Space Flight Mechanics

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Module No. 01

Lecture No. 38

Attitude Dynamics (Contd.)

We have been discussing about the attitude dynamics. So, we will continue with that topic. So, then, in the last lecture, we are looking into the Euler angles.

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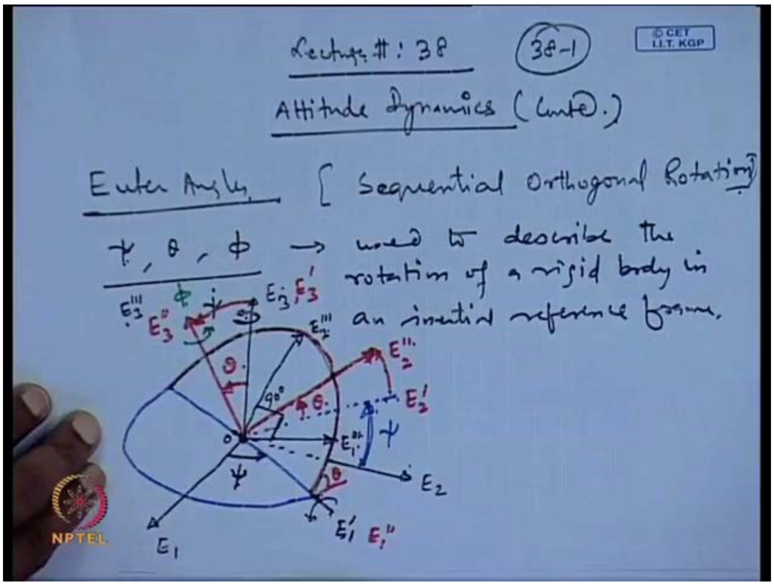

So, in the Euler angles, basically, we had the angles psi, theta and phi, which we had use to describe the rotation in an inertial reference frame. So, in that context, we are looking into this picture. So, the first rotation was given about this E 3 axis and here, we got psi dot and this angle, we wrote as psi. Then, the next rotation was given about this axis and this axis, we can write as E 1 prime. So, after the rotation, the resulting axis was…So, the rotation from here to here, this is theta. Once we are rotating from here to here, so, your E 2 axis will also rotate by the same amount, and this angle, from here to here, this is also psi. So, once you rotate, so, this, this is your E 2 prime, and we have the E 3 prime here, in this place itself. So, once you rotate it, it will rotate up from this place. So, this is rotating and going from here to here. So, it will move from this place to this place, and this, you are rotating by angle theta; you can write here E 2 double prime. Similarly, here, this will be your E 3 double prime and in this place itself, you can write E double prime, E 1 double prime.

Now, the third rotation, we give about the E 3 double prime axis, and that rotation, we give by angle phi. So, if you rotate by angle phi, so, you can see that…Now, I will make it solid; make this line solid, to represent that, this is lying in this, the plane constituted by this blue line and this green line. So, basically, the inclination of the, this plane, which is shown by the green line, this is with the E 1 E 2 plane; this is nothing, but theta. And, maybe we can make it little…This is lying in the same plane. So, this will be better represented like this. Now, once you rotate it by angle phi, so, the next rotation will be, this line will, the line from O to E 1 prime, this will rotate by angle phi. And, this we can show somewhere, let us say, this is your coming to E 1 triple prime, and similarly, this will rotate also by angle phi. And, it will move away from this place to this place, and this E 2 triple prime. And, these are the orthogonal axis. Remember, E 1 prime and E 2 prime and E 3 prime will be here, in this place itself. So, E 1 triple prime, E 2 triple prime and E 3 triple prime, they form a, an orthogonal axis. So, the angle from here to here, this one is, this black angle is 90 degree. So, what we have done, we have given 3 rotations. So, the first rotation we give about the E 3 axis.

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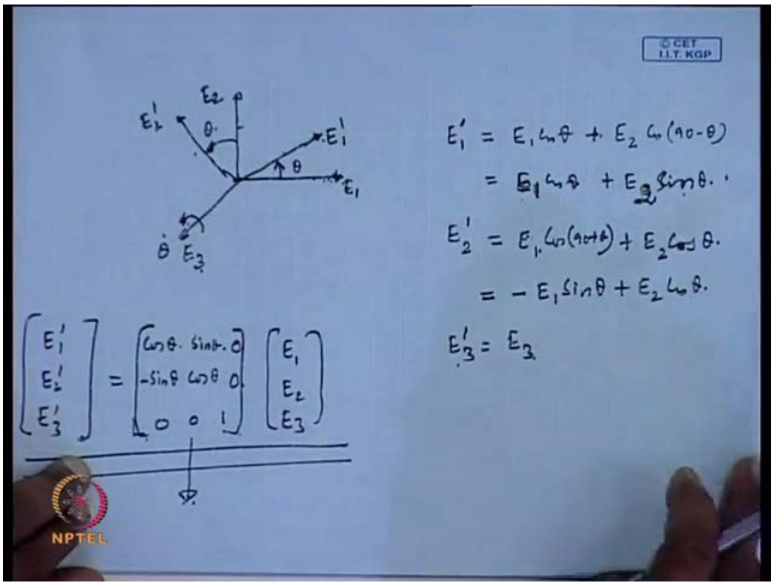

So, any rotation, let us say that, we have the E 1 and E 2 here, and if you rotate it, rotate this axis and write this as E 1 prime and this as E 2 prime, and if you are rotating it by some angle, say theta, then, it is possible that, we can describe the transformation from 1 reference frame to another reference frame and we will be rotating about the axis E 3. So, we are rotating about this by theta. So, theta dot is in this direction, and this two are rotating and coming into this place. So, the resulting components in this direction, you can write as E 1 prime is equal to E 1 Cos theta plus, we have to take the component of this, E 2 Cos 90 minus theta. This is the angle from here to here, 90 minus theta; E 3 sin theta. Similarly, you can write E 2 prime, this is E 2 prime; so, E 2 prime will be E 2 Cos theta and we have to take the component of this, along this. So, this will be E 1 Cos 90 plus theta; and, this gives you minus E 1 sin theta plus E 2 Cos theta; while E 3 prime you can simply write as E 3, because you are rotating about this, so, there is no change.

So, basically, any vector whose components are E 1, E 2 and E 3,or say, if they are unit vectors, so, the components of the unit vector, these are E 1, E 2 and E 3. So, you can rotate them and describe them, in other…If you rotate them, so, it can be described as, the another components can be written as, after rotation, so, E 1 prime, E 2 prime, E 3 prime. So, from here, you can see that, if you have rotated about the E 3 axis, so, E 3 axis there is no change; E 3 prime is just E 3. So, from here, we can write, E 3 prime is equal to E 3. Completing this matrix, E 1 prime can be written as E 1 Cos theta. So, here, you have Cos theta and E 3 sin theta; and then, we have, this is E 2; sorry, this is E 2 sin theta. So, we have sin theta here, in this place. And, E 3 term is not there; so, here we put 0. And then, from this place, we have minus E 1 sin theta; so, this is minus sin theta. And then, E 2 Cos theta; so, we put Cos theta here; this term is 0. So, this is your rotation matrix, or the transformation matrix which rotates from one reference frame to another reference frame. So, we have given rotation about the E 3 axis.

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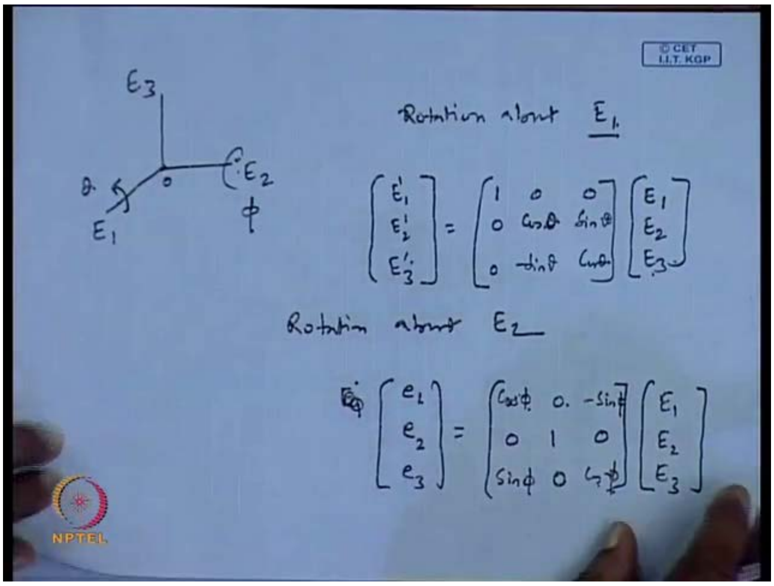

So, in most of the cases, we have chosen the axes like E 1, E 2 and E 3, in this way. So, if we rotate about E 1 axis, E 2 axis and E 3 axis. So, simply, we can write as, rotation about E 1, it can be written as E 1 prime, E 2 prime, E 3 prime. So, if you are rotating like this, using, if you are rotating about E 1,so, you will have 1 here, in this place, because this vector will not change; this component will not change and we will write here E 1 and rest other things, you can write as…Say, if you are rotating first by the, about the E 1 axis by, let us say, by theta; you are rotating about this. So, if you are rotating by theta, so, this becomes Cos theta, sin theta, minus sin theta and cos theta. Rotation about E 2, similarly, we can write as, let us say, another notation we use here. Let us say, e 1, e 2 and e 3, and now, we are rotating about E 2. And therefore, E 2 will remain unchanged; it will, E 2 will be equal to E 2. And, let us say, we want to rotate about this, by some amount phi. So, we will write here as Cos phi. See the sign change we have done here; here, this is minus; in this case, it was plus here, diagonally for the sin; for the Cos, it remains same; but sin here, for the sin, it has changed. Again, if we rotate about…

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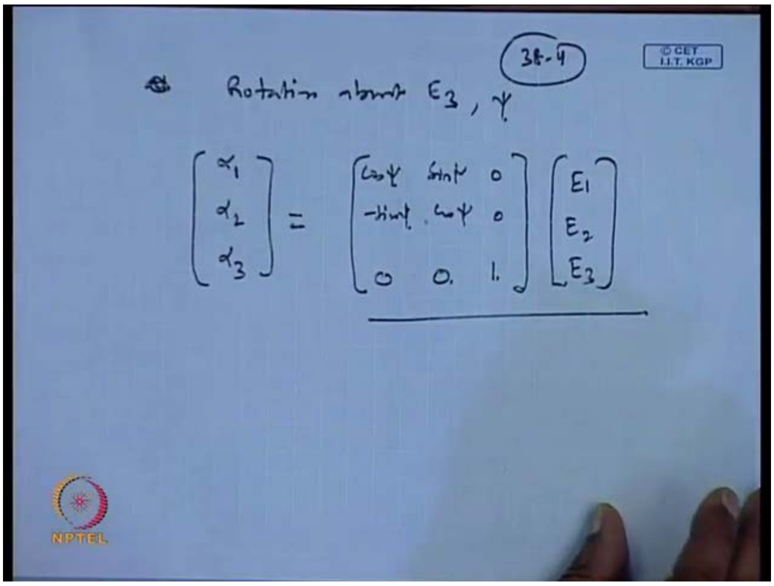

Let us indicate this vector as alpha 1, alpha 2, alpha 3, and this we rotate by psi. So, we are rotating about E 3, now. So, this remains unchanged and rest other will change. So, we will write here, Cos psi, minus sin psi here; Cos psi, 0, 0. So, you can check this; this is very simple; you can do it as an exercise yourself.

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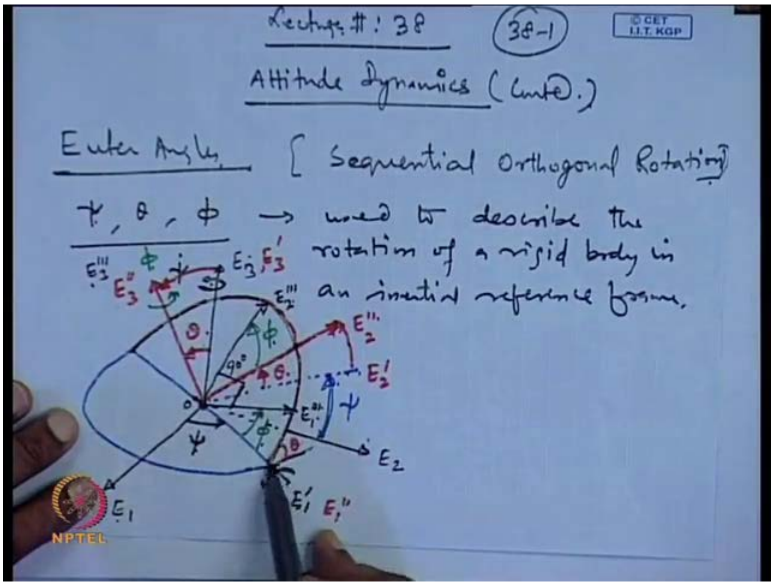

Now, coming back to this figure, so, what we did in this figure that, we first rotate it about the E 3 axis by angle psi, then, about the resulting, this E 1 rotated, and came from here to here. So, the resulting x axis, this is now, this is basically the x axis, and this is y and z axis. So, we have replaced that notation by E 1, E 2 and E 3. So, after rotation, it comes to this place. Now, if you give the rotation by theta, so, this theta is shown to be here, and this angle comes to, this E 3 rotates and comes to this place. And, the next rotation we are giving by phi. So, once we give the rotation by phi, so, this we have not shown the angle here; this is your angle phi. And, the angle from here to here, this is phi. You should remember that, theta and phi, they are not in the same plane. Theta is the inclination of this plane, with respect to the E 1, E 2 plane; while phi is lying in the plane over which I am running my plane here, over these lines. So…

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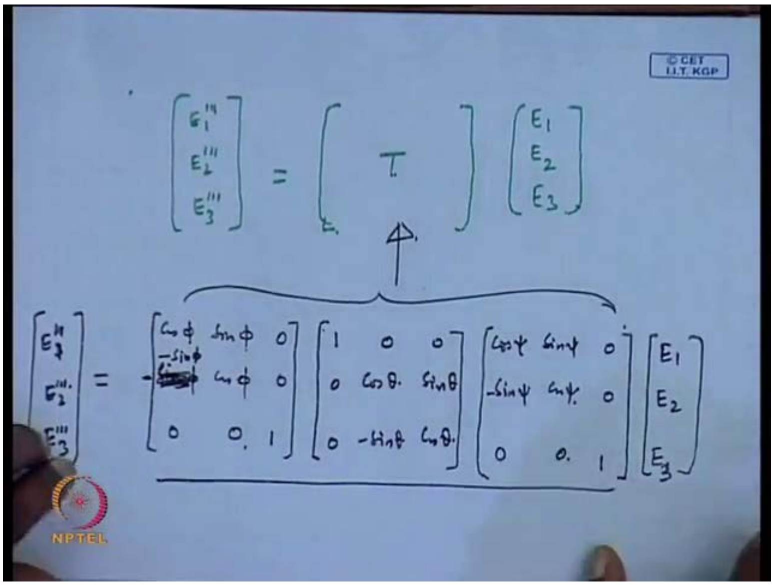

Now, we have the three rotations. So, the same way, we are start with the E 1, E 2 and E 3 vectors. So, finally, what we need that, after the rotation, what will be this transformation matrix, which will be giving us the final configuration, where we have written this as E 1 triple prime, E 2 triple prime and E 3 triple prime. This we need to find out. So, for this, the first rotation you gave about the psi. So, first, it is about the E 3 by angle psi. So, E 3, you are rotating, about the E 3, if you rotate by psi, so, this will be Cos psi, sin psi and minus sin psi, Cos psi, E 1, E 2, E 3.So, the resulting vector, now, it will be operated by another rotation, or the transformation matrix, which will be result of this rotation theta, about this resulting line E 1 prime. So, once we rotate it here, so, the next rotation…This will be operated, this is basically, giving you a vector. So, this vector will be operated by another rotation, which you are giving about the E 1 prime axis. So, here, this will be 0, 0 and you are rotating by theta angle and the finally, this will give you again a vector. Now, once again you need to rotate, to come to this position E 1 triple prime, E 2 triple prime and E 3 triple prime. So, here, you are rotating and this is your phi dot and in this direction, you have already got theta dot; psi dot we have already written here; this is psi dot. So, once you rotate by this angle phi, and that rotation again, we are giving about the E 3 triple prime axis and all these are the orthogonal rotations. This rotation is by phi, angle phi; this is sin, minus sin here, minus sin phi, Cos phi, 0, 0, minus sin phi. So, this gives you the final vector, which is E 3 triple prime, E 2 triple prime and E 1 triple prime. And, once you multiply this, so, this is the total transformation vector that was I was looking for.

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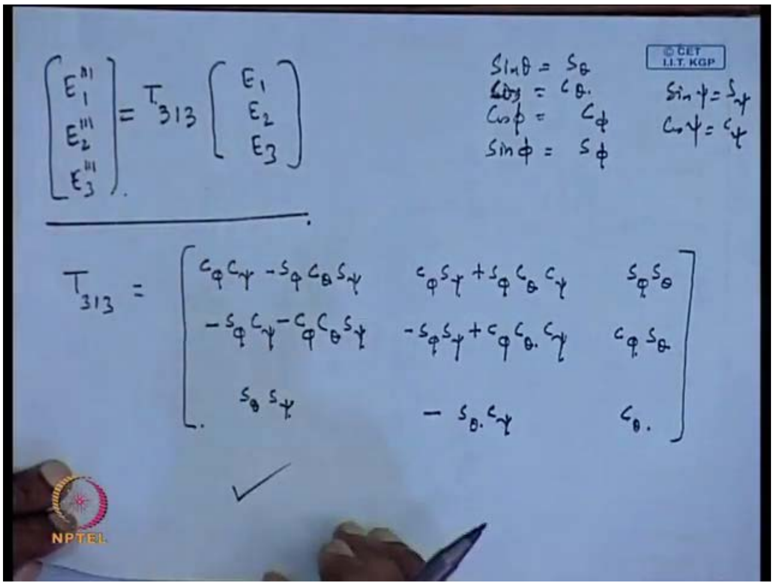

So, the transformation matrix T, what the rotation, the sequence of rotation we have given, this was 3, 1, 3 and this is being operated upon E 1, E 2, E 3, these are the vector components and then, you are converting into E 1 triple prime, E 2 triple prime and E 3 triple prime. So, T 3 1 3, after the matrix multiplication, if you multiply the matrix and we use the short cut notation, this Cos phi can be written as c phi; similarly, sin phi you can write as s phi. On the other hand, we can write sin theta as s theta, and Cos theta as c theta. Similarly, we can use the notation for sin psi as s psi, and Cos psi as c psi. And, if you multiply, so, this will yield you, c phi, c psi…So, this is the transformation matrix, which will transform from here to here.

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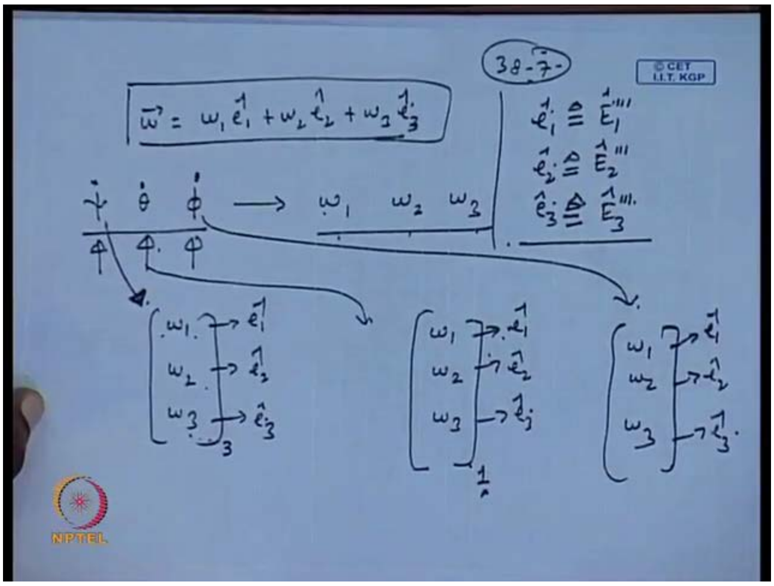

So, now, we have the rotations here. Already we have written that, omega, this is equal to omega 1 e 1 cap plus omega 2 e 2 cap, omega 3 e 3 cap, where e 1 is identical with E 1 triple prime cap; e 2 cap is identical with E 2 triple prime cap, and e 3 cap is identical with E 3 triple prime cap. So, going back into this figure, now, you can see that, in this figure, we have psi dot along this axis; theta dot is along this axis, and phi dot is along this axis. And, we want to convert into, all these along the, in terms of omega 1, omega 2 and omega 3. So, omega 3 is being taken here. omega 2. So, omega 3 is along this direction, and omega 2 is along this direction; omega 3 here, and this is the, your omega 2 and omega 1 is along this direction. So, these are the three perpendicular directions. So, an angular vector is broken along this three perpendicular directions, which are the final directions; after all the three rotations, these are the final orientation.

So, if you want to convert from psi dot, theta dot, phi dot to omega 1, omega 2, omega 3,so, you need to convert each of the rotation, this each of the rate, individually into omega 1, omega 2 and omega 3; that is omega 1, we can write as omega 1, omega 2, omega 3. We will have a component, once we, once we decompose this, take the components along the e 1, e 2, along the e 1 cap direction, e 2 cap direction, e 3 cap direction. So, we will have the components omega 1, omega 2 and omega 3. And to indicate that, this is due to the rotation psi about the e 3 axis, we will put here, the 3 notation. Similarly, if we have the theta dot, so, theta dot will have the components omega 1, omega 2, omega 3 and because we, this we are rotating about the e 1 prime axis, so, here, we will put the 1 subscript, to indicate that, this is because of the rotation of the, because of the rotation theta about the e 1 prime axis. And, this will be the components along e 1 cap, e 2 cap and e 3 cap directions. And similarly, for the phi, you will have omega 1, omega 2, omega 3 cap and this will be along the e 1 cap, e 2 cap and e3 cap directions.

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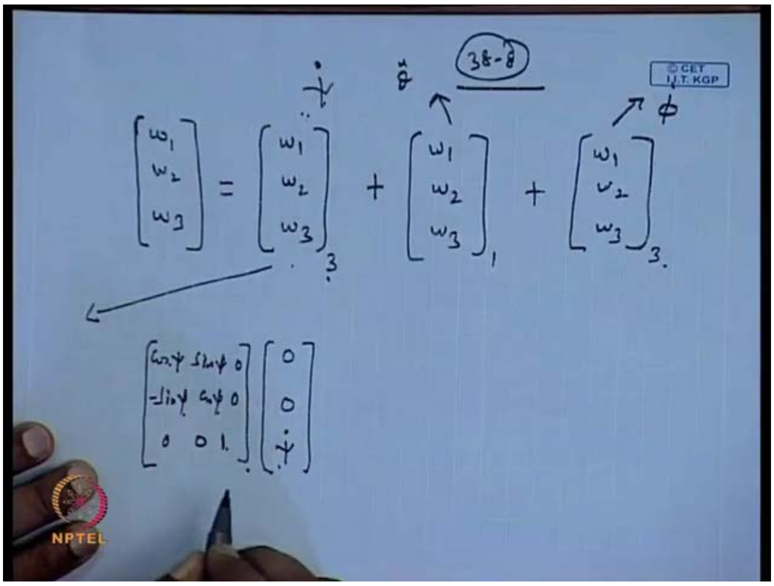

So, ultimately, what you need to do that, omega 1, omega 2, omega 3, this you can write as with subscript 3, here. Remember, this subscript, we are putting here; omega 1, omega 2, omega 3, this is for 1; and again, omega 1, omega 2, omega 3, this is the final rotation. So, this rotation, this is for, you are rotating for the psi. This you are converting for the psi dot; this you are converting for the theta dot, and this you are converting for the phi dot. So, this we will take it up and this will be shown by 0, 0,psi dot. So, we make it a vector and then, this is to be operated upon, we are giving the rotation psi. So, this becomes Cos psi, sin psi, 0, minus sin psi, Cos psi, 0, 0, 0, here, in this place. So, this is the rotation about the E 3 axis you are giving. Now, next rotation. So, you are rotating this vector, psi dot is a vector in the present reference frame, which is shown by this line, this line, this line and it is shown by…So, first, we rotate it by psi. So, it resulted, the reference frame came to this place, and from here, it came to this place, E 2 prime. So, the next rotation, you will give as theta, which will bring into this place; we will have to stop again.

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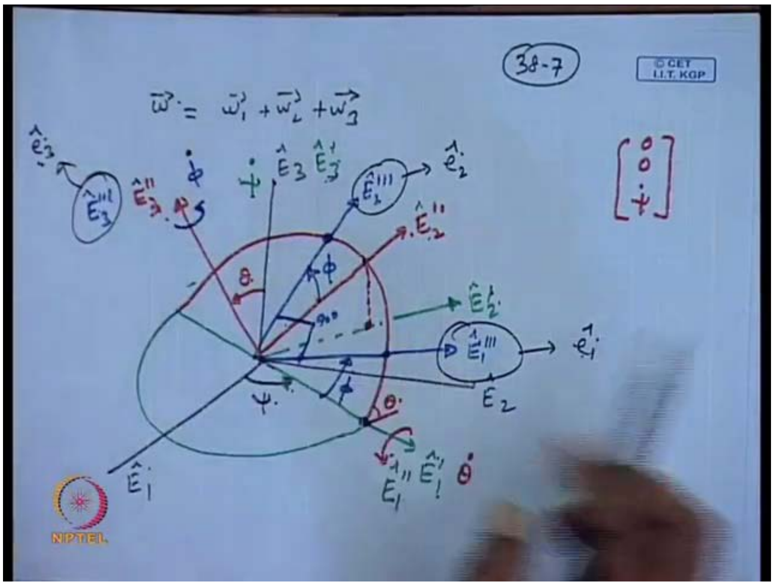

Working out this omega angle, we will make another figure to make the concept clear; that figure has become too complicated to indicate anything now. So, we have this E 1, E 2 and E 3; first rotation we give by psi, and therefore, the resulting axis, what we are getting here, this is E 1 prime, and this E 2 will move, and we can show it on the back here; E 2 prime and E 3 prime will be here, in this place. So, this has rotated by angle psi here. So, psi dot will be present here, in this place. The next rotation we are giving by…So, remember, this is a, psi is a vector, psi is directed along the E 3 direction and we can put the cap to indicate that, these are the unit vectors. So, we can put cap here, to indicate, these are unit vectors. Once we do this, so, you can see that, psi is directed along the new axis; psi dot is directed along the new axis, E 3 prime. So, we have the reference axis now, E 1 prime cap, E 2 prime cap and E 3 prime cap. And, along which the third direction, which is the equivalent to the z direction here, so, your psi dot is pointing. And, you want to convert it into the omega 1, omega 2, omega 3. So, we need to work little more further.

So, the next vector is, once we rotate next by angle theta about this, so, the theta dot turns out to be in this direction, and this will rotate by theta. So, this will move from this place, and finally, you can show it in this plane. So, basically, any line which was lying here, any point, so, it will move up and come here, in this direction, and in this direction, you will show the, this as the E 2 double prime. And similarly, this point will move away from here, and this will be your E 3 double prime, and this is also, the angle here theta.

So, psi dot, we can indicate as a vector; psi dot we can write as, psi dot here, 0, 0. So, this is a vector, which is in E 1 prime, E 2 prime, E 3 prime axis and that has to be converted along the final axis. So, we have, one more rotation is remaining. So, the next rotation, we will give about this axis, by angle phi, and this is the rotation that we are giving. So, after the rotation, this E 1…Here, we have the, finally, once we give this rotation, so, E 1 cap double prime is also in this direction. So, this will move away from this place to somewhere else, and that we will show by red and green and blue line, we are using for this. So, this point will move from here, and it will come here, in this place. So, here, we will write as E 1 triple prime cap. Similarly, this will move in this plane shown by this red line, and it will come here, somewhere in this place. So, this, we show as E 2 triple prime. And, the third rotation we have given about this axis. So, this is E 3 triple prime.

Now all these…So, the final orientation is E 3 triple prime, which we will indicate by E 3 triple prime cap; this, we will indicate by E 3 cap, to indicate this is the final body axis. This is also called sometimes the body axis, because with respect to an inertial reference frame, we are trying to orient the body. So, this is called the body axis. So, this is the third, third direction E 3 cap and the E 2 direction is coming here, in this place. So, here, we will have E 2 cap and this is giving us the E 1 cap. So, now, you can see that, after giving this rotations, here, this is the angle from this place to place, this is your phi angle; from here to here, this is your phi angle; this angle is 90 degree; this blue to blue line, this is 90 degree. So, this psi dot vector, need to be converted, finally, into E 1 cap, E 2 cap and E 3 cap as it. So, to convert it, the first rotation you need to give is theta; another rotation you need to give is phi. So, it will get oriented with the, this E 1 prime, E 2 prime, this E 2 prime is here, and E 3 prime is here, in this place itself. So, this will get oriented with the, next one E 2 double prime and E 1 double prime and E 3 double prime. So, this is one rotation required and the next rotation required is by angle phi.

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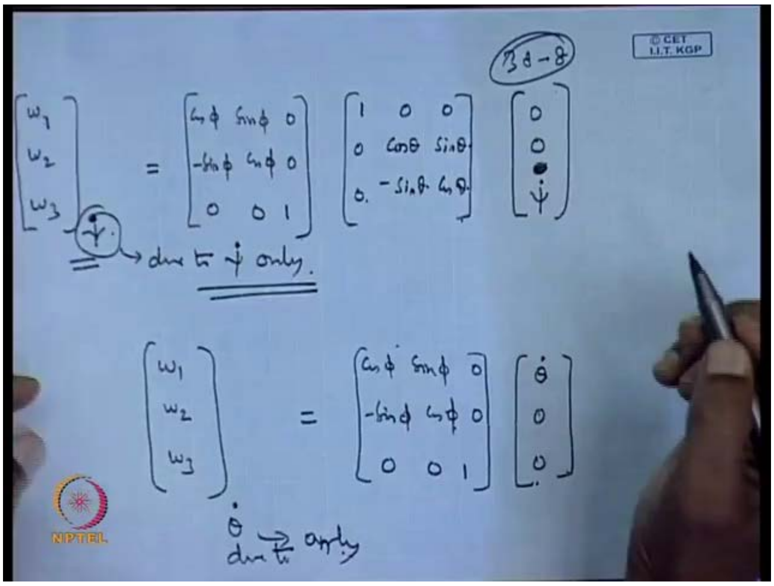

So, we will write this as…We have the psi dot vector; this is psi dot; this is 0. First rotation, we give by, give about the x 1 axis by theta angle. So, this is Cos theta, sin theta, minus sin theta, Cos theta and this is 0, 0. So, the next rotation you need to give is, about this axis. So, this is the rotation you are giving here, by phi. So, this is your phi dot, basically. So, by rotating by phi, about the 3, so, this will be Cos phi, sin phi, 0, minus sin phi, Cos phi, 0, 0, 0. And, this will give you the components along the E 1 body axis, E 1, E 2 and E 3, which is the final body axis. So, we will write omega 1, omega 2, omega 3, these are the components, but this is only due to psi, to indicate this due to psi, due to psi dot only; due to psi dot only. Finally, the next one is our theta dot. So, theta dot, we need to convert. And, theta dot the rotation we are giving about the one axis, we are rotating about this axis. So, here, we write theta dot, 0, 0. And, theta dot, if you have rotated now, about this axis, so, you can see this, this is along the E 1 double prime direction, and this axis can be described as E 1 double prime, E 2 double prime cap and E 3 double prime cap. So, and theta dot is directed along the direction 1.

Now, this is to convert it into the final axis, which is E 3 the body axis. This is to be converted along the body axis. So, if you try to convert it, what rotation is required; there is only one rotation is required; this is by angle phi and that, you are giving about the E 3 axis; the third axis basically, you are giving. So, third axis means, here, this is Cos phi, sin phi, 0, minus sin phi, Cos phi, 0, 0, 0. And, this gives you omega 1, omega 2, omega 3, due to theta dot only; due to theta dot only. Similarly, we can do the final orientation.

So, final orientation, we have the phi dot here, which need to be converted into omega 1, omega 2, omega 3. You can see that, the final orientation, this phi dot is nothing, but it is a oriented along E 3 cap direction. So, we need not do any transformation, or either you can say that, you are multiplying a vector by a unit matrix.

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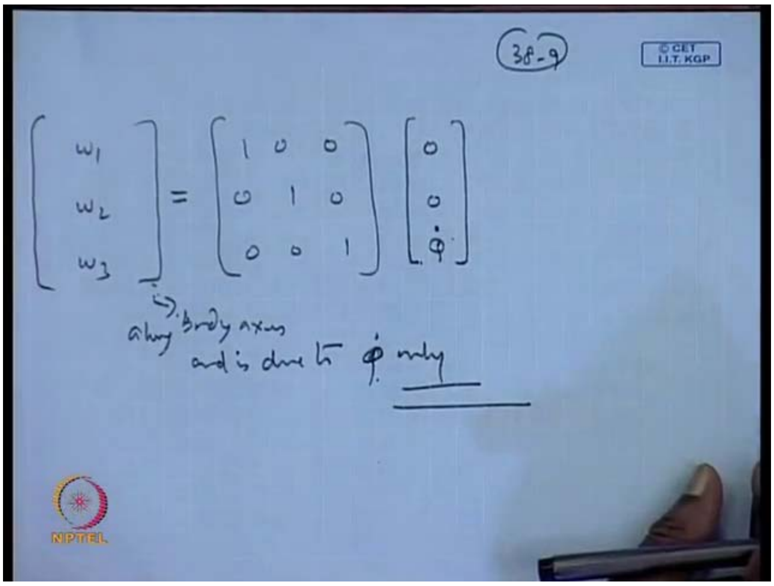

So, finally, you have, you have the rotation which is given along the third direction. So, this phi dot, you can represent like this; phi dot, 0, 0; and, this need to be just multiplied by a unit matrix, and that will convert it into omega 1, omega 2, omega 3, along the body axis, along the body axis and is due to phi dot only.

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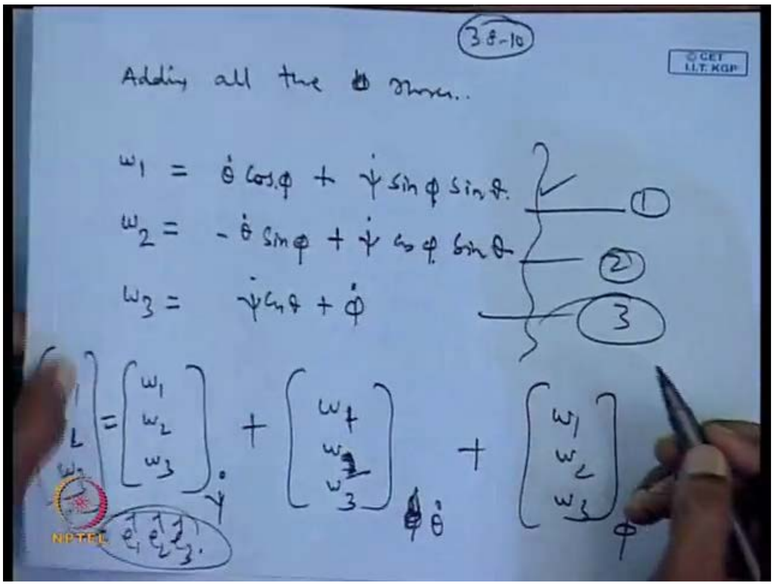

So, at this stage, finally, what we need, is to add up all the components. So, adding all the three... So, we have this two here, and this is the third one. So, we need to add it up. So, you can see that, omega 1 can be written as…First, we need to multiply this. So, if you add them up, this will give you theta dot Cos phi plus psi dot…So, this is the first equation. The second equation we can write as, omega 2 equal to minus theta dot sin phi…And, the third one can be written as psi dot Cos theta plus phi. So, what we have done in getting this, basically we added omega 1, omega 2, omega 3, due to psi dot only, plus omega 2, omega 3, omega 1, omega 2, omega 3, due to phi, theta dot only and 1 omega 1, omega 2, omega 3 due to phi dot only and this is giving you the final value of omega 1, omega 2, omega 3, which is oriented along the body axis, e 1 cap, e 2 cap, e 3 cap . So, this gives you the final orientation. Now, these three equations, 1, 2 and 3, you can write in the matrix format.

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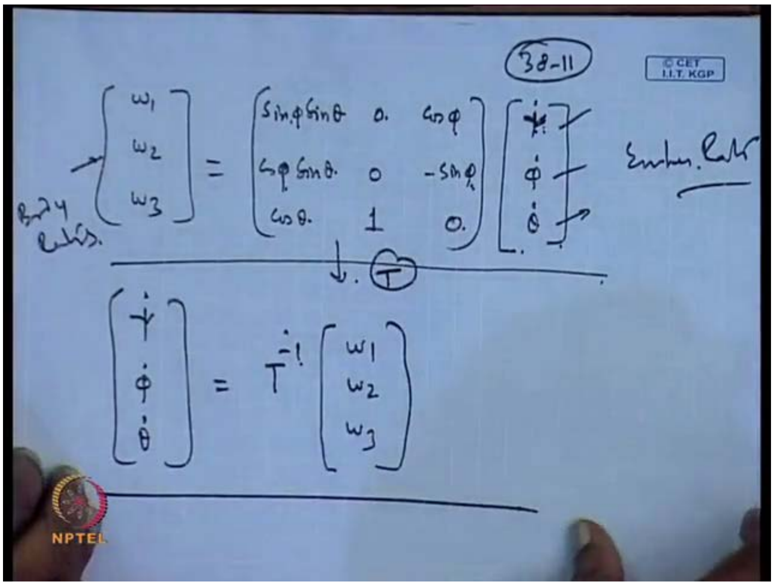

So, we can write it as omega 1, omega 2, omega 3; these are the three components, the Euler components, so, psi dot, phi dot and theta dot, we can write it like this; for any sequence, you can make it here, in this place, and, accordingly, the entries here, the elements of this matrix will change. So, this will become, omega 1, you can see from this place, this is theta dot Cos phi. So, theta dot is the last entry. So, here, we can write Cos phi and phi dot is not picturing here. The another one is psi dot sin phi times sin theta. So, this is sin phi times sin theta and theta, phi dot is not picturing, so, for this, we put here 0. So, the next one, omega 2 is minus theta dot sin phi. So, theta dot again, and sin phi; theta dot is the last entry. So, we will put here as minus sin phi. Then, we have psi dot Cos phi times sin theta. Psi dot is the first entry; so, we put here Cos phi times sin theta; and, phi dot is not appearing anywhere in this equation. So, we put again here in 0. Lastly, the omega 3, which is psi dot and phi dot. So, psi dot is multiplied by Cos theta, this is the first entry, and phi dot, this is only 1 will appear in this place, and last entry will remain 0. So, this is easily constructed. So, this is how psi dot, the Euler rates are converted to the body rates. These are called body rates, the Euler rates. And vice versa, to get psi dot, phi dot, theta dot, you need to invert this matrix. So, if we write this as the T matrix, so, this can be written as T inverse, omega 1, omega 2, omega 3.

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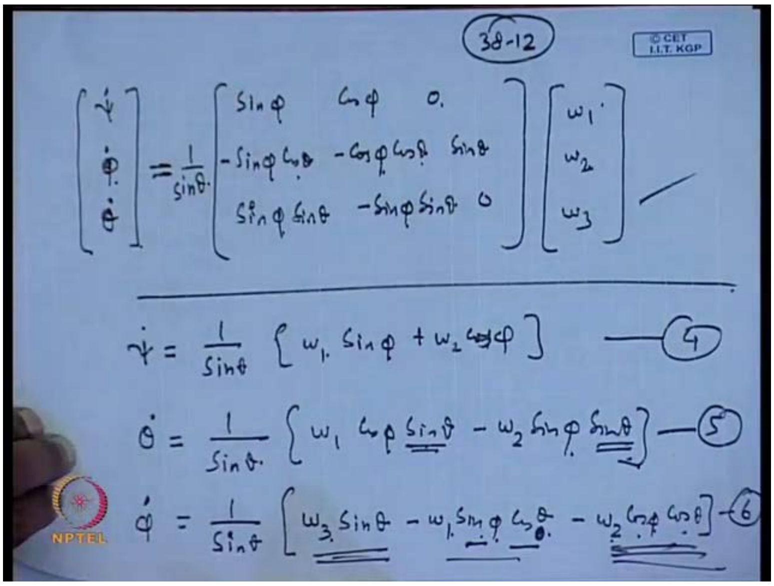

So, through inversion you can do it, or either simply, these equations are there. So, you just have to do a little bit manipulation and write this equation in terms of omega 1, omega 2, omega 3, instead of theta dot, psi dot and phi dot. So, if you do that, so, psi dot, theta dot, phi dot and theta dot, this can be written as 1 by sin theta times sin phi, Cos phi, 0; and then, minus sin phi, Cos theta and minus Cos phi, Cos theta, sin theta, sin theta, minus sin phi; these to be multiplied by omega 1, omega 2, omega 3. So, through, this is the conversion equation from…This will give you all the conversion equations. And, alternatively, you need to describe the psi dot, theta dot, in terms of omega and you will finally, get the same kind of equation. So, if you try, you will see that, the psi dot can be written as 1 by sin theta times omega 1 times sin phi plus omega 2 times Cos phi. Let us say, this is the equation number 4; and similarly, you can write…This step I am skipping, to save time. Theta dot can be written as 1 by sin theta times omega 1 Cos phi sin theta minus omega 2 sin phi sin theta. If here, I, to make this 1 by sin theta common, I introduced sin theta inside. This step, you carry out yourself; this is not very difficult.

And finally, the phi dot also, you can write as 1 by sin theta. So, here, in this case also, I am taking in the denominator sin theta outside and therefore, I will be introducing inside the sin theta factor, and this can be written as, omega 3 sin theta minus omega 1 sin phi minus omega 1 sin phi and Cos theta minus omega 2 Cos phi into Cos theta. So, in all the places, sin theta is not appearing here; those are the terms which get cancel out; they will not appear here, in this place. So, you can check this equation yourself. Now, if you want to check with the equation that we got after matrix inversion, here, this is the equation. So, you can check here. Let us say, this is the, this contains all the three terms and this constitutes your phi dot. So, phi dot you can see, this is 1 by sin theta, multiplied by minus sin phi times Cos theta minus sin phi times Cos theta multiplied by w 1; this is the first element here; then, minus Cos phi times Cos theta multiplied by w 2; minus Cos phi times Cos theta multiplied by w 2; this term is here; and, the last term is sin theta multiplied omega 3. So, you see, sin theta multiplied by omega 3. So, this just needs to be rearranged and this is easy to work out; but this is how, the conversion is done from one reference frame to another reference frame.

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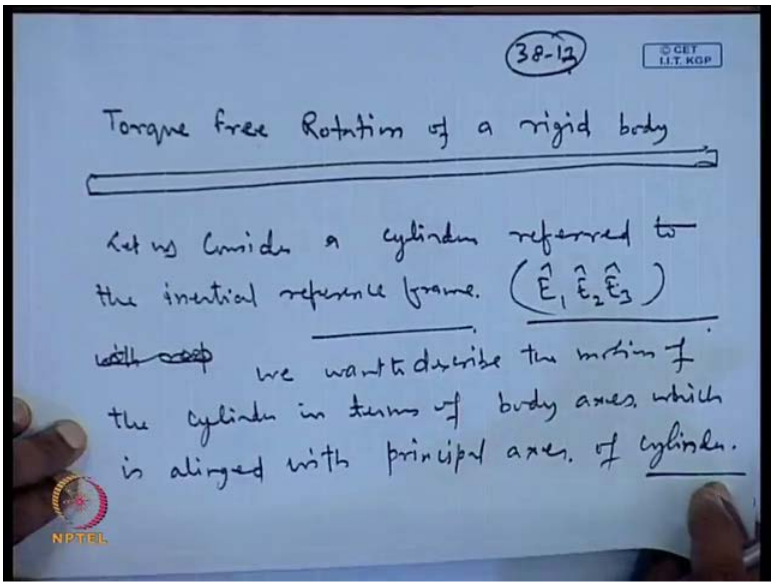

So, finally, we are ready for doing some elementary treatment, which is a, in the scope of this course. So, what we will try to do, do the torque free rotation of a rigid body. So, we will look into the dynamics of a rigid body, once it is torque free. So, let us consider that, let us consider a cylinder, referred to the inertial reference frame E 1 cap, E 2 cap, E 3 cap, as earlier we have indicated, with respect to…So, let us consider a cylinder referred to the inertial reference frame. We want to describe the motion of the cylinder, in terms of body axis, which is aligned with the principal axis, principal axis of the cylinder. So, the body axis is aligned with the principal axis of the cylinder. So, in that situation, you describe the, and describe the motion of the body.

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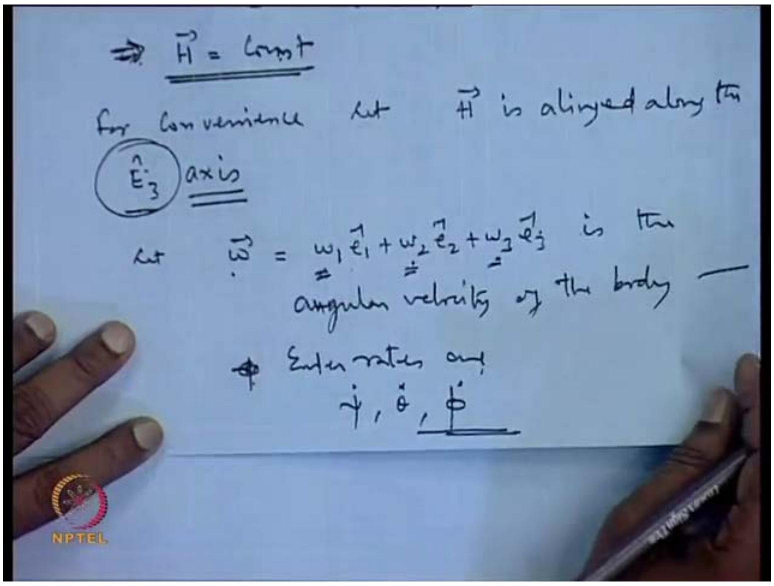

So, what here assumption is that, it is a torque free; that is, T equal to 0; this is torque free. So, this simply implies that, the angular momentum of the body H will be a constant. So, for convenience, let H is aligned along e 3 cap. So, it is aligned along the e 3 cap axis, where e 3 is the inertial reference frame, z direction. Let omega is equal to omega 1 e 1 cap plus omega 2 e 2 cap omega 3 e 3 cap is the angular velocity of the body, where omega 1, omega 2, omega 3, these are the body components, the components of the angular velocity taken along the body axis, which is, in this case the principal axis. And, Euler rates are, Euler rates are psi dot, theta dot, phi dot. Now, we can construct the figure, and look into the, how the cylinder is oriented with respect to the, how the cylinder is oriented with respect to the body, with respect to the inertial reference frame.

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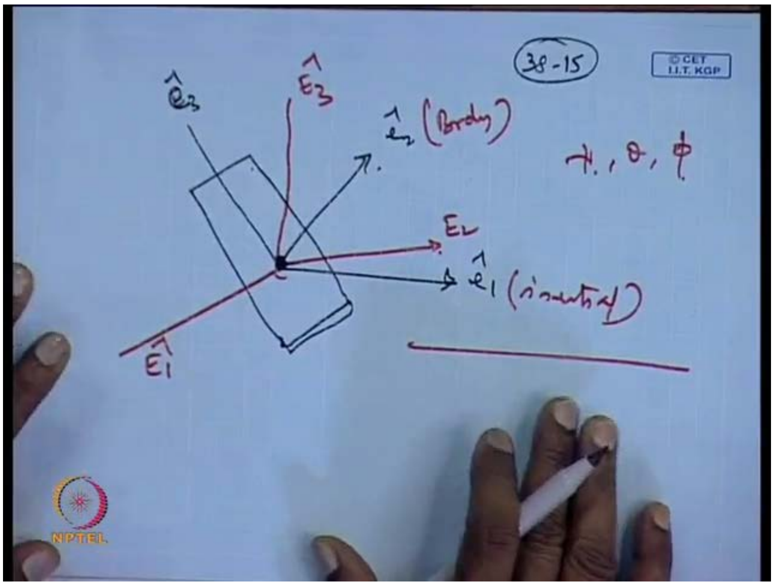

So, time is getting up. So, we will consider it next time, but whatever the one, two minutes it is remaining, we will try to look into this. So, we can have a cylinder like this. So, the cylinder, the body axis can be oriented, let us say, one along this direction and other along this direction; this is the center of the cylinder. So, this is your e 1 direction; this is e 2 direction and this is the small e 3 cap direction; and, the inertial axis you can show along any of the 3 directions, like the E 1 cap, E 2 cap and E 3 cap. So, by giving this three rotations psi, theta and phi, this has been oriented from the red color directions to the black color directions; red color axis to the black color axis, which is, this is constituting the body axis and this is your inertial axis. So, next time, we will elaborate on this, and look into the torque free rotation of the body, and also the stability of such a system. Thank you very much.