Space Flight Mechanics

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Module No. # 01

Lecture No. # 42

Propulsion (Contd.)

So, today we will start with the propulsion topic in this. Still now, we have gone through the attitude dynamics. So, for attitude dynamics or even for the or vital maneuver, we require the propulsion part. So, in the earlier lecture while doing the trajectory transfer, we looked into the propulsion part, but it was very rudimentary. So, we go into the little more details, and see if how the rocket it works.

So, our main objective here will be to put the satellite in the orbit. So, for putting the satellite into the orbit, we need to find out. How much the burn out velocity will be there, and at what altitude with the given propulsion our rocket system can take? So, we do an elementary analysis it is not a very deep analysis, because we have a very limited number of lectures.

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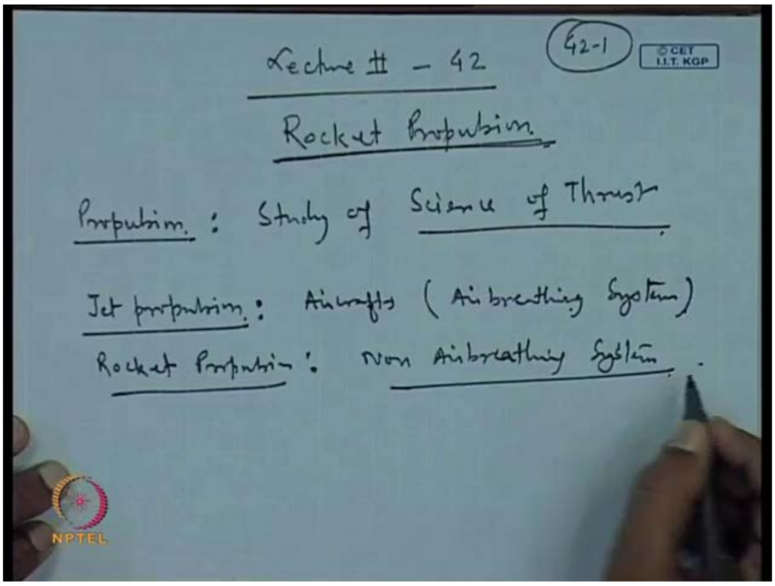

So, in the propulsion we define this as the study of science of thrust. So, we can have various types of propulsion like we can say the jet propulsion, which is nowadays used on aircrafts, and this is basically air breathing air breathing system. While on the other hand the rocket propulsion, this is non air breathing system. Still now, but scientists are trying for air breathing rocket propulsion also.

In which as the rocket, goes through the atmosphere. So, it will collect the air, and it will dissociate or the air contains both the hydrogen and the oxygen. So, mostly the oxygen part is larger. So, the oxygen part will be taken out from the air, and then the hydrogen will be stored in the cylinder in the rocket it, in the beginning itself at the time of launch.

So, using these two; using the hydrogen and the oxygen, so the bond can take place, and after bonding the hydrogen in a regulated manner. So, the propulsion can be achieved. So, that type of rocket propulsion, it will be called the air breathing rocket propulsion. So, but till now in the rocket propulsion it is not air breathing, because whatever the oxygen or hydrogen is required for the propulsion part.

So, it is already stored in the cylinders inside the rocket and it is used, and it is a called basically the cryogenic part, because the hydrogen and oxygen both are to be stored at low temperature. So, it is also called the cryogenic stage, and this is the last stage for used for inserting the satellite into the orbit.

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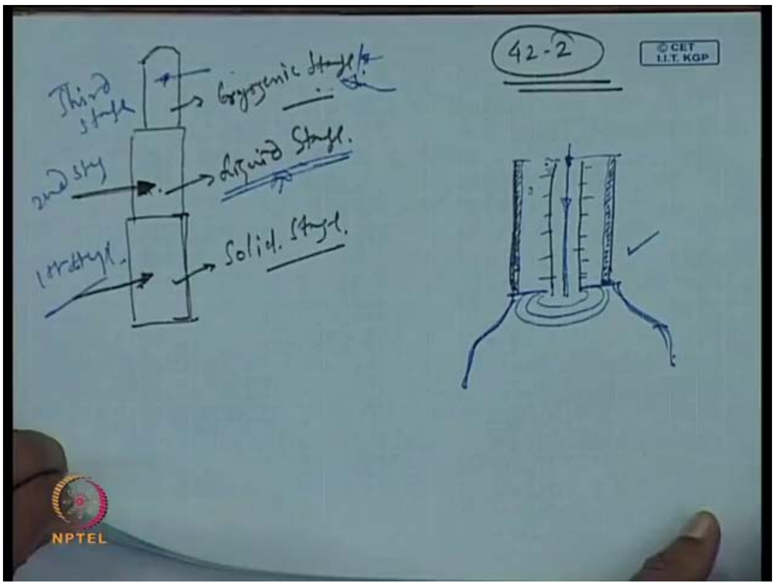

So, a rocket can have a number of stages. So, in general we will see that, three stages may be applied. So, this stage may be the solid stage or the solid rocket and this is the liquid stage and this is this the cryogenic stage. In the liquid stage some different materials can be used, some hydro carbon or other materials can be available. While in this case the solid propellant means; in the rocket the solid propellants are first mixed with some blue, and then it is packed inside the rocket.

So, you will have this kind of a structure. There will be lining inside the rocket this is the old lining. And, which is sent rated and this will basically used for to prevent any gate loss. You can have nozzle here in this place. Then a charge is fired from this place. So, as the charge is fired. So, this charge will go from here to here, and then the burning takes place radically throughout the rocket. This should be unique form, and at most care is taken to device this solid rocket. So, a server has mastered this technology, it is a given a small word should not be present in inside this solid material solid propellant.

Otherwise, this may cause different behavior of the rocket means the propulsion will not this bonding will not be uniform, so as the time progresses. So, these diameters will keep increasing, it will increasing the bonding takes place radically still, it still the whole all the propellants are bond. So, it is a packed inside and it is dried and kept in a safe place.

While, in the liquid stage, the liquid things this also we can keep a in a packed stage, but this one is at the time of launch only, this is the loaded on the rocket. So, this is your third stage, see this is the second stage and this is the first stage. So, cryogenic material which is the hydrogen and the oxygen, they are loaded or pumped in to the cylinder at the time of the launch, before the launch itself.

And then, the launch is done. Before this you cannot keep an packet here, because it continuously requires very low temperature to store it, as the temperature changes the pressure inside the cylinder will rise. And, that will be detrimental for the storing part itself. So, low temperature storage is a necessary part, and therefore, the cryogenic stage is ready only at the time of launch, while other things can be kept ready before the launch.

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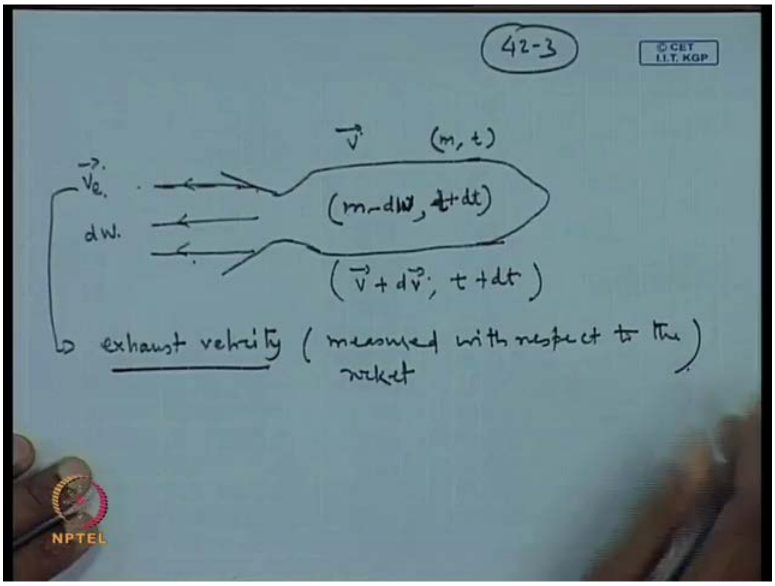

So, first we will go through the singular stage rocket, and thereafter we will go for the multi stage rocket. So, in this rocket propulsion let us, first go through a very simple derivation earlier we have done the derivation, but we did not take care of the sign and other things here, we will take care of all those signs.

So let us say, mass of the rocket at time t it is m t, and after time d t a mass d w or say this is the indicating the some weight. So, d w is ejected and therefore, mass of the rocket changes to m minus d w, and at that time t plus, this is d t this is taking place. Initially, it is velocity was v and after the ejection of the mass its velocity becomes v plus d v, and that happens at time t plus d t. And, the ejection of mass, it is done at the rate of v e. So, this is called exhaust velocity, and this is measured with respect to the rocket.

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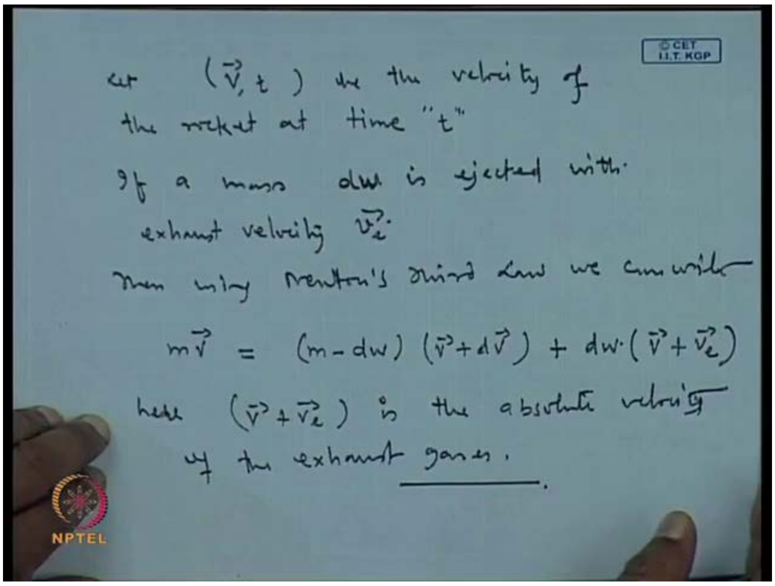

So let, V t be the velocity of the rocket at time t. If, a mass d w is ejected with exhaust velocity v e then, using Newton's third law. We can write m v this is the linear momentum in the beginning at time t. So, this will become m minus d w time v plus d v, this is the change mass, and this is the changed velocity plus the linear momentum of the ejected mass. So, it is velocity will be given by v plus v e. So, to a start with suppose, this is the v, and let us forgets about the direction about, this v either it is here or in this direction or that direction. So, if it is with respect to this rocket, if the velocity of ejection of the mass is v e. So, the inertial velocity of the ejection of mass will be v plus v e is not it.

This rocket is moving with suppose the velocity v, and then a mass is ejected from this, and this starts moving this velocity v e. So, how do we add get the inertial velocity of this mass, we will just add to the velocity of this. So, we have to really react, and we get this.

So, this is v plus v e, we are not taking care of in which direction the mass is being ejected later on. We will put the proper things, and we will get the proper result also. So, if the mass is ejected from this side suppose. So, it is a called retro rocket, if the velocity is here velocity of the say, the satellite is moving in this direction, and if you eject mass from this side. So, this will be called retro rocket. So, this is often used to de orbit the satellite. So, like in the specific of re experiment, which the sire conducted for retro rockets were fired to de orbit it and bring it to the earth.

So, that case the satellite was moving in this direction, and rocket was fired from this direction. So, the exhaust velocity was here in this direction, while in our case in the normal rocket. You will have the exhaust velocity opposite to the direction of motion of the rocket or the satellite. So, this is the difference between the normal rocket, and the retro rocket.

So, here v plus v e is the absolute velocity of the exhaust gases, and we are assuming exhaust velocity is in the same direction as the rocket velocity. So now, we can simplify this. So, we will have m v equal to v break the bracket m v plus m d v minus d w v and minus d w times d v plus d w v v e.

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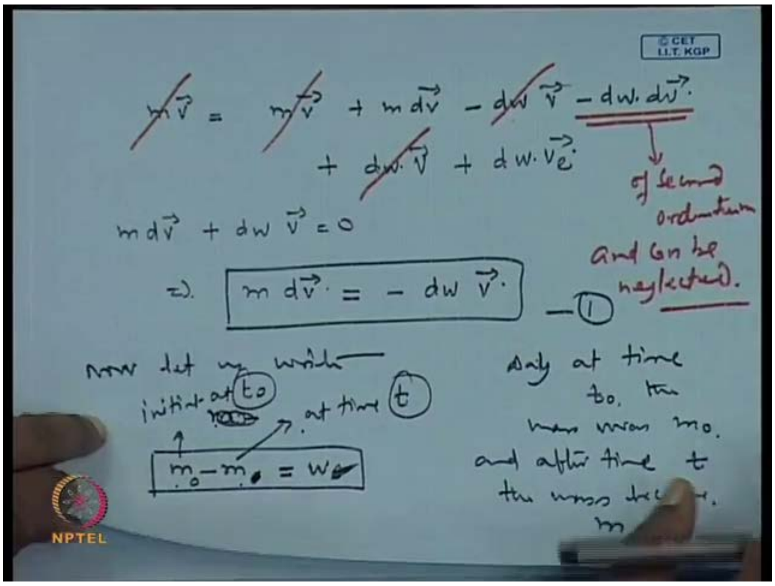

Now, we can see from here this can be cancelled, and d w v is here with minus sign. So, this can be cancelled out this quantity, this is of second order. So, d w is also infinite simply a small, and d v is also infinite simply small. So, both of them together it constitutes second order term, and can be neglected. So, what we get from this place m d v plus d w times v this is equal to zero, and this implies m d v equal to minus d w v.

So, this is our equation number one. Now let us, write we know the mass of the rocket is m. So, if am mass w is ejected from this. So, mass of the rocket will change. So, at time t we assume that, the mass of the rocket is m. So let us, make it little more a specific say at time t 0 the mass was m 0, and after time t the mass became m.

So therefore, m minus m 0 this is indicating the amount of well, which has been bond. this is m 0 minus m this is the initial mass initially at t 0, and this one is at time t. So, m 0 minus m equal to w. Now, differentiate this, now differentiating this we see that m 0 is taken at a particular instant t 0. So, we can consider that, this is the mass in the at the time of the launch. So, this quantity is a constant. So, if we differentiate this that will become equal to 0 and what we get.

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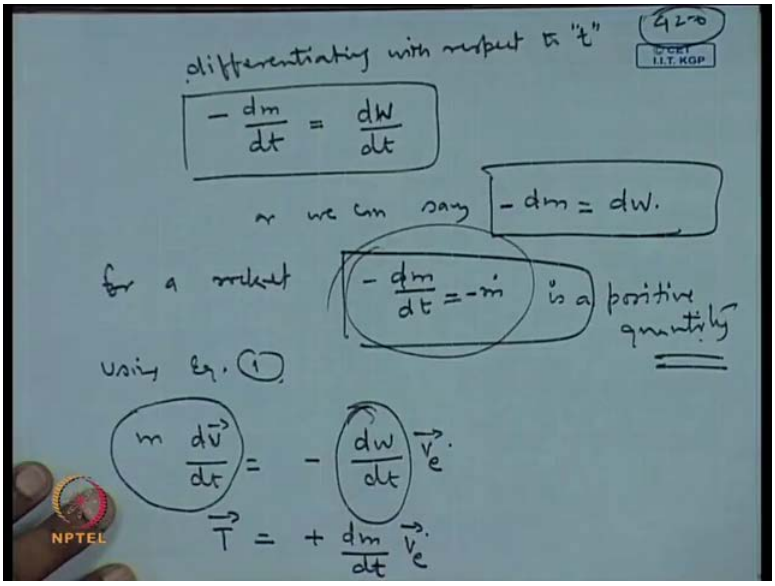

So, differentiating with respect to t differentiating with respect to t, we get minus d m by d t is equal to d w by d t or we can say minus d m this is equal to d w. So, for a rocket minus d m by d t, which we often write as m dot is equal to minus m dot, this is a positive quantity. So, d m by d t, this is for a rocket. This is a positive quantity right hand side therefore, why because d w by d t, this is the rate at which the mass is been ejected. So, the mass of the rocket is continuously decreasing. So, d m by d t, this is a negative quantity and therefore, minus d m by d t this becomes a positive quantity.

Now, we can go back into this equation, we have m d v equal to minus d w v, so using equation one. So, using equation one. Now, we can write m d v by d t this is equal to minus d w by d T v. So, you can see from this place this quantity is nothing, but thrust.

So, we can write thrust equal to minus, and d w by d t we can replace in terms of d w by d t we can write as minus d m by d t.

So, this becomes d m by d t, and this is d w this is v. Quantity we have missed the subscript here, this is v e, which we are carrying it from this place.

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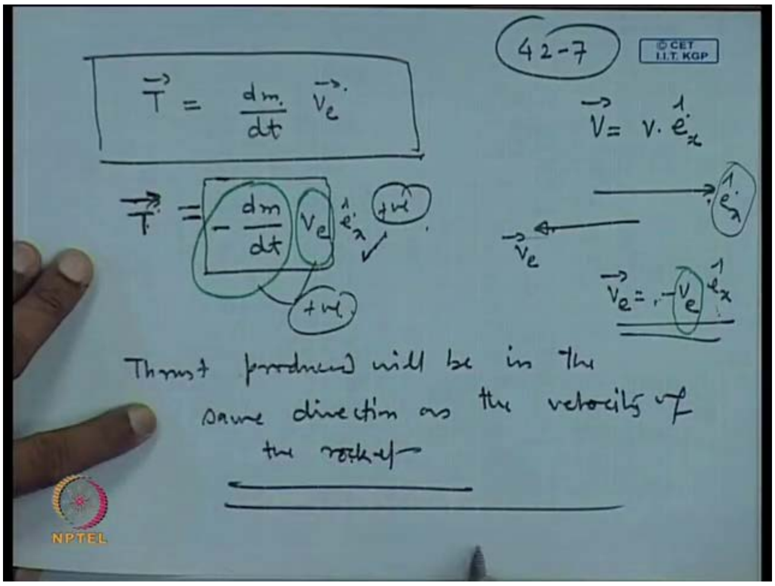

So, this quantity is here v e putting here, the v this is our equation. So now, we get the thrust equation now, we if we write v is equal to suppose v times v cap x. Which is indicating the positive x direction this is the x direction, and if we assume that v e is opposite to this. So, we can write v e equal to minus v e times e x cap, this is in the position direction of this. So, we have the e x cap vector, this is the unit vector in this direction. So, following this notation, we can change it here. And, we can write d m by d t is equal to v e with A E x sign cap here, and we put a sign here in this place.

So, what does it indicate? So, this indicates that thrust will be in the direction of e x why, because the quantity d m by d t. We have already seen this quantity is a positive quantity v e is we have taken the magnitude of this, we have taken outside. And, here the minus sign is appearing, because this is in opposite direction and therefore, this minus sign we have put it. So, this is minus e x cap basically the unit vector in this direction. So, we have done the replacement for that. So, this whole quantity is positive and the e x is already, we have chosen as a positive quantity.

So, what does it say that the thrust produced will be in the same direction as the velocity of the rocket? So, this is opposite to the basically the exhaust velocity. So, systematically we have come to this stage, where we can write t. Now, we can get rid of the this with the vectors here it is the e e x cap.

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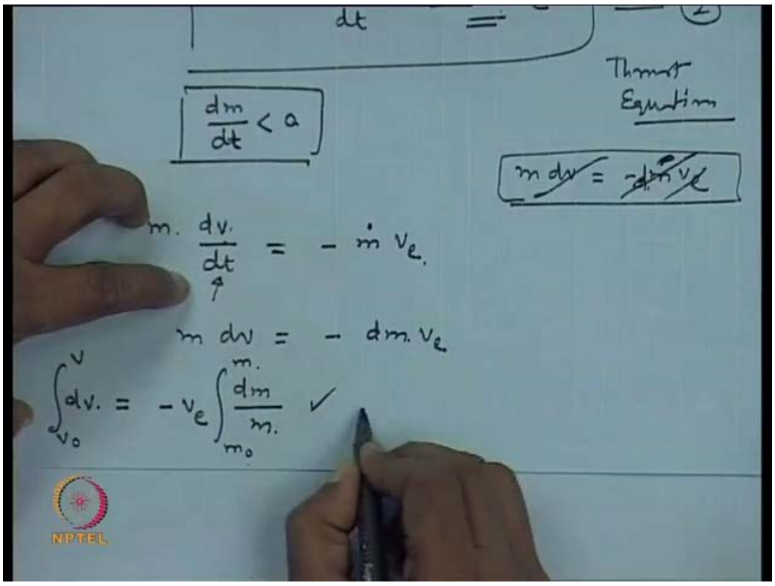

So, we can delete it and here also, we can delete it and simply. We can write T equal to minus d m by d t time is v e. So, this is minus m dot v e where minus m dot this is always positive quantity, and this is the thrust equation. So, this is a very systematic derivation, which gets rids of all the confusions of the sign or any other thing. So, here we have d m by d t this is a quantity, which is negative. Now, if we want to know the velocity of the rocket, after the bonding of some mass. So obviously, we can integrate this equation.

So, we can write it say, we have m times d v by d t thrust equal to m times d v by d t. We have already got rid of the vector sign. So, we can write this as m dot v e. So, using this equation, and or may be the in the elementary form, we derived this as minus m dot times v e here itself not m dot this is d m.

So, we have to get rid of this d t from both the side. So, we will get rid of the d t from both the side. So, we can write m d v equal to minus d m v e. Now, put it in a proper format. So, we can write d m by m times v e with a negative sign, this will be equal to d

v. Now, integrate this between mass m 0 to m, and this from v 0 to v. So, this implies v minus v 0 this will be minus v e time is l n m minus l n m 0.

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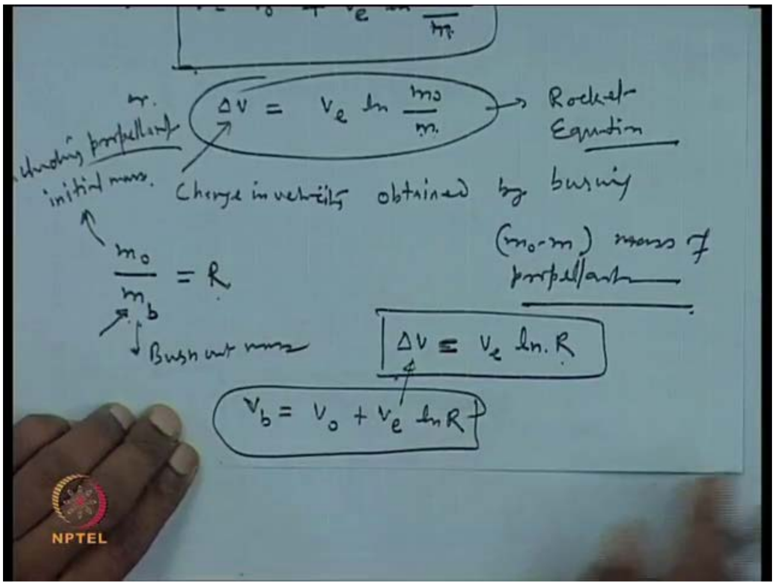

. So, this implies v equal to v 0 plus v e times l n m 0 by m or simply. You can say delta v is equal to v e times l n m 0 by m, this is the change in velocity obtained by burning m 0 minus m mass or propellant, and this is called rocket equation. And, m 0 by m if say m is the m b. So, this is the mass of the rocket, at the time of burn out means all they have been a spin. So, at that time what is the mass of the rocket, we write it as the m b this is called the burn out mass. And, this is the initial mass of the rocket including propellant including propellant, and this often is written as r. So, this is called the mass ratio.

So, this gives you delta v this is equal to v e l n r. So, what it says that larger this mass ratio the larger will be the change in the velocity, or you can say that larger will be the burn out velocity, which you can write as v b equal to v 0 plus v e l n r.

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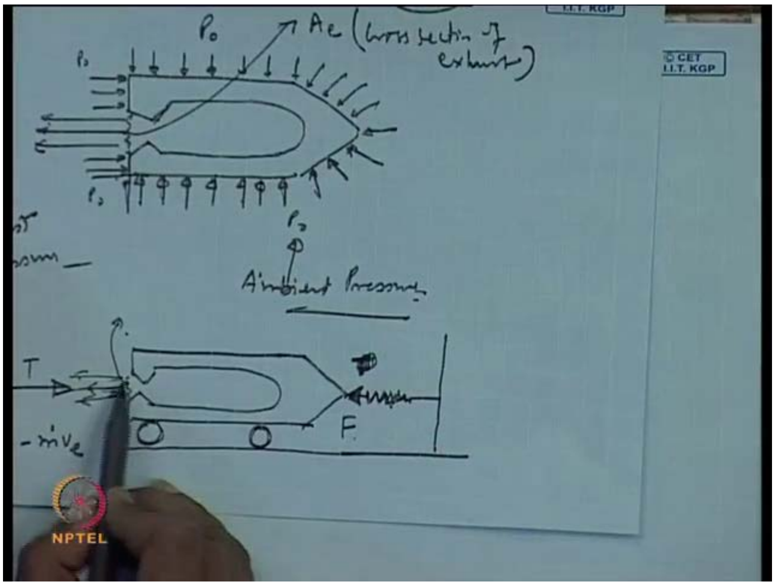

Now, once we have done this. So, we do little more explore exploration of the rocket behavior. So still now, we have just seen that how the rocket is equation is derived. Now, say mu rocket my rocket is moving in a atmosphere.

Once, it is a launched from the surface of the earth. So, it is a basically passing through the atmosphere. So, in the presence of the atmosphere the rocket behavior it changes little bit, because of the ambient pressure. Which is acting from outside this, we can write as p 0 and the. So, we have the here all these are p 0. So, ambient pressures acting over this these are all p 0.

While in the exhaust let us say, here in this portion the exhaust pressure is P E. So, this is your exhaust pressure, and this is your ambient pressure. So, this modifies your rocket equation little bit. So, if you visit any testing site of the rocket, you will see that rocket is mounted on a slag, and then it statistics can be carried out to find out, how much thrust it develops. So, it will be mounted on a slag. So, where this will be restrained by a spring, which will be measuring the thrust developed.

So let us say, that this spring flies the thrust t or t or we can write it as any other notation. We can choose let us say, this is F which is being applied by the spring in this direction opposite to the what, the exhaust thrust will be developed. So, already we know that, there is exhaust thrust equation is given by T equal to minus m dot v e. So, in this figure we can see that, the f is the force being applied from this side.

And, t is the thrust you are the, because of the exhaust which will be applied on the rocket. So, this is t and besides the pressure difference, which will be pushing it on that side. So, the p zero is acting from this side, and you can consider that the cross section of this is A E. So, if we consider this cross section to be A E A E. This is the cross section of exhaust then, the pressure difference also it will curve certain thrust in this direction.

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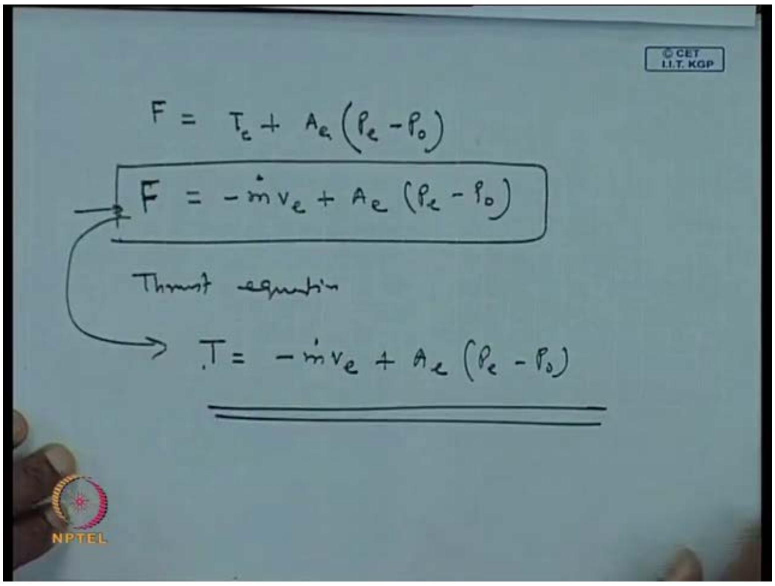

So, combining them together, we can write F equal to F, which is acting from this side, and the all the forces, which are acting from this side. So, this will be T plus A Exhaust, this is the area of this exhaust the cross sectional area cross section area of exhaust area of exhaust A E times. Now, p is the pressure which is acting here, on this the p e the pressure will this. We have shown here, the v e direction this is not the p direction. The p e can be shown by let us say it by we are showing by the green line this is your p e.

So, we can write it as p e minus p 0, so all the forces being balanced. So, t you can replace by m dot v e, and this becomes here. So, this is the thrust that the rocket will be developing, and let us put here, e mark to characterize that this thrust is due to the exhaust. So, we will put it T e to distinguish it from the normal thrust or the total thrust, we will put it here T e. So, this is the force, that the rocket is developing the total force.

So, the thrust equation thrust equation, now it becomes T equal to minus. So, instead of f now, we can write this as t, and this becomes minus m v e plus A E times p e minus p 0.

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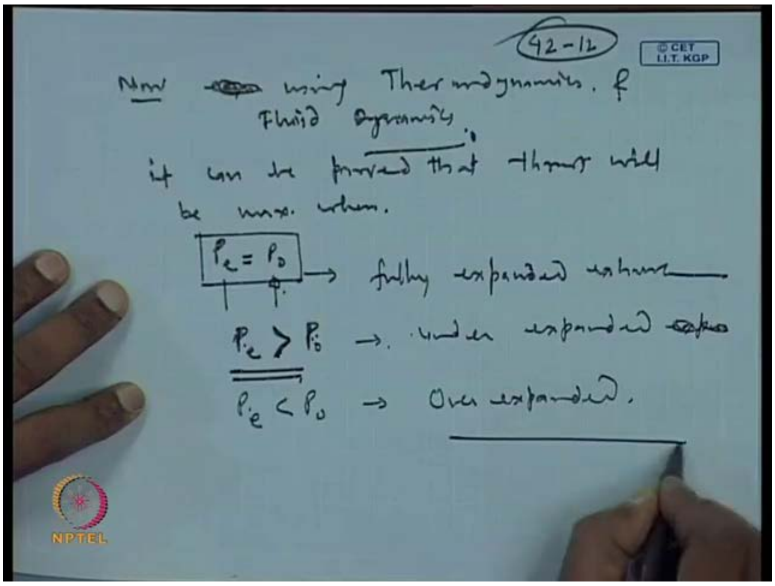

Now, using thermodynamics and fluid dynamics, it can be proved that thrust will be maximum. When p e equal to p zero, that is exhaust pressure equal to the ambient pressure, and this is called fully expanded exhaust. If, p e is greater than p zero then, this becomes your under expanded. So, still it is not fully developed the flow is not fully developed.

The simply it implies. So, you get the maximum thrust for the situation, where p e equal to p not. And, this can be proved thermo using thermodynamics, and the fluid dynamics. So, we are avoiding this, because we do not have enough lectures to include all these details. Now, there can be situation where p e is less than p zero. So, in this situation it will be called over expanded. So, fully expanded or we say we can also say this is correctly expanded and in this situation only we are getting the maximum thrust

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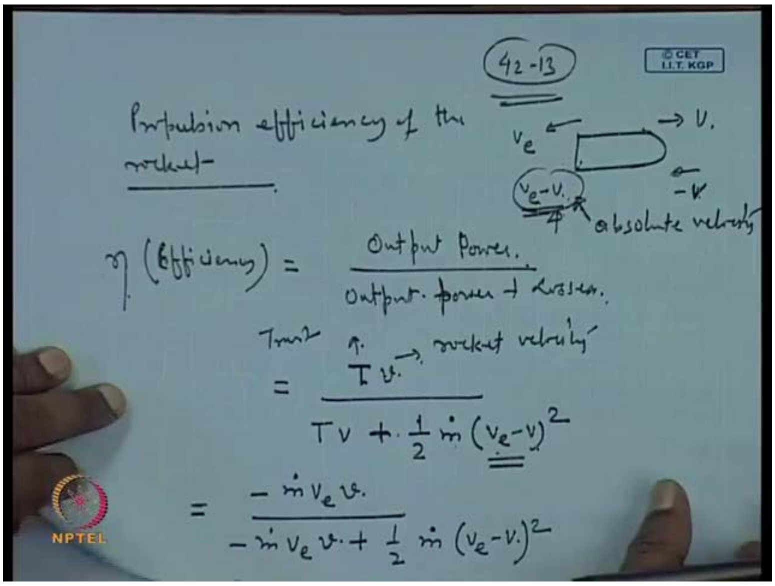

So, another equation in this connection, we can write the propulsion efficiency of the rocket propulsion efficiency of the rocket. So, this we write as, it eta this is the efficiency and this is the ratio of the output power by the total power. So, this is output power total input power, so output power plus losses.

So, output power; obviously, we can write as t time is v. So, this v is the rocket velocity, and t is the thrust developed. So, this is the power output power. So, here also we can write T v, and the losses how do. We compute the losses are basically once the propellant are is skipping. So, it takes away some kinetic energy.

So, we will just now see how the propellant is speed if it is optimum. So, you get the maximum efficiency. So, we can write the propellant loss the kinetic energy loss due to propellant as one by two m dot. So, m dot is appearing, because of the fact that per unit time the mass m is lost, because we are dealing with power not the energy. And, therefore, the time factor must appear and this minus v e minus v square.

Now, here you can see that, the rocket is moving with a speed v and the propellant. You are the propellant is exiting with velocity v e. So, what will be the inertial velocity of the propellant? So, if you apply minus v in this direction. So, the rocket will come to a stop and also.

We impose on this minus v. So, this is v e minus v. So, rocket becomes comes to halt. So, it is in a stationary condition, and then with respect to that how the propellant mass is moving at with what velocity. So, that gives you v e minus v. So, this is the propellant is skip velocity with respect to an inertial reference frame.

So, this is it is an absolute velocity, basically absolute velocity. So, it is a magnitude we are taking. So, v e minus v square and T v already, we have written in the simplified format, because without taking any ambient pressure condition. So, this is m dot times v e minus m dot times v e times v divided by minus m dot v e times v plus one by two m dot v e minus v.

The minus sign, we are inserted here in this place. So, this is for the thrust now, once we are including computing the power. So, we must get rid of this signs, because we are considering the power. How much power is lost the thrust is a vector, but once we are considering the power. So, we get rid of this sign. So, we are putting it a rather than a minus sign, we are making it a plus sign. Now and the m dot is also appearing here in this place.

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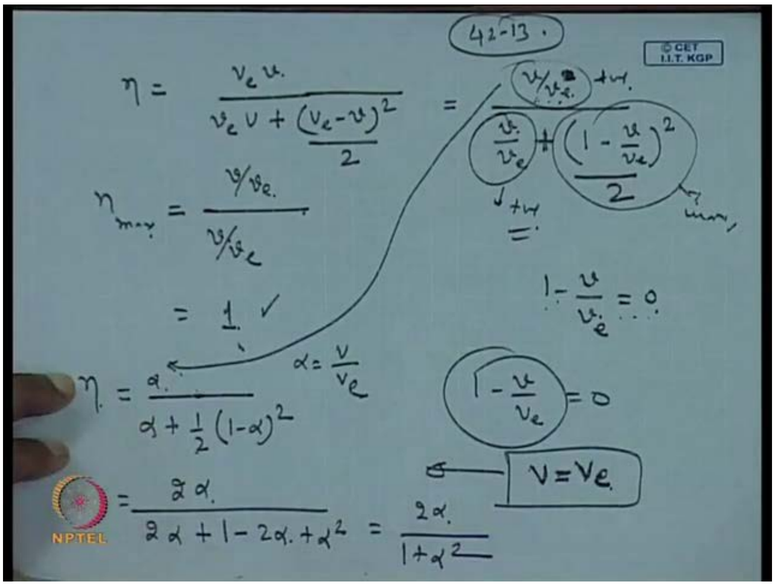

So, m dot from the numerator, and the denominator we can cancel it out. And, we can write v e v divided by v e v plus v e minus v whole square divided by 2.

So, this is the efficiency of the rocket, and if you want you can simplify it little bit more by dividing it by v. So, this becomes v by v e, and here if you divide by or maybe we can divide it by v e square. So, we are dividing by v e square. So, we will put it in this final format like v by v e, and here also you will have v by v e plus one minus v by v e square and this divided by 2. So, we got rid of the minus sign already. So, output power irrespective of the sign, this is the amount of power being produced here, while we write it in this form. So, here we will not put the sign of m dot the anything minus sign will not appear with this similarly, here in this place here also now here the minus sign will appear.

So, this is the output power this is output power, and this is the power loss and from here. You can see that this quantity, this is a positive quantity this quantity this is a positive quantity and efficiency will be maximum this is also this is square quantity. So, this is also a maximum. So, efficiency will be maximum only if, this quantity term becomes 0.

So, for that you need that, one by one minus v by v e should be equal to 0, because this is a positive quantity. And, this is always going to add to this, and only if this equal to 0 then, you are going to get the maximum value of the efficiency. So, this comes to a term max and at that time, you can write mu by mu e as mu by v by v e. So, this becomes equal to 1.

So, in this situation what is exactly happening your loss, which is one minus v by v e. This is signifying the lost term this is becoming 0. So, you can see that, at that time what is happening here, from here v is coming equal to v e as I told you earlier that once v e . The velocity of the rocket is equal to the exhaust velocity then, the efficiency is maximum, and this turns out to be equal to 1.

And, the same thing you can also do using the differential calculus. So, for that you need to write this as let us say, alpha plus alpha and here you can write one by two one minus alpha square, where alpha is equal to v by v e. So, if you do this kind of insertion. So, this becomes two alphas and the numerator. And here you get 2 alpha plus one minus two alpha plus alpha square and this silts you to alpha divided by one plus alpha square.

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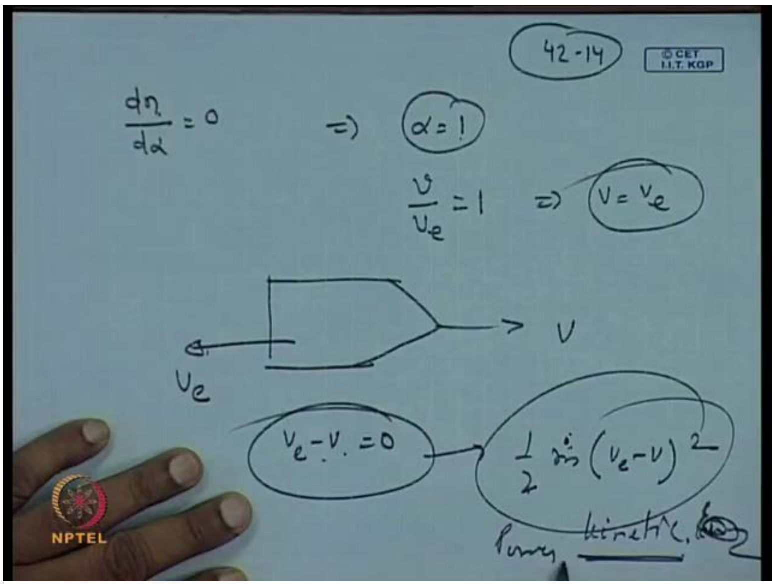

And, for maximum efficiency then, dou eta by dou alpha. You set it to zero differentiate this quantity, and this will imply you alpha equal to one. So, alpha where set as v by v e. So, this is equal to one this implies v equal to v e, which is the same thing just by observation, what we have written here. So, in this situation what is happening that rocket is moving with velocity v in this direction, and exhaust is coming out of this.

So, the absolute velocity of exhaust v e minus v this is becoming zero means. So, this is what is your constitutes, your kinetic energy on y two times m dot times v e minus v square. So, this is your kinetic energy, which is the going as loss kinetic energy or the kinetic power we will write here, because dot term is there time term is there. So, kinetic power. So, this is going as a loss, and this will vanish if an only v e equal to v, and this is what gives you the maximum efficiency.

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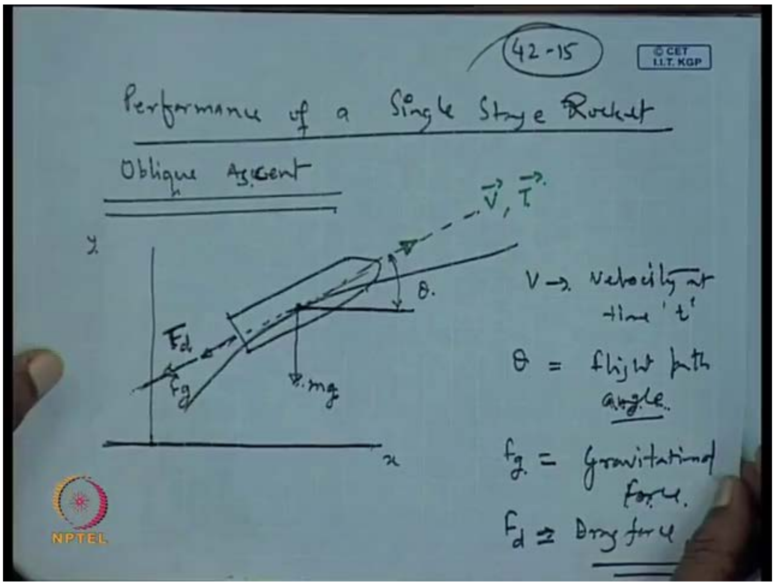

Now, once we have done this. So, we can now develop the equation for rocket performance in single stage. So, single stage rocket performance of a single stage rocket, and we write a general equation that is for oblique ascent say this is our inertial x.

For our convenience we assume that, the velocity vector and the thrust vector both are directed. In this direction eta is the angle that, it mix with the local origin with the launch origin, and we can write this as a F drag, which is acting in this direction the component of this weight acting in this direction. So, this is tangent to the trajectory. So, we can also write here l g.

So, the equation of motion of the rocket or we can write let us say, v is the velocity at time T eta is the f g, this is the gravitational force. So, this is taken along this direction not along this direction. So, we will be computing the motion along the trajectory, and similarly, F d this is the drag force.

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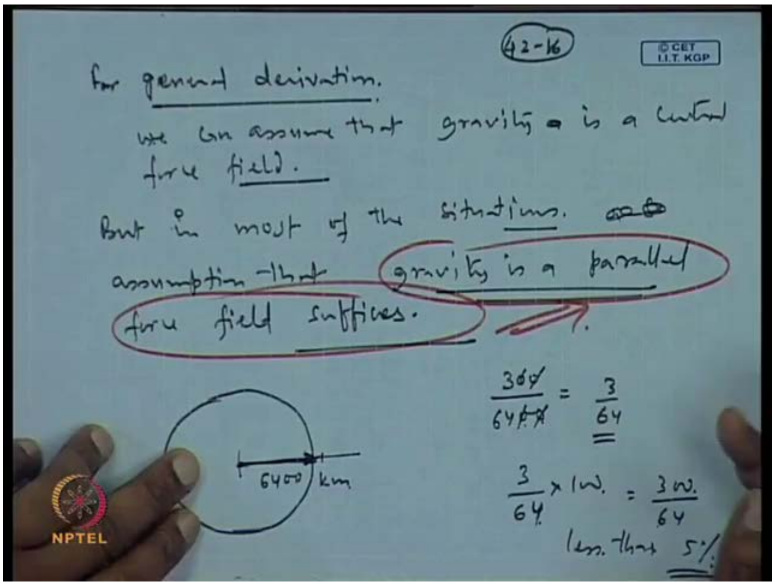

So, for general derivation, we can assume that gravity is a central force field, but in most of the situations assumptions most of the situations or assumption in most of the situation or assumption that assumption that gravity is a parallel force filed suffices. And, reason for this is say your earth, is it is a radius is around 6400 kilometers. While you will be launching, and inserting the satellite at an altitude of 252 to 300 kilometers, so you add an altitude of 250 to 300 kilometers.

So, you can see here let us say, we take 300 kilometers. So, 300 divided by 6400 how much it comes. So, three by 64, and if we write this in terms of percentage, and this is 300 by 64, so less than 5 percent. So if, suppose this is implying the actual, this is the scale. So, this is the actual distance of the radius of the earth from here to here. So, the 5 percent of this, you can assume 5 percent of this will be just looking something like this. It is a very small; it will be just near the surface of the earth. So, for such an altitude you can assume the gravity to be a parallel force field.

And, you do not do much error while you do such kind of assumption. So, this is a very good assumption, and it does not create a very large error that, will totally defy your computation or your whatever, you derive after working out either doing the mathematical computations.

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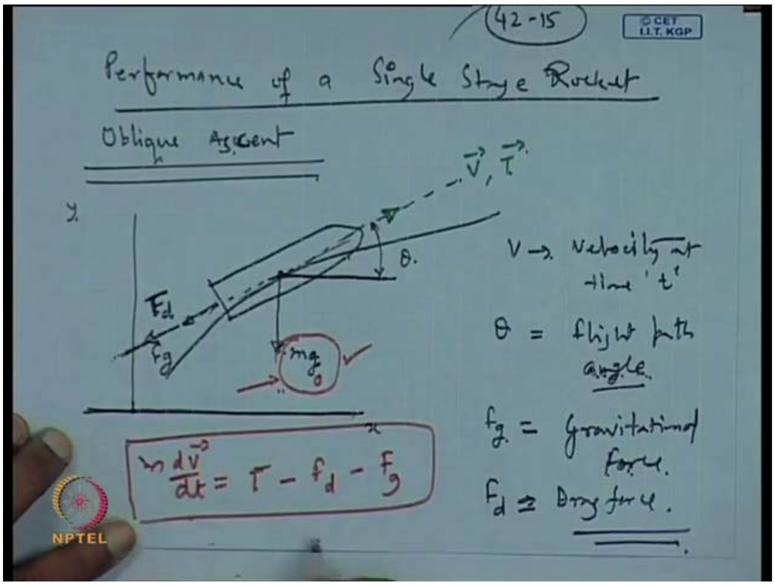

So, proceeding with this assumption that the gravity is remaining constant, so here we will put instead of g. We will put here g 0 to indicate, this is the gravitational acceleration at the surface of the earth. Now, we can see that as the as the rocket goes up. So, this the value of the g, becomes small, but here the distance change is not much. So, that the change will be no appreciable, and we will not be making any mistake or doing a very large mistake by making this assumption.

So, we will assume this and thereafter, we can write the equation of the motion. So, the equation of motion; obviously, you can write as m d v by d t and thrust is acting in this direction minus F drag minus f g. So, this is along the trajectory equation, this is along the trajectory. So, we have start with this equation, and we will continue in the next lecture, and then we will develop the complete equation for the single stage rocket. And, then we will go into the multi stage rocket, and see if how by using very simple equation, we can get a fill and A even the practical applications can be done. So, for the initial computation about, the performance of the rocket this computation will suffice.

And then, we do the sophisticated computation taking into account, the very accurate things. So, that while you are injecting the satellite into the orbit, you do not incur much error, because they are a small velocity change. It means a large change in the orbit situation. So, for that we need that the rocket performs well to the assumed value.

So, whatever we have; we wanted the rocket to perform the same thing the rocket should perform, if it happens and that is the ideal situation. So, thank you very much we look forward for the next lecture.