Space Flight Mechanics

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Lecture no.# 34

Attitude Dynamics (Contd.)

Last time, we have been discussing about the attitude dynamics. So, we continue with the same topic. So, we looked into the active and the passive control. In the active control, we had the rocket control, the reaction (( )) control, then, dual (( )) satellite, (( )) satellites we saw; while the passive control was the gravity gradient control. So, but the gravity gradient control, it is a cheaper, easier to design; but what happens, its pointing accuracy is very poor; it is around 10 to 20 degree.

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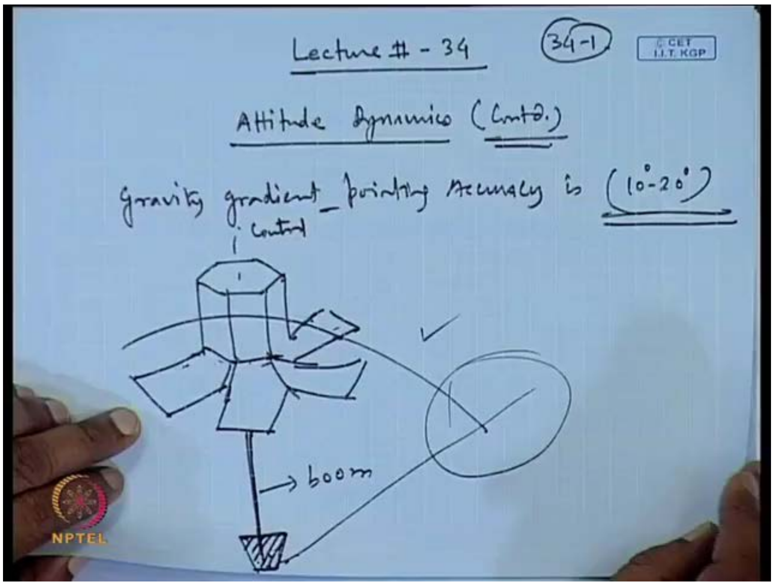

So, the gravity gradient control, pointing accuracy 10 to 20 degree. So, it is not good. It may assist; you can design a satellite, where the reaction (( )), other things are available. And, if you have to make a earth pointing satellite, so, in that case, it can assist, provided, you design your satellite in that way. So, many of the satellites maybe designed like, say, you have a satellite here, which look like, these are the solar panels; these are the connected solar panels and you may have a gravity gradient boom, where a mass is attached to it; this is the boom, and one mass is attached to that. And, instead of there being a mass, you can have certain instruments stored in this, if this is heavy. So, this will provide you the constant gravity gradient. So, this will assist in stabilizing the satellite, which will, and where you are designing it, to always point toward the center of the earth. So,the. So, this is called the favorable gravity gradient, in this case, because it is a favoring aid to a stabilize it.

But as a whole, using just the gravity gradient controls satellite, for our various scientific purposes, is not feasible. You can see that, the accuracy is not very large. Moreover, the orientation and other problems are there, which you must do. So, therefore, either we have to do the rocket control, reaction (( )) control; these are the active controls. Other thing like the spin stabilizers, and those are only for the stabilization purposes. But if you want to change the orientation of the satellite, so, ultimately you have to come down to the, using the reaction (( )), or the rocket motors, or what we call as the magnetic actuators.

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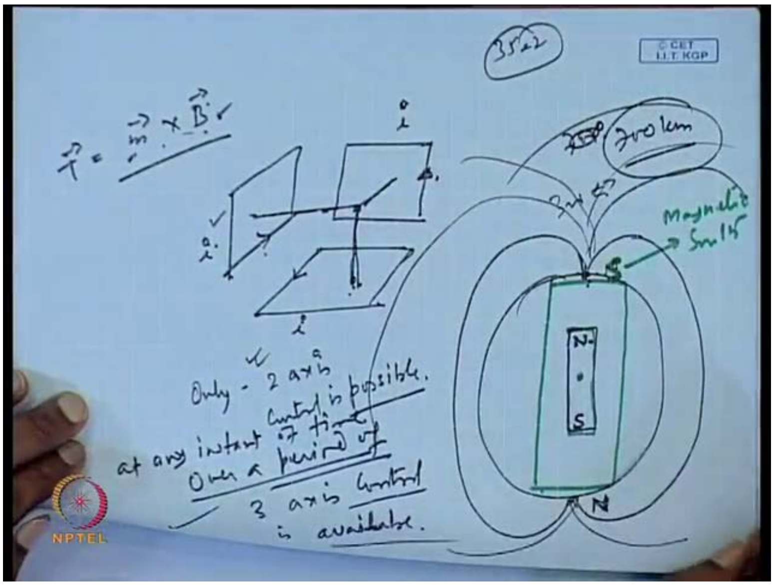

So, in the magnetic actuators we have three coils. So, this is one current carrying coil and another carrying coil will be perpendicular to this; another will be lying like this. So, these are in three different planes. So, we can assume that, the center is here. So, this is one phase of the coil; this is another phase, second coil; this is the phase of the third coil and this wires, they carry the current. So, if you, if this, this is put on the satellite on board and the satellite is at low altitude, say up to 700 degree, 700 kilometers, 300 to 700 kilometers. So, as we go up, magnetic field of the earth it dies out fast. So, magnetic of field of the earth, it is a directed like this; say, if my, if our magnet is pointing to you, if the north of the magnet and this is the south of the magnet. So, it will point, what the magnetic south of the earth. So, earth is basically, acting as a big dipole; this also is acting like a magnet. So, naturally, if you have artificial magnet, or the, and if you hang it about this point, so, it will try to orient like that only. So, in that case, this will be pointing you the south, magnetic south. So, the north pole of the earth, which you call the geometric, geometric north, so, that is basically, the magnetic south of the earth, because here the north pole, north pole will only orient towards the south pole, and south pole will only orient toward the north pole of a magnet.

So, earth can be assumed to be a big magnet. So, in this magnetic field, if your satellite is moving…So, you have the magnetic field, which is spreading like this and that is coming and terminating over this pole. So, if your satellite is interacting, this coils are interacting with this and these are carrying currents; so, they will produce dipole movement. So, this also, these coils, they act as the dipole. And, we are not going to get into the equations for them, but you assume that, this are acting as a dipole and therefore, once they are interacting with the magnetic field, a torque will be produced. So, at any time, in the case of the magnetically actuated satellites, only two axis control is possible, at a particular instant of time. So, again, we need not go into the details, but it will be suffice to say that, over a period of time, over a period of time three axis control is available. So, the torque is written as the dipole movement cross the magnetic field. So, whenever the magnetic field, it coincides with the, it is parallel with the dipole movement, so, no torque will be produced. So, we have the three axis coils here. So, one of them will be coinciding with the, or say that, the one of the component of the total magnetic, total dipole movement this three coils constitutes, so, one components will always lie along the B vector and that gets neutralized.

So, only two components are available, and therefore, we say that, only two axis control is possible, at a particular instant of time. So, only two axis possible, at any instant of time. However, over a period of time, three axis control is available. So, as the satellite is moving in the orbit around the earth, so, three axis control can be carried out. So, for small satellites, this is very advantageous and nowadays, when the micro, nano, and Pico satellites are being designed, so, in that case, it can be utilized and it is a low in cost, easy to design, reliable also, active control can be done in using this; but few limitations are there that, if the satellite is rotating at very high speed, it is not feasible to use this magnetic actuators; because it can control the satellite at lower angular velocities, at lower rotational rates. So, this is our description of magnetic actuator control.

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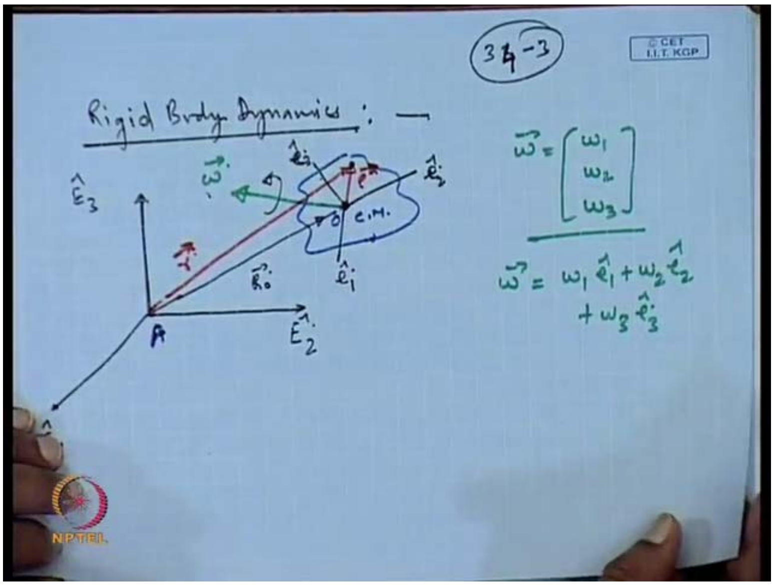

So, this is enough background for our purpose, for this course, because we are not going into the…Only six lectures are assigned for this particular topic. Rest of the things, we devote over the attitude dynamics. So, now, we consider the rigid body dynamics. Suppose, this is an inertial reference frame and we have another reference frame e 1, e 2, and e 3; this is a rotating reference frame. Some rigid body is there and this is the center of mass of the rigid body. This point, let us write this point as A; this point, we can write as O. The angular velocity of this rigid body, let us say, it is 1 by omega, where omega is a vector, whose components are omega 1, omega 2, omega 3. So, omega can be written as omega 1 times e 1plus omega 2 times e 2 plus omega 3 times e 3. So, this body is rotating at the angular velocity, with angular velocity omega. Now, what we need to do here, describe the rotational motion of the satellite. So, for describing the rotational motion of the satellite, we need to proceed from the scratch, and work out the whole thing. So, let us say that, the radius vector of the point O, this is R 0 and there is a point in the body, whose radius vector is rho. So, the distance from here to here, this can be written as r. So, while we are working with this, the rigid body dynamics, so, we will utilize the result developed in the few lectures that I gave, at the start of this course.

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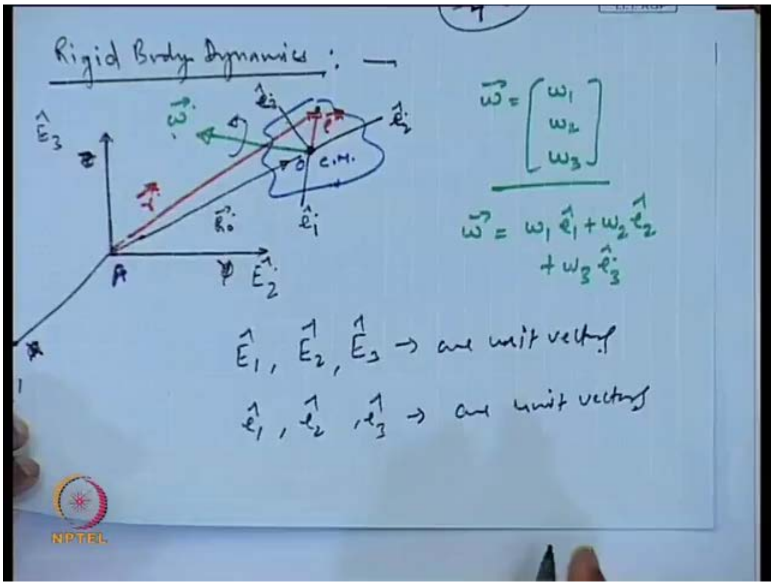

So, O is the center of mass of the rigid body and also origin of the reference frame e 1, e 2, e 3; e 1, e 2, e 3, these are the unit vectors.

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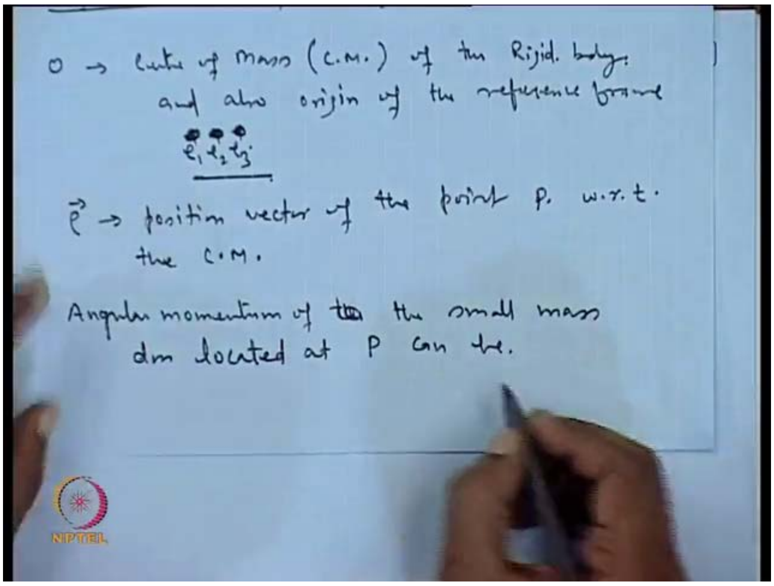

They can write it as x, y and z, and the unit vectors in this directions are E 1, E 2 and E 3; or either we can remove this altogether and we can, imagine that, E 1, E 2 and E 3, these are the three axes and E 1 cap, E 2 cap and E 3 cap, these are the unit vectors. So, E

1 cap, E 2 cap and E 3 cap are unit vectors. Similarly, e 1 cap, e 2 cap and e 3 cap, these are also, are unit vectors and the axes are e 1, e 2 and e 3. So, here, we should, the reference frame, we should remove the cap here; because that indicates, the unit vector. rho is the position vector of the point; let us say this point is P, point P, with respect to the center of mass. So, angular momentum of the small mass, there is a small mass which is located here, in this point. So, let us say, its mass is d m. So, the angular momentum of the small mass d m, located at P can be…

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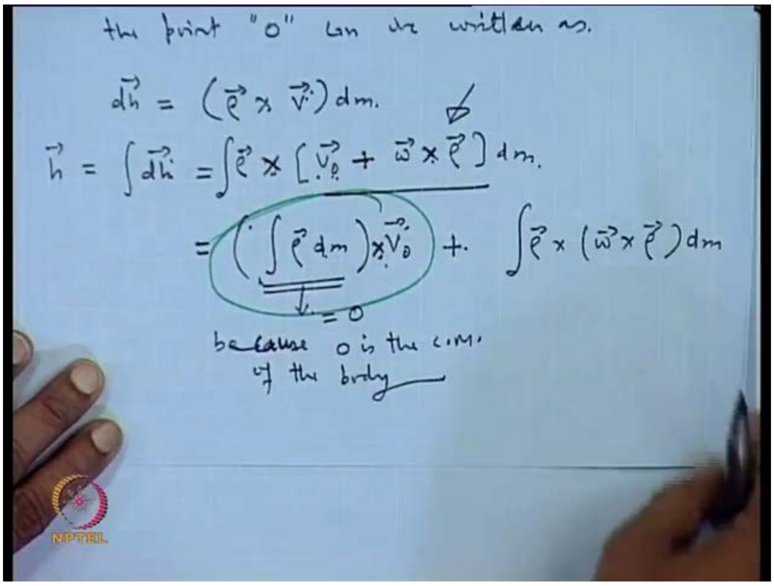

So, for the angular momentum of the body, the small mass P, that I need to find it out. So, angular momentum, angular momentum of P, or the mass d m, about the point O can be written as d h is equal to rho cross v times d m, where we will see that, we are finding the angular momentum about this point. So, this is the momentum. Suppose, our velocity v is directed like this; for this particular mass, this is the velocity direction v. So, if this is the velocity direction, so, rho cross v multiplied by m, that represents the angular momentum vector. And, v, obviously, can be written as, the velocity of the center of mass which is v naught plus omega cross rho. And, this we have worked out in our, few initial lectures, at the start of this course. So, if we integrate this, this we have to multiply it by d m here and if we integrate it, we get h. Now, this we can, this can be written as rho d m times v 0; this is the cross product; this is the cross product, here; this is also cross product, plus rho cross omega cross rho d m. So, now, if O is the center of mass, so, this integral will vanish. So, this integral, this quantity, this is equal to 0, because O is the center of mass of the body.

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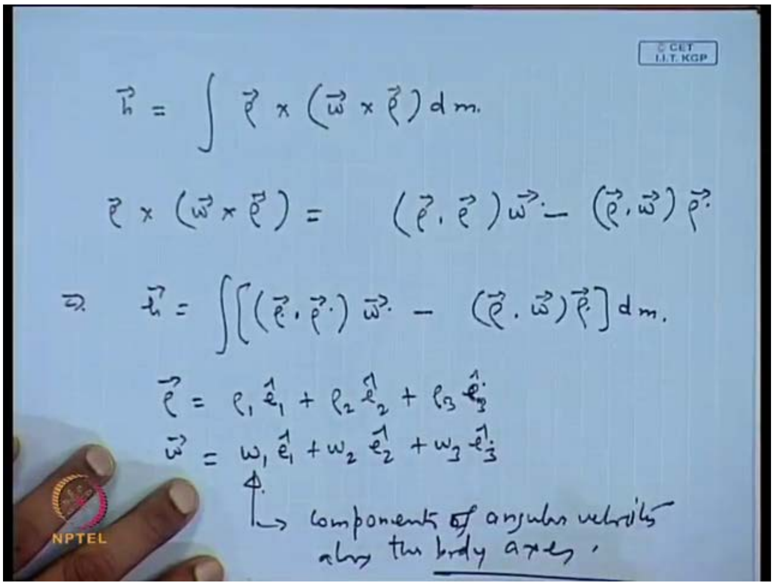

So, this quantity as a whole, it is not contributing to the angular momentum of the body. So, what we are left with, h is equal to rho cross omega cross rho. So, this is a cross product and the, this can be written as rho dot rho times omega minus rho dot omega. So, therefore, h will be…Now, we can expand it and write in a proper way. So, we, we can write, rho equal to rho 1 e 1 plus rho 2 e 2 plus rho 3 e 3 and omega, already we have written in terms of omega 1 e 1 plus omega 2 e 2 plus omega 3 e 3. Remember, this omega, these are the components of angular velocity along the body axes. So, in this figure, this is the velocity vector omega. So, we are taking the components of this velocity vector along the direction e 1, e 2 and e 3. Similarly, this rho vector, its components are taken along the e 1, e 2 and e 3 directions.

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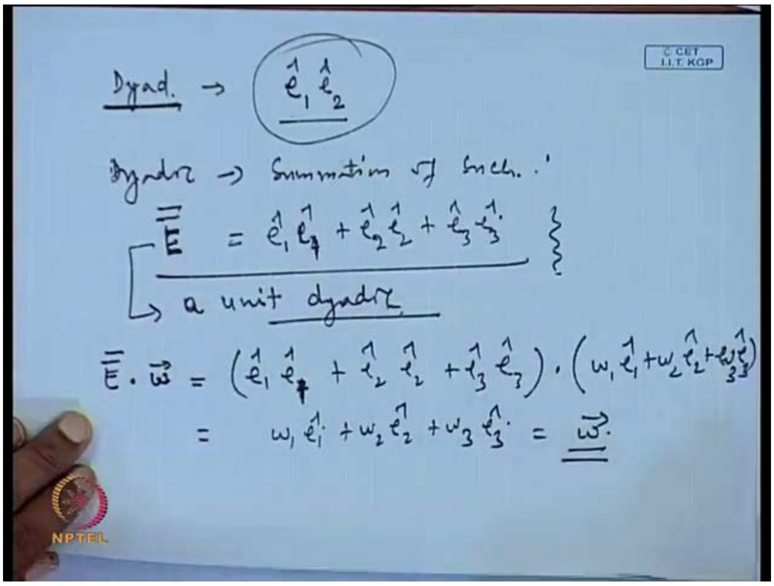

Now, we defined that Dyad as, this Dyad, this is defined as the product of two vectors without considering dot or cross product. So, if we write e 1 times e 2, so, this constitutes a dyad. So, now, the, we define a quantity, what is called the dyadic; so, this dyadic will be the summation of, summation of, such dyads. So, the, if we write e 1 e 2 plus, e 1 e 1 times e 2 e 2 and assign this, the symbol E double bar. So, this is called a unit dyadic. Now, we utilize to, utilize this to rewrite this equation.

So, we can number this equation as A. So, e double dot, w r, we can see that, this can be written as e 1 e 1 plus... So, this will get reduced to omega 1 e 1, if we take the dot product, which is nothing, but omega. So, if we are taking the dot product of a vector with a unit dyadic, so, ultimately we get the same vector.

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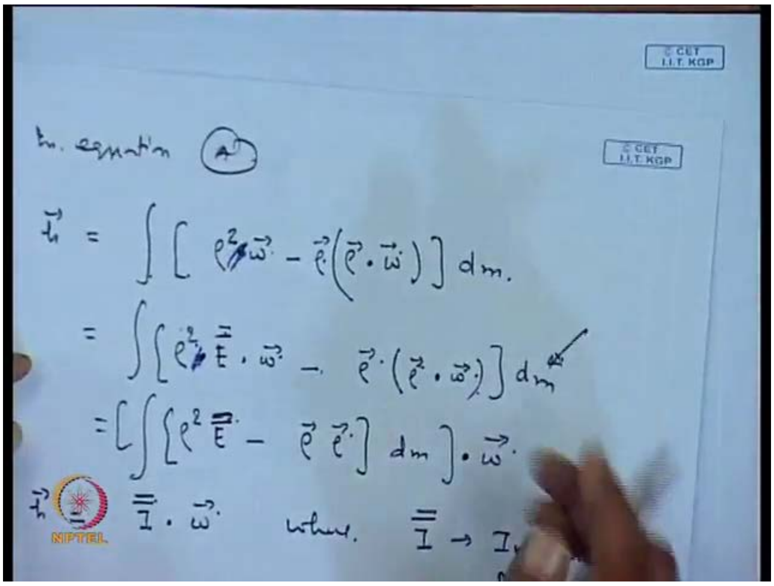

So, we utilize this property, in equation A, we can write h equal to rho square omega minus rho times, this is the dot product, omega dm. Now, the omega, we can replace in terms of a dyadic. So, writing this as, we have written as E double bar dot omega; this is not a dot product here; this one, this is not a dot product; simply a multiplication here, in this place. So, rho square omega, we are taking it from this place. So, this rho dot, this two vectors multiplied together, this becomes rho square and there is no dot product; so, it is just a point I indicated. So, I removed now. Now, if you look into this equation, omega dot can be taken out of the integration sign, because integration we are doing with respect to the mass; not with respect to the omega. So, therefore, we can write this as, rho square E double bar minus, this multiplied by omega. And here, the dot we are bringing in. So, ultimately, h can be written as, where I double bar, this is called the inertia dyadic, inertia dyadic.

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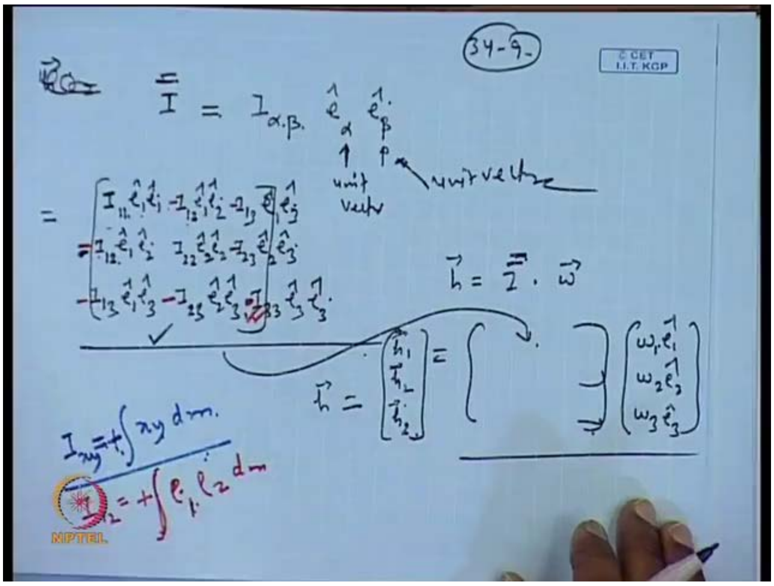

Once we have done this, so, this h, this is can be expanded and it can be written in a format. So, inertia dyadic, let us first write this, further. First, we will do this and we will come to the expansion of the h. So, this can be written as, I alpha beta times e alpha cap e beta cap, where e alpha cap and e beta cap, these are the unit vectors, unit vector. So, basically, this is written in a ((tensure)) notation. So, inertia ((tensure)) can be, this inertia dyadic can be written as I 1 1 e 1 cap e 1 cap, I 1 2 e 1 cap e 2 cap and I 1 3 e 1 cap e 3 cap. Similarly, other terms can be filled in and this is basically symmetric. So, this also becomes, I 1 2 e 1 cap e 2 cap, I 2 2 e 2 cap e 2 cap and I 2 3 e 2 cap e 3 cap. Similarly, here, in this place, you will have I 1 3 e 1 cap e 3 cap, I 2 3 e 2 cap e 3 cap, I 3 3 e 3 cap and e 3 cap. So, this is your inertia dyadic and what you are doing, you are writing h as the product of this inertia dyadic. So, in the matrix notation, the same thing, this thing you need to place here, in this place, and your omega is omega 1 e 1; you write omega 2 e 2 in this way; omega 3…Take the matrix product and you will get the result for h. So, h 1, h 2 and h 3 components, we get it from this place.

H can be expanded and finally, you can write in this way. Now, one thing more, this, the, you can put the sign here, if you want to you can put a negative sign here, before the I 1 2 and here, you can make minus I 1 3 and similarly, a negative sign before this off diagonal terms. So, why I am suggesting that, because once you write the cross product of inertia, say you write I x y, so, this is x y times the mass.

So, here, if you write it in this way, then, you need to put a negative sign before this. But if you absorb the sign in this place, then, you can keep the positive sign here, in this place. So, if you are keeping a negative sign here, in this place, then, all these terms will turn to be positive, with positive sign. If you make this as positive, so, here, the half diagonal terms, they will have the negative sign. So, it depends on the choice, what kind of the presentation you want to do. So, let us make this negative. So, all this half diagonal terms are negative. So, the cross product, this is your diagonal term; this is half diagonal term, and this is half diagonal term. So, here the, we have the negative sign here, in this place, this place. So, only the diagonal terms will not have any negative sign. So, similarly, so, in this case, you have I1 2. So, I 1 2, we will be writing as, the rho 1 rho 2 times d m, where rho 1 and rho 2, these are the components of the rho vectors. And, because you have absorbed the negative sign here, in this place, so, all this will contain a positive sign. So, ultimately, the equation showed here, in this place, this particular equation, you can expand it in terms of this vectors, and do the integration and get the result.

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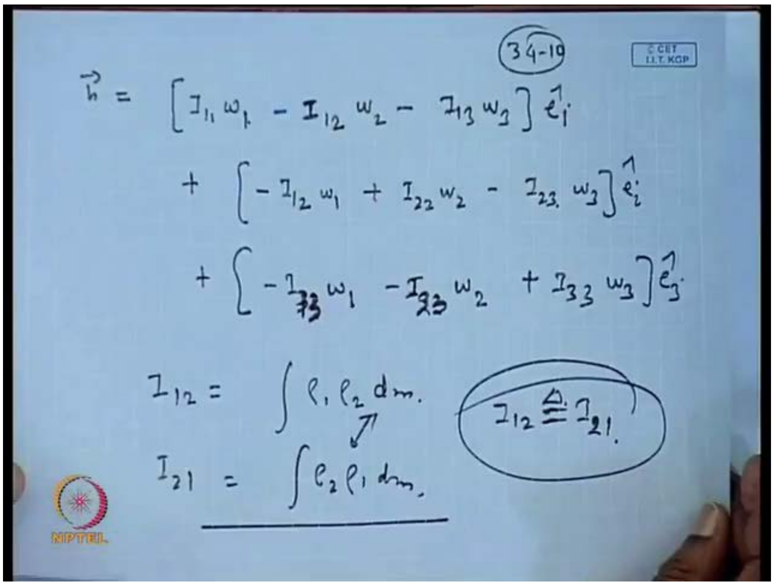

So, maybe, we will postpone it little bit, for the next step and we will right now see the results, and later on, work with the more details. So, right now, our concern is with looking into the configuration, how does it look like. So, this is I 1 2 omega 2 minus I 1 3 omega 3; this is I 1 3 I 2 3, because it is symmetric, so, we write in the same way. See, this I 1 2, this indicates your integration rho 1 times rho 2 d m. And similarly, I 2 1, this indicates the integration rho 1 times rho 2 times d m. So, there is no difference between this. So, I 1 2, this is equal to, this is identically equal to I 2 1.

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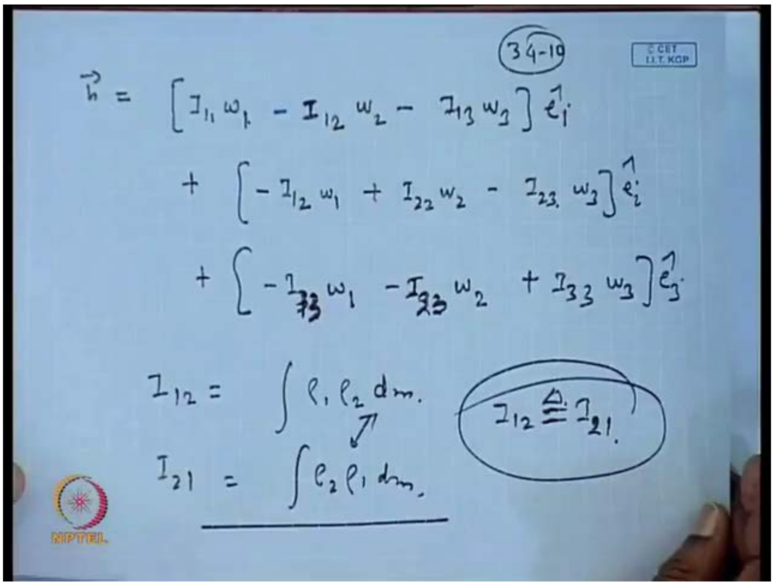

So, in the matrix notation, you can write h 1, h 2, h 3. Now, we have got this. So, this is your angular momentum equation and this is your angular momentum vector, whose components are shown here.

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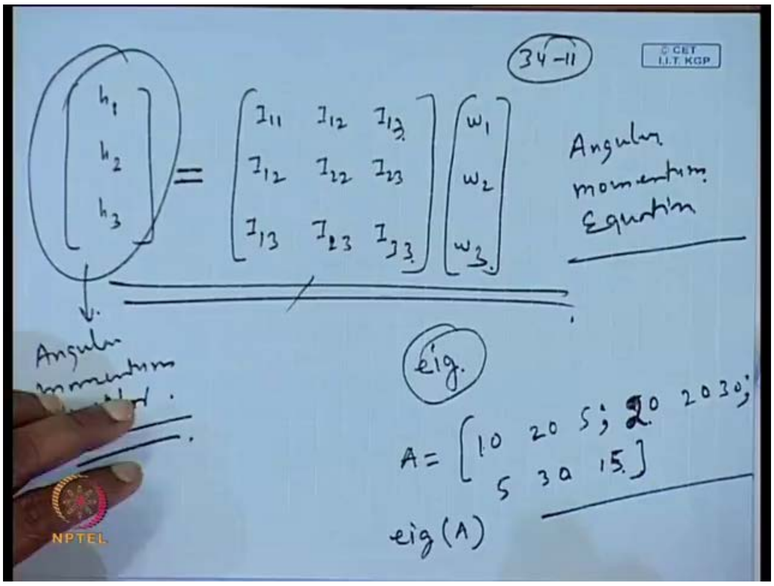

Now, before we do anything further, let us look into the, some of the properties of this real symmetric matrix. So, properties of real symmetric matrices. We are going to utilize this properties in developing the equation of motion. So, all real symmetric matrices…Eigen values are, Eigen values are real. So, you may wonder, what does it mean by the Eigen values of this matrix? In Matlab, there is a command e i g. So, if you have a matrix, if you type a matrix in the format, say, matrix you define as A, where the components of this matrix are written as something, 10, 10, 20 and 5 and they are separated by this semicolon and then, you can write say, 5, 15, 20 and 5; so, this is symmetric matrix, this will be, this will be 20 and then, let us say, this is again 20, and the last term is 30. And again, separated by a semicolon and then, say, 5, 30 and let us make this as the 15; and, if you give, in Matlab, this command e i g A, so, it will give you the, list you the Eigen values of this matrix. So, this is the moment of inertia matrix. These are the diagonal terms; these are the half diagonal terms. So, half diagonal terms are nothing, but basically, the product of inertia. So, the Eigen values of this matrix, it signifies the principal moment of inertia. So, it is possible that, if you diagonalize this matrix, so, you get the principal moment of inertia and the corresponding Eigen vectors they indicate the direction of this principal moments, principal moment of inertia direction.

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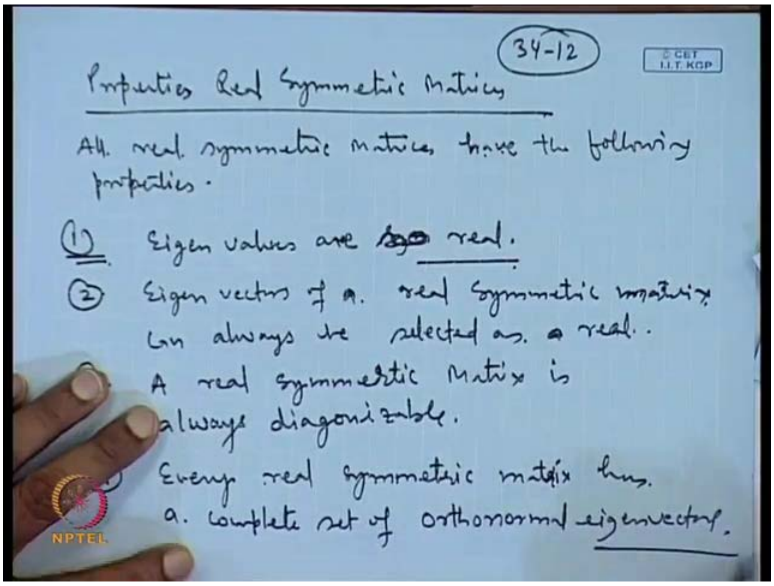

So, in which direction it is lying, so, that direction, you can obtain with respect to certain axis. So, we list the first property as the Eigen values are real. So, Eigen vectors, Eigen vectors of a real symmetric matrix can always be selected as real. A real symmetric matrix is always diagonalizable. So, a real symmetric matrix is always diagonalizable.

So, what does mean by diagonalizable that, if you give this e i g A command, if this is, obviously, here, this is a real symmetric matrix. So, if we give this command, so, you will see that, this matrix is having only the diagonal entries; this three entries; rest other entries are zero. So, this implies diagonalization. So, only a real symmetric matrix is having this property, that is, it is always diagonalizable. So, every real symmetric matrix has a complete set of orthonormal Eigen vector. So, in this case, this matrix, this is a inertia matrix, and because this is real and symmetric, therefore, this will have three orthogonal, or ortho, orthonormal, orthonormal Eigen vectors.

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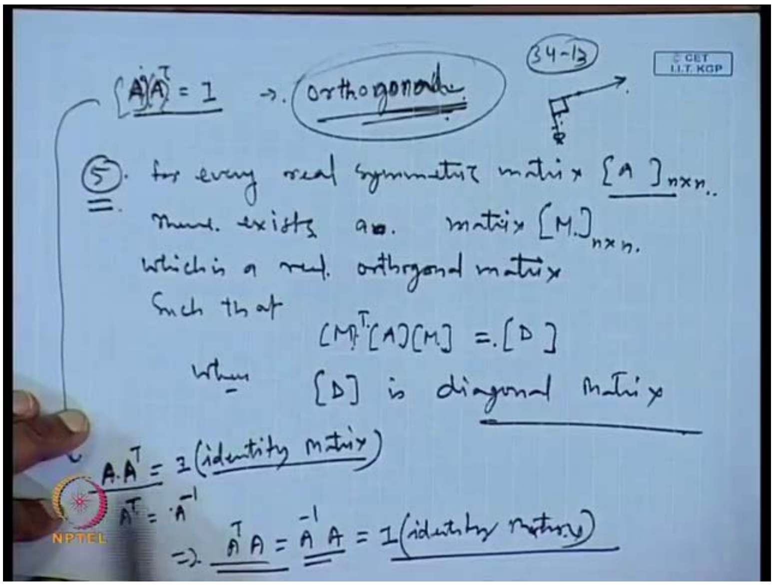

So, if you have a matrix A, so, A transpose A, if this is written as I, so, you call this as the orthonormal. So, now, we take the next property. For every real symmetric matrix A, obviously, here, we are considering a matrix of size n into n, the n rows and n columns, so, there exists a matrix of size, or matrix of size, a matrix M, which is again of size n into n, which is a real orthogonal, orthogonal matrix, such that, M transpose A equal to D, where D is a diagonal matrix. This two matrices are orthogonal, if A, A times A transpose equal to I, we can put here in this bracket. So, this is orthogonal, we write. And, if two vectors are orthogonal…So, two vectors are orthogonal means, they are perpendicular to each other. So, they will not have any component of this, in this direction; while in the case of the matrix, we write this that, if they are orthogonal, then, they have this property that A times A transpose equal to I. And, you can see from this property, if A times A transpose, this is equal to I, so, A transpose, this implies that, this is equal to A inverse. So, if we take the product on the right hand side, like, we, we write A transpose A, so, this becomes A inverse A. And, A inverse A is nothing, but your identity matrix. This is identity matrix; I is a identity matrix. You should not confuse this with the inertia matrix.

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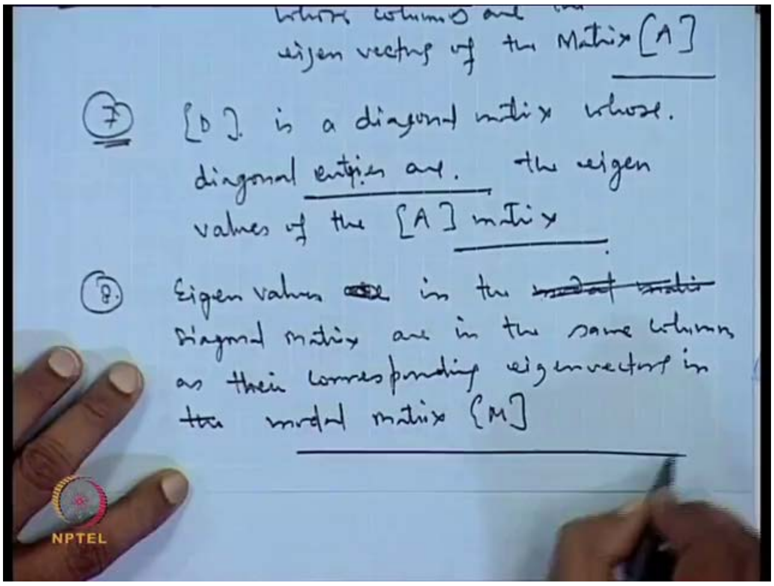

So, we have seen that, the orthogonal matrix, it will satisfy this two equations, what we have written here. So, next, we go to the next property. So, the matrix M, this is called modal matrix, whose columns are the Eigen vectors of the matrix A. D is a diagonal matrix, whose diagonal entries, whose diagonal entries are the Eigen values of, of the A matrix. Eigen values in the modal matrix, Eigen values in the diagonal matrix are in the same column as their corresponding Eigen vectors in the modal matrix M. So, with this eight properties, we will be able to analyse the inertia matrix.

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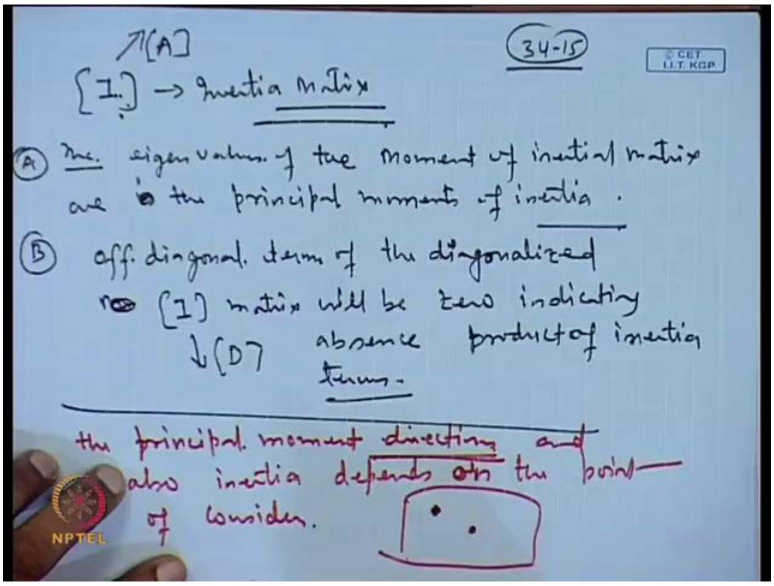

So, we list few other properties also. So, if we are given the I matrix, which is the inertia matrix. So, we can list three principal directions of this, which will be corresponding to the…In the three principal moment of inertia, which will be corresponding to the diagonal elements of this. So, we have again, the Eigen values of the moment of inertia matrix is the principal, are the principal values, principal moments of inertia. If this happens, so, if your matrix gets diagonalized, the inertia matrix A, which we have been earlier using with notation A; so, if your inertia matrix is diagonalized, then, the half diagonal terms, half diagonal terms will become zero; half diagonal terms of the diagonalized I matrix, which we have indicated as D earlier, will be zero, indicating absence of product of inertia terms. One important thing you should note down that, the principal directions, the principal moment of inertia, the principal moment of, principal moment directions, and also inertia, depends on the point of consideration. So, if in, if you take any body, so, if you are considering what will be the principal moment of inertia about this point, so, it will be different and the corresponding principal moment of inertia direction will also be different. If you consider another point, which is located here, so, principal moment of inertia will again be different; and the corresponding direction also becomes different. This is a very, this is the characteristic which need to be taken care of while developing the equation. So, we stop here, and in the next class, we can, we will start developing the equation of motion. Thank you very much.