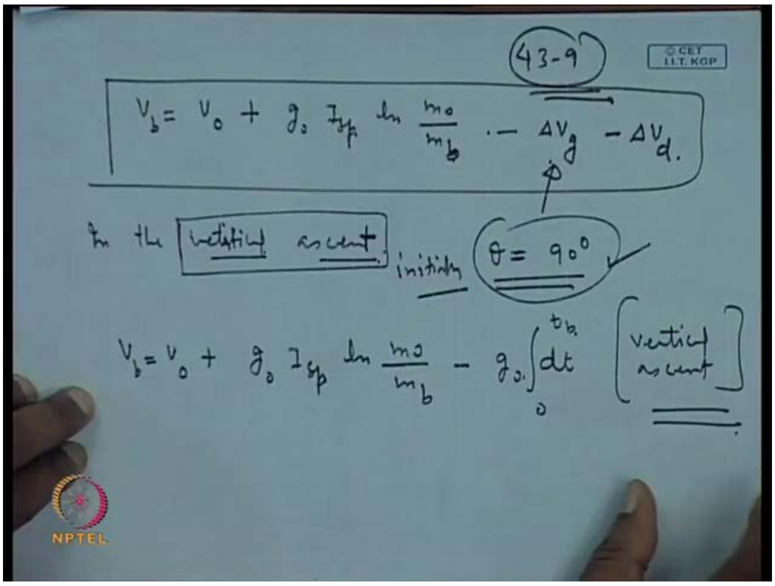

So, somewhere again the compromise is required and v b, we can write as v zero plus g zero I s p l m m zero by m b and other terms. Which are here, we have written as delta v and gravity and this is the correction required due to gravity. And, delta v due to drag and this correction terms are nothing, but the terms present here. So, these are the correction terms required, this is due to the drag and this is due to the gravity. So, further gravity term at least for this gravity term, we saw that if theta is 0 theta remains zero then, this term will vanish or either the time of flight should be small. Where theta is the flight path angle that, we have already stated.

So, once we go for the vertical ascent. So, in the vertical ascent we have the theta the quantity, this is equal to ninety degree. While in oblique ascent, this is not equal to ninety degree initially. So, in the vertical ascent initially theta is equal to ninety degree, and if we assume that the rocket is just vertically ascending throughout. So, we can assume this quantity to be ninety degree throughout.

So, in that case just by removing the drag term, we can write this as s p l m m 0 by m b minus g 0 d t 0 to t b, this is for vertical ascent. So, this is not the real situation as, we are writing here this is just to compute that, how much velocity is possible if the rocket ascents vertically.

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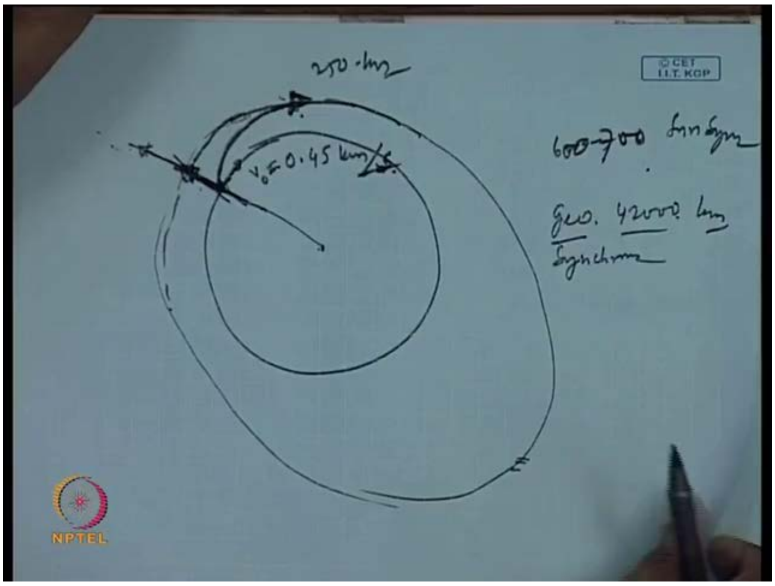

So, as you can see this is the earth, and somewhere near the earth, you are going to inject the satellite in this orbit. So, if you are starting from this station. So, ultimately your velocity vector, you are a starting vertically in this direction. So, this will call as the vertical ascent if it ascents throughout vertically, but this has to be bent to inch to inject the satellite. This has to be tilted to eject, the satellite into the orbit as the orbit is this. So, we cannot fire like this and leave it.

This will not put the satellite in the required orbit. So, orbit is this, in which we need to put the satellite. So, the in reality rocket it ascents vertically, and after ascending it is started, it is rotated. So, the orbit it is a trajectory is tilted and slowly it is a brought closer to the injection point, and at that time. It is almost tangential to the injection point, there itself the rocket the satellite is injected and thereafter it will follow its own trajectory.

So, it will follow electrical trajectory in the beginning, and if it is polar orbit say 700, 600 to 700 kilometers altitude, and this is some synchronous orbit, let us say some synchronous orbit. So, then you need to boost up from this 250 kilometers to 700 kilometers, so perigee boosting is required and if there is difference in the apogee.

So, apogee boosting is also will be required unlike this for geostationary satellite. You need a lot of boosting, because here this is 250 kilometers. While the radius of the geo synchronous orbit is around 42000 kilometers geo synchronous, because so in that case also the boosting is required.

So, here our objective is just to calculate the velocity of the rocket. If, it is just sending vertically like this, we are not trying to tilt or do any other thing. So, rocket already we have seen that, if the rocket is at the equator. So, it will have a component of v 0, which is horizontal and this value is around. I told it is a four point four five kilometer around kilometer per second, but this does not add to the vertical velocity.

So, while we compute for the vertical velocity, we can assume that the vertical velocity at this point is 0. And, then it is a sending directly up .So, the orbit just a trajectory tilt is required to inject, the satellite into the orbit without this the satellite injection is not possible.

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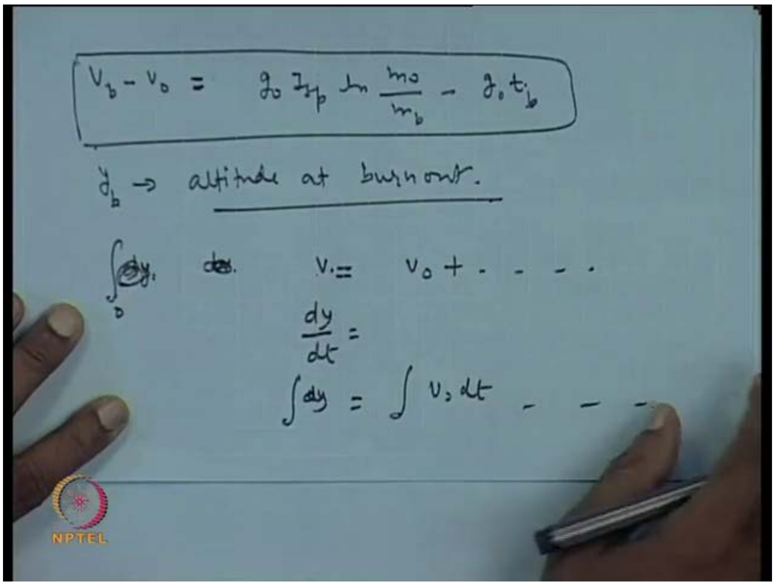

Now, we go for the vertical ascent performance of the rocket in vertical ascent .So, from the previous equation that, we had written after removing the drag term and for the vertical ascent us already. We have put theta is equal to ninety degree throughout, and therefore, sin theta is one. So, the sin theta has been taken out of the bracket by putting v equal to one, and we have this quantity.

Now, v b minus v 0 then can be written as g 0 I s p l n m 0 by m b minus g 0 t b. So, once we integrate it to once more. So, we get the y b the altitude at the burn out.

So, y b this is the altitude at burn out. So, this d v by d t we integrate this v is given to us. So, v equal to v 0 and plus other terms are there. So, we just, we will write as d y by d t and do the integration. So, d y we can integrate it, in this way v 0 d t and plus other terms.

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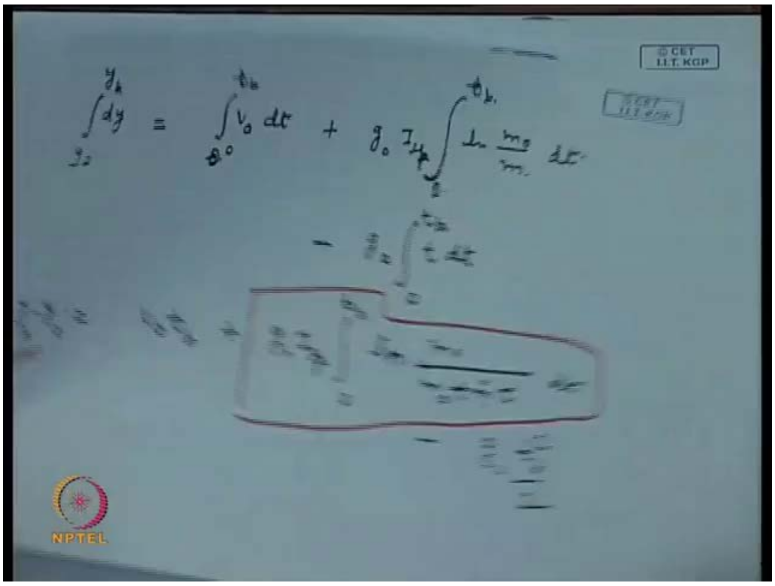

So, writing this d y from y 0 to y, and we can put a b to indicate. This is the burn out altitude this will be v 0 from 0 to t v and plus g 0 I s p l n m 0 by m. So, while we are doing the integration. So, in that case m b we are replacing just with them, because m b is the at the termination or when the final burn has taken place. So, here we will put t equal to zero and up t v, and the last term remaining with us is g 0 t. So, this becomes g zero T d t to t b

So, this equation we have taken here, and just written g zero I s p l n m zero by m b minus g 0 t. So, this term we are removing; the b we are removing to represent a general condition and then, we are integrating, and this is a simple. We just write this equation as y b minus y 0 this is equal to v 0 t b plus g 0 I s p, now this term requires little more manipulation. Here, we can write this as m 0 m, we can write as m 0 plus m dot t this is the mass at burn out m dot. We know already that, this quantity is negative for a rocket g 0 t square b by 2.

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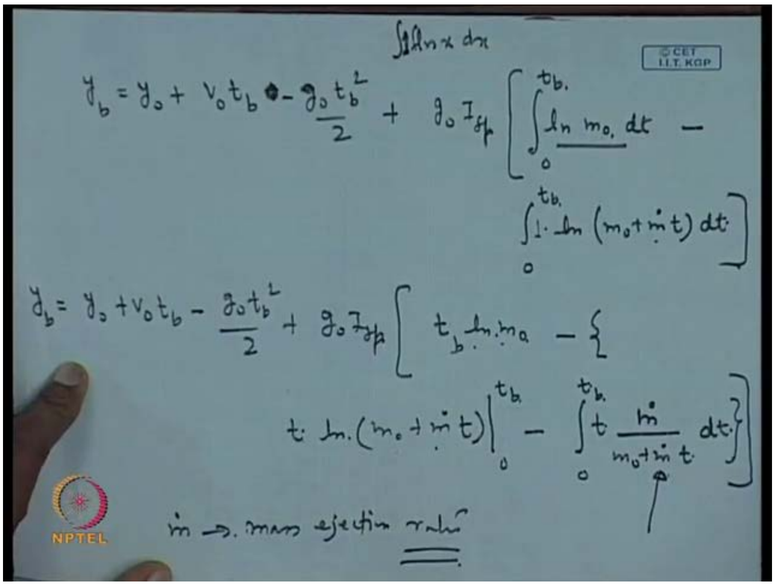

So, we have to further work out this term, and this will give us the altitude at the burn out. Then, we write here g 0 t b square divided by 2 this term, we need to first break it out into two portions. This is l n m 0 d t 0 to t b minus 0 to t b l n m 0 plus m dot T d t. This can be integrated as keeping you know, that this kind of integration. How do, we do l n x d x if we have to integrate it. So, we assume that, there is one is present and this is multiplied by l n x and then we integrate it. So, following the same notation same procedure. We can integrate and write here as l n; obviously, here in this case l n m 0 this is a constant.

So, we need not worry this will be simply t b times l n m 0. This can be taken out of the bracket, but this is the general procedure that we will apply to this particular integrant. So, t b times l n m 0 minus t times and differentiating next time, we are assuming here to one to be present, so integrated. So, this gives you t and this is l n m 0 plus m dot t, and this is to be evaluated between 0 and t b. We will put inside the curly bracket here, and then minus 0 to t b. This is the t term appears here differential of this term.

So, differential of this term will be 1 by m 0 plus m dot t, and once we differentiate this term. So, here m dot will appear and this is d t. And this is inside the curly bracket, and then the bigger bracket closed. Now, you should remember that, m 0 m dot. This is a mass ejection rate mass ejection, and this is basically acts as a constant for a rocket this is also a constant

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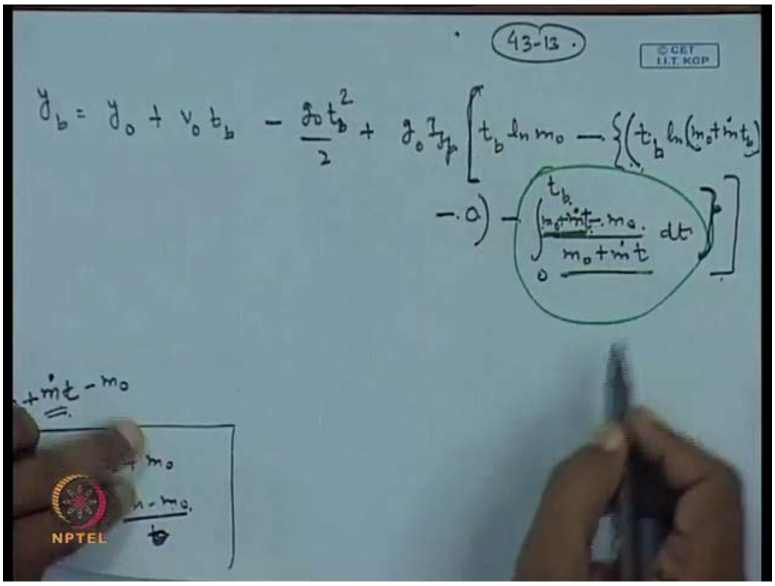

So therefore, this is m 0 plus m dot t b minus 0, there is small bracket closed this, we are working for this part. So, once we put for t b. So, t b tangent 0 m dot t b, and for 0 this part becomes 0. So, we put here 0, and then again which minus sign we have 0 to t b. Now, m dot m is nothing, but m dot t plus m 0.

So, m dot can be written as m minus m 0 divided by t or either, you can say that m dot t is nothing, but m minus m 0. So, in this equation this m dot t is present. So, we can replace this in terms of m minus m 0. So, this becomes m minus m 0 divided by m 0 plus m dot t, or another way of working with the same thing is they say m dot t. We cannot write as m 0 plus m dot t minus m 0. So, you can see that m 0 m 0 cancels out after living out m dot t. So, if we follow this notation. So, in that case this can be written as m dot t, and here m 0 plus m dot t minus m 0.

So, this two are cancelling out living us with the original term, which was present here. So, this notation is little more comfortable. And here, you can see that this term and this term the term this term and this term they will cancel out. So, we can divide it, and do little more simplification. So, we will carry this term separately this is inside the curly bracket, and then the capital bracket closed.

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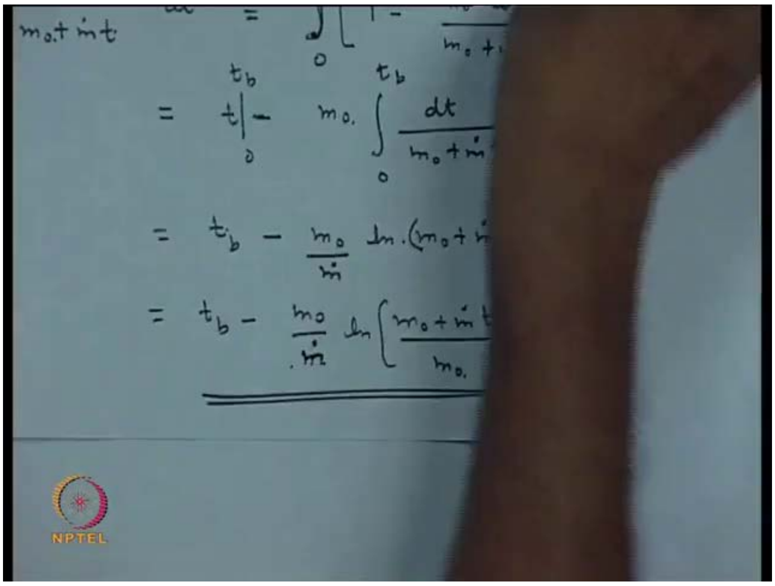

So, work out this term separately zero to t b m 0 plus m dot t minus m 0. So, this gives us t minus 0 to t b, this will come t b minus m 0 l n m dot, and this evaluated time t b. So, it is a just in a very simple format, we can write it as. So, we have got the integration of this term.

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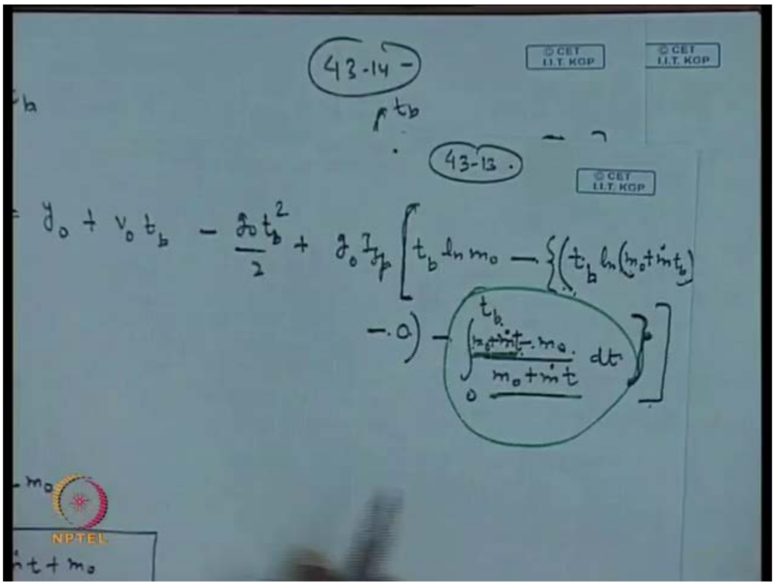

So, next time is getting over. So, next time we will insert in the original equation and work out rest of the expression. So, this will give us the altitude at altitude rate at the final burn out thank you very much, we will continue in the next lecture.