Space Flight Mechanics

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Lecture no. # 31

Trajectory Transfer (Contd.)

We are started last time with the rendezvous problem in elliptical orbit which was the co planner maneuver. So, at the end of the lecture we went into discussing about why the non co planner maneuver is required and we were discussing about that.

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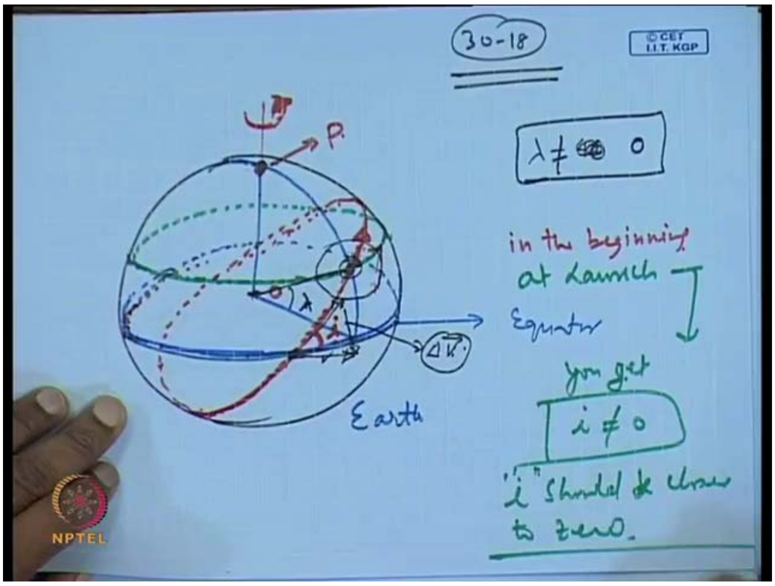

So, in that context we made a figure and in that figure we saw that suppose this is we were discussing that we can we may be forced to launch the satellite from some certain latitude lambda, because this is our country our country say the located here in this place . So, we cannot go here and lunch in this place we cannot occupy some somebody else's space. So, we are forced to launch here in this place, if we are forced to launch here in this place. So, we need to launch along this circumference direction and that naturally gives the initial inclination I. And the situation was that we want to put the satellite in the geosynchronous or the geostationary orbit.

So, for that, I need to be either or the small having a small value. So, if we are doing that then; obviously, in our figure we can look that this I need to be reduced. So, for reducing this I we need to give impulse here in this place. So, this initial velocity say in this direction and then the velocity the impulse must be given in this direction to bring the satellite in this plane. So, this is out of plane things then we need to put here in this delta b is given by the line, which is shown here. So, this is your delta v and this is the original direction and this is a new direction.

So, this is a change required. So, therefore, by applying out of orbit impulse we will be able to change the inclination of the orbit So, if you do this you apply the impulse here in this place then we will be able to change the inclination, if you apply at some other place then you are capital omega, which is the nodal angle will also get effected. So, let us start discussing about this.

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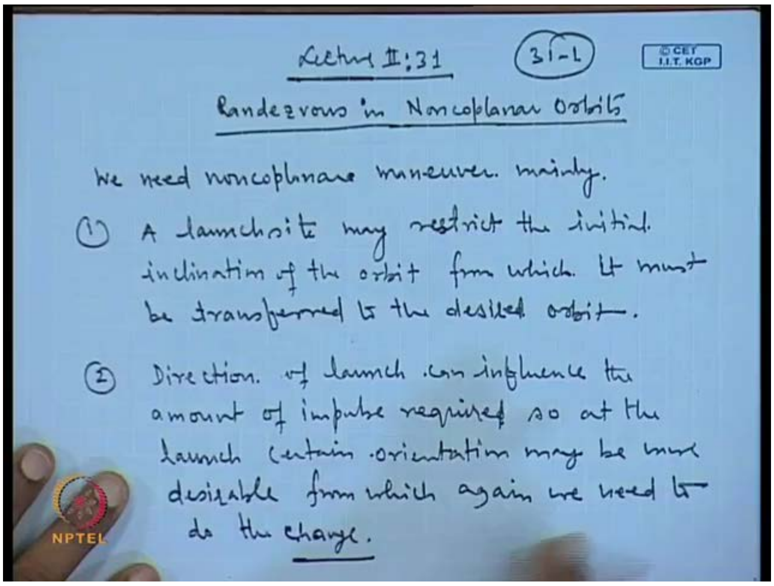

So, we have a the non co planner maneuver requirement a long sight may restrict may restrict the initial inclination of the orbit from which it must be transfer to the desired orbit sometimes it may happened at the direction of launch can influence. The amount of impulse required. So, at the launch certain orientation may be more desirable from which again we need to do the change.

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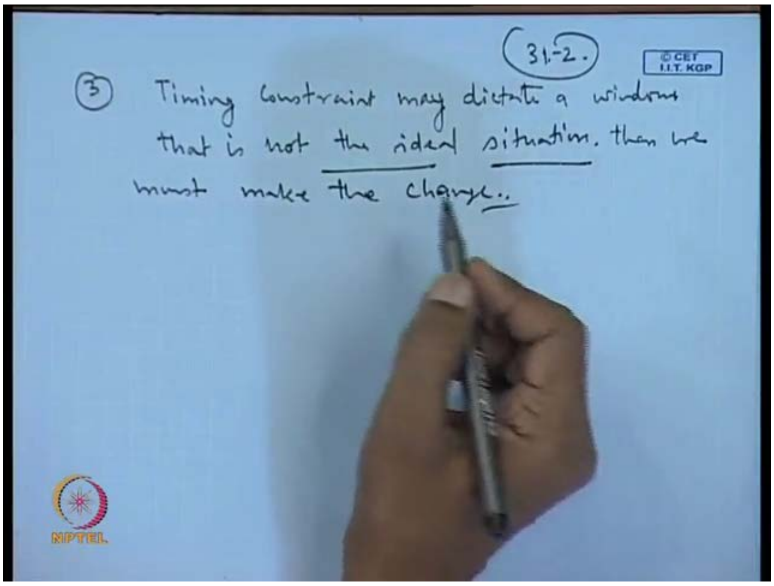

Sometimes timing consent may dictate a window that is not the ideal situation. So, timing consent may holds you to launch along a certain direction and that may not be the desired situation then you must make the change.

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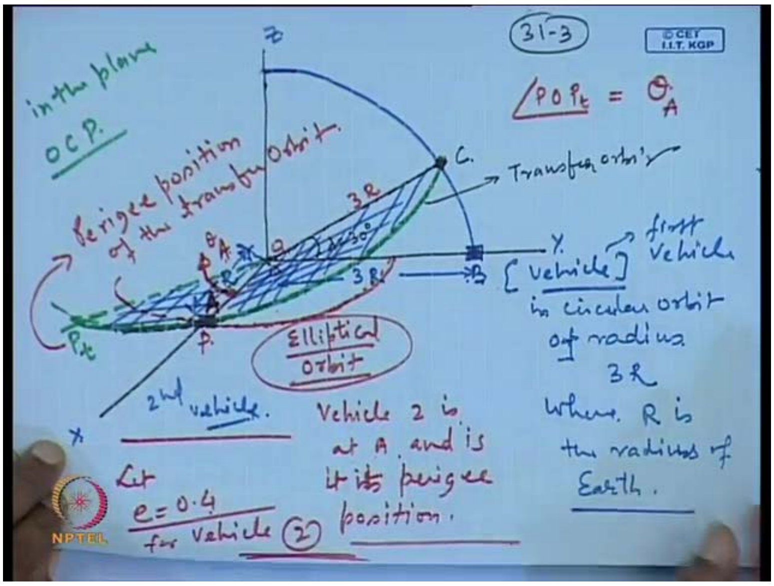

So, for this reason we have to change the orientation of the orbit suppose we have a vehicle here in this place at b. So, this is the space vehicle and this is moving in an orbit those radius is. So, this is in circular orbit of radius 3 R where r is the radius of R so, this distance is basically wrote here from here to here and this axis let us say this is X this is y and this is z. Another satellite you have, which is a line here in this place this is at point a and the distance from here to here from this point. And this is r if this is the situation and here you have this vehicle, which is the first vehicle say first vehicle and here you have the second vehicle.

And this is moving along these are circular orbit of radius r. And this vehicle let us say this is moving in an elliptical orbit in the X Y plane. So, here this is moving in X Y plane in elliptical orbit and for convenience. We can assume that this is the perigee position of the vehicle. So, vehicle two is that a and is it is perigee position. Now, as this vehicle B is moving in this orbit. So, at certain latitude let us say this angle is lambda equal to 30 degree. So, this is the new position of b, which we are writing as c. So, we have to give impulse to the vehicle to in such a way that it goes here and catches up c catches the vehicle one at the position c.

So, this becomes our transfer orbit this is your and the vehicle two it is moving in an elliptical orbit. So, vehicle two is at t a and it is perigee position and let e equal to 0.4for vehicle two. Now, you device your plan, how to send vehicle two from position a to positions c where the randhibo has to take place. So, for this and the randhibo take place here in this place at c. So, this trajectory must be designed, but this designing this trajectory now, you can see that here this radius is R while the distance from here to here this is three r so; obviously, this is not going to be a circular orbit this is bound to be a elliptical or the hyperbolic orbit.

So, we what we are going to do? We will assume that let us assume as earlier we have done. So, in the plane OCP. So, in the plane OCP which is given by o c and then this trajectory and this p somewhere the perigee of the transfer orbit lies. So, this we show as P t which is the perigee position for this; obviously, this whole thing is co planner. But here it is not appearing this way it is a not very possible to. So, this three dimensional thing and; obviously, the inclination of this the new. The new elliptical orbit from this fold elliptical orbit this will be give 30 degree, because if we rotate it and bring it into this position.

So, we have one orbit here in this plane and we are rotating it by 30 degree like this. So, it will match with the new transfer orbit. So, we have the new transfer orbit, which we can as it like this. So, this is the new transfer orbit you can catch it like this is constituting one orbit. And P t is the perigee position of the transfer orbit this is the perigee position of and what is it is location? So, location of p from o t so, this is o and this is P t from o P t. So, we can write here the angle P O P t. As let us say theta p r the here the, because this we have named this as a position a. So, we will write as theta a so, this angle from here to here this becomes theta a. So, now, we have described the problem what we need to do?

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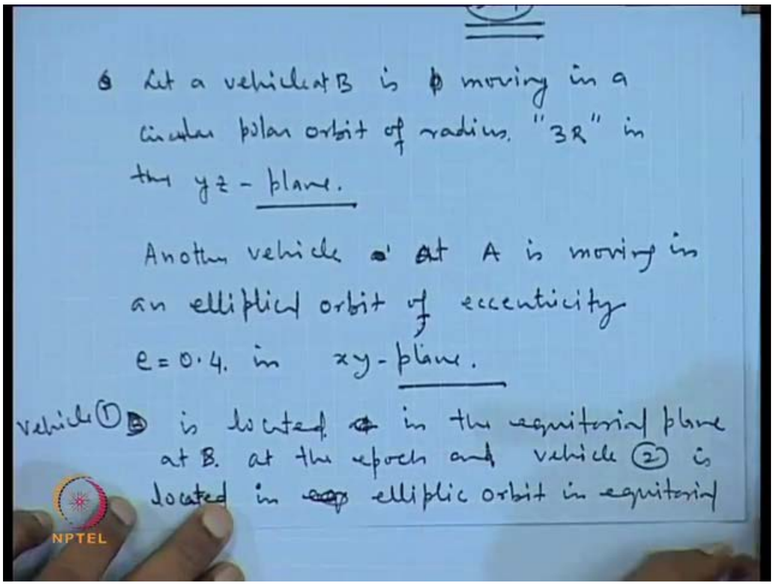

So, formally we can state the problem like this. Let a vehicle b is moving in a circular polar orbit of radius three r in the why is it plane? Another vehicle at a is moving in an elliptical orbit of eccentricity a equal to 0.4 and x y plane.

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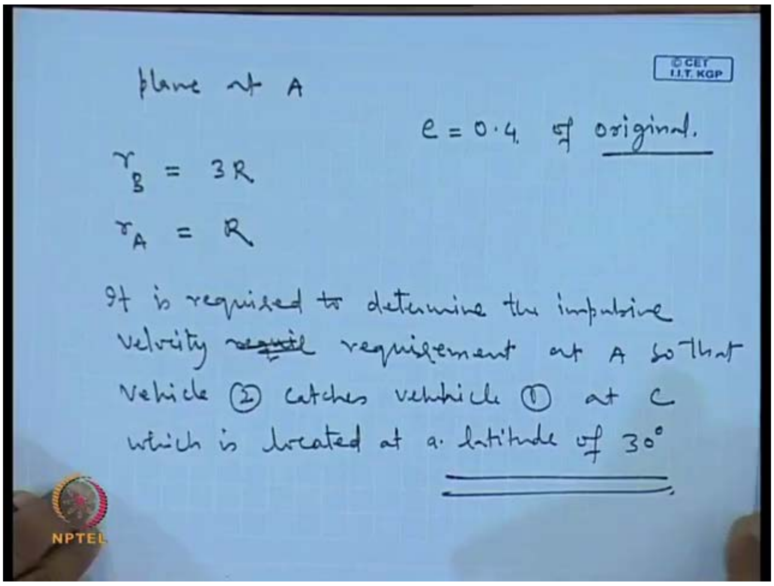

So, v is located in the equatorial plane vehicle to we write here vehicle one is located in the equatorial plane at b at the epoch. And vehicle two is located in elliptic orbit in equatorial plane at a. In fact, the vehicle two is a moving in the equatorial plane only and this is an elliptical orbit. So, r v is three r and r A is r it is eccentricity is 0.4 original orbit and for polar orbit; obviously, the eccentricity will be zero. It is required to determine the impulsive velocity requirement at a. So, that vehicle two catches vehicle one at c, which is located at a latitude of 30 degree. So, this is the formal statement of the problem.

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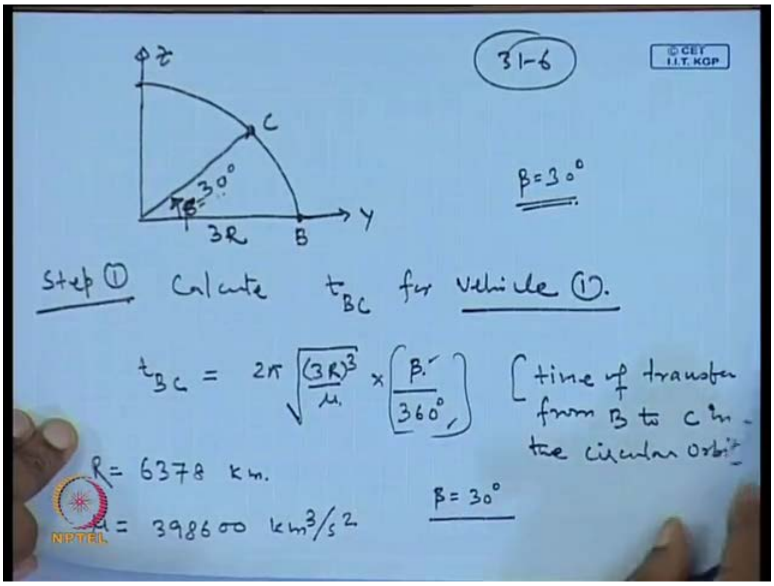

Now, how to proceed? So, we need to go exactly in the same way as we have done earlier. So, all the states are involved are exactly same nothing different here. So, first of all we will work out in the y z plane the time period of so, the step one calculate t B C time taken to go from b to c for vehicle one. So, this is the position b and this is the position c, which is at 30 degree and this is along y direction and this is along z direction this distance is given to with 3 r. So, t B C this will be given by this whole time period for going in one circular orbit.

And that can be proportionally scaled for that 30 degree angle. So, this is two pi times three r By mu three r whole cube under root and then; obviously, whatever this angle is so, let us say that we write this as angle beta. So, we write this angle beta. So, this will be beta divided by 360 degree where beta is in degree beta equal to 30 degree. So, this gives you the time of the transfer from b to c in the circular orbit. Now, this values are known to us so, we have the radius of earth is six three seven eight kilo meters mu is given to be three nine eight six zero, zero kilo meter q v per second square beta is here 30 degree.

So, we are just proportionately scaling it this you would have written in terms of the radian also, if you write in radian this also, you need to express in the radian, but ultimately the ratio will be remain same this ratio is not going to change.

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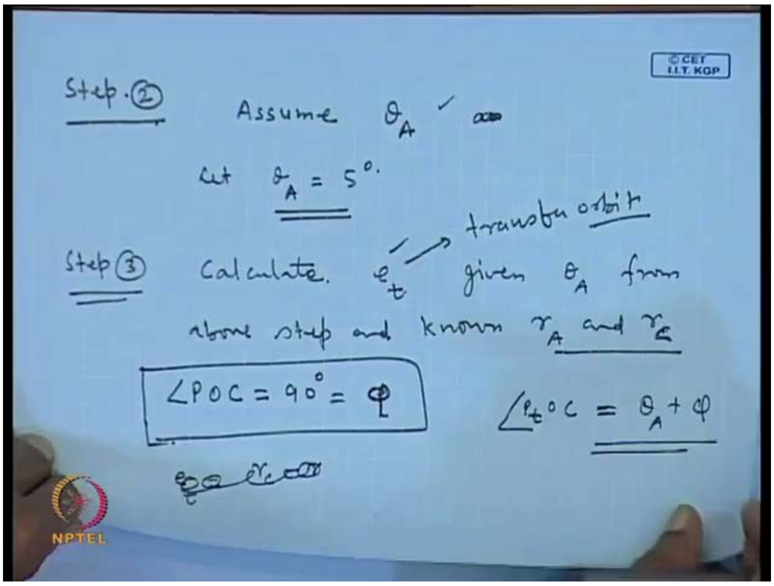

So, if we now put this value. So, t B C will become two pi times 3r to the power three by two under root mu times 30 degree divided by 360 degree. So, this becomes 5 by 6 mu under root times 3 r to the power three by two and inserting the value of r. So, t B C gets will get reduce to three times 63678 this will give you approximately 2195. Step to assume theta a so, theta a we have assumed here in this figure as shown here this is your theta here from this place to this place. This is angle from here to here so, assume theta a. So, let theta a equal to 5d next calculate e t given theta a from above step and known r A and r C. So, this is further transfer orbit and this angle from here to here.

So, the angle P O c angle P O c we will show as P O c is nothing, but 90 degree this we will show as five e. So, c is located from o p by this angle five by two or ninety degree. So, we need to calculate e t so, earlier we develop the equation of e t. So, e t we wrote as r C minus r A in a particular elliptical orbit. So, we have now the shown by this green line is here in this place o this is constituting one transfer orbit thought it is a at appearing little non planer, because of this black line. But it is a say one plane in which the transfer orbit is located so, the angle from this perigee line or preiaxis which is o P t to o c is nothing, but theta a plus 5. So, the angle P t o c this is theta a plus 5. So, we can utilize our old relationship that we developed or either we can face facely work it out.

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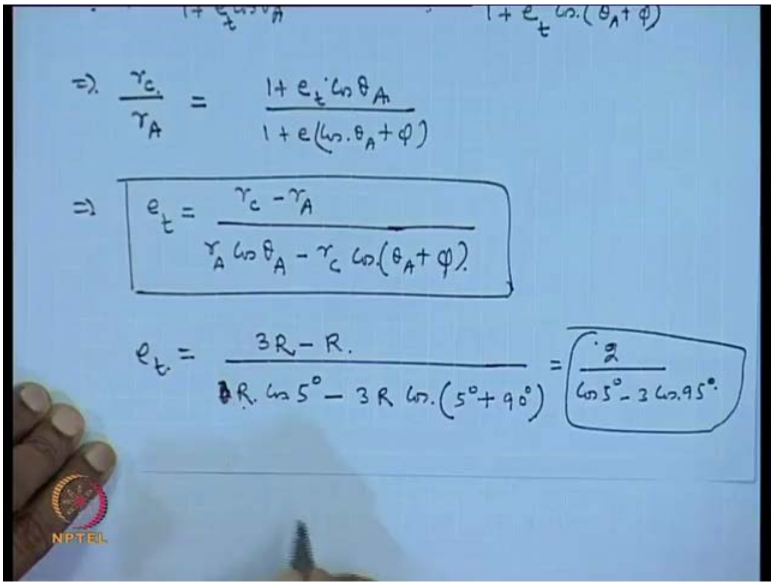

So, we have r A by r A equal to l by one plus e cos theta a and r C equal to l by one plus and here t we will put here to indicate this is eccentricity of the transfer orbit, e t Cos theta a plus phi therefore, this implies r C by r A. So, we divide r C by r A here and I usual we have done. So, this will get reduced to eighty equal to r C minus r A divided by r A cos theta a minus r C cos theta a plus phi. So, in the third step we can calculate this value now, r C and r A these are given to us. So, r C is three R and R A is r this becomes 3 r and cos theta a so, theta a we have assumed to be 5 degree. And r C is again r A is r and r Cos 5 degree and this is 3 r Co theta a is now, 5 degree and 5 is 90 degree. So, r will cancel out in the numerator and denominator so, this will be two divided by cos 5 degree minus 3 cos 95 degree.

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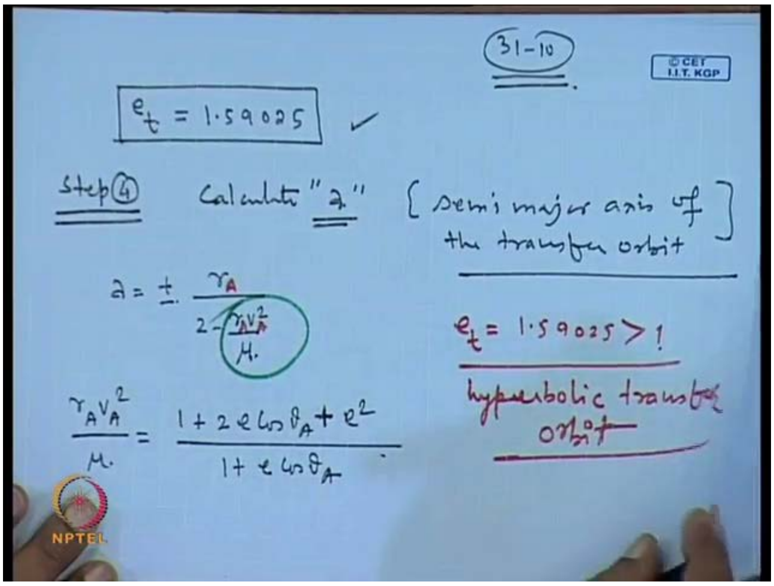

So, this is a value of 80. So, 80 can be computed and it will turn out to be 159025. So, this is the first stage we are working out here once we have done this. So, the next step will be this was a step number three. So, step four calculate a. And this is a semi major axis or the transfer orbit, but before calculating a using the relationship plus minus r by two minus r by v a square by mu you need to calculate this. So, here e t is given to be e t is just now, we have worked out 1.59025, which is greater than one. So, therefore, hyperbolic transfer orbit now, compute r v a square first and therefore, then you will be able to compute the value of a and this we need to work it out at the position a. So, r A y 2 minus here we can put the subscript a and all the places so, at a we are calculating the semi major axis of the transfer orbit. So, r A v a square by mu.

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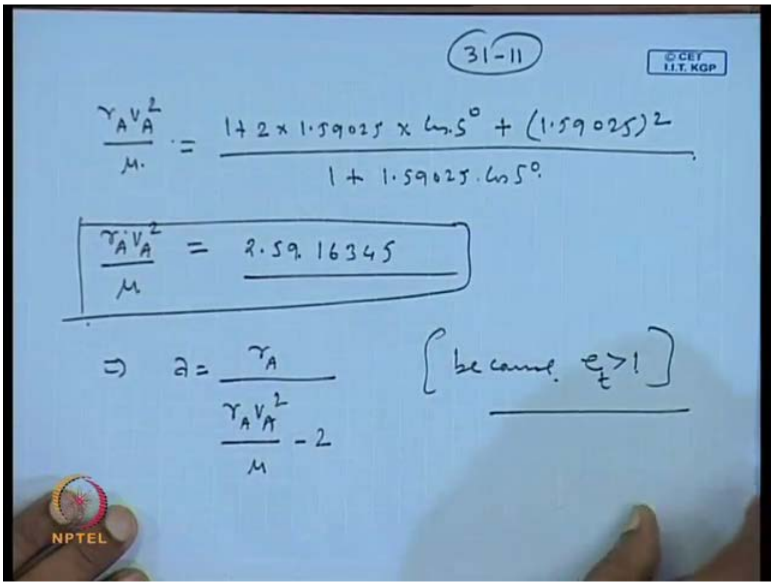

This we have written as one plus two e cos theta a plus a square, just in the lecture we have derived this equation. Now, compute this values so r A v a square by mu this will be 1 plus 2 and e is 1.59025 this cos theta is 5 degree and plus 1.59025 a square 0 2 5 cos 5 degree so computing this quantity so, we get the result 2.5 91 63 45. So, this is your r A v a square by so therefore, a will be r A y r A v a square by mu minus 2, because e t is greater than one and therefore, we have to consider the negative sign in this equation.

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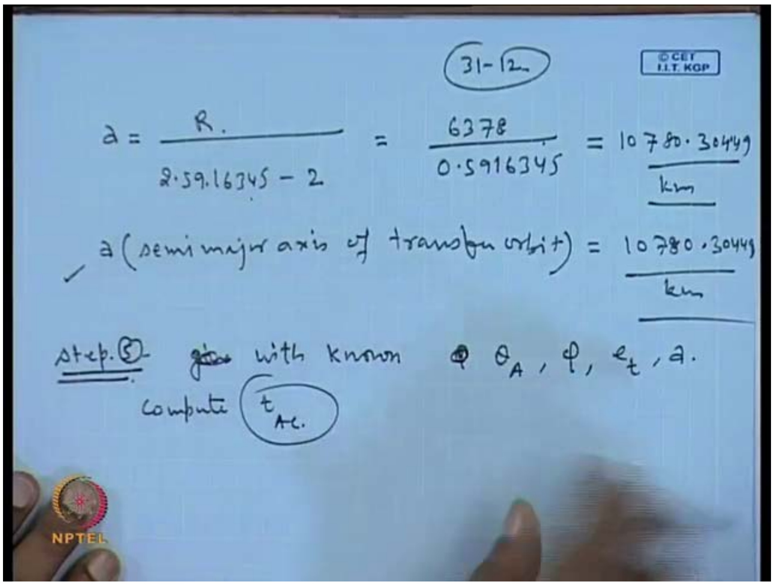

So, this will get reduce into this form and inserting all the values here. So, r A equal to nothing, but so this becomes equal to r A equal to r and r A v a square is 2.591634 5 divided by mu this quantity we have copied here this quantity we are talking in this place this minus 2. This value is given to us 6378 divided by 0.5916345 and this will turn out to be 10780.30449 kilo meters. So, a the semi major axis transfer orbit this becomes 107 80.30449. So, here in this for the non co planner maneuver the instead of this writing the equations, we are taking a problem and demonstrating.

How the whole thing can be worked out? Basic equation we have already developed. So, there was no need of repeating the same thing rather than a problem would have done this problem can so this concept in a much better way so we are following that. Once we have done for a so, the next step 5 now, given orbit known theta a phi e t A. Compute t A C so going where into this equation. Now, t A C you have to calculate so, first you have to let us defeat this transfer orbit in the elliptical from.

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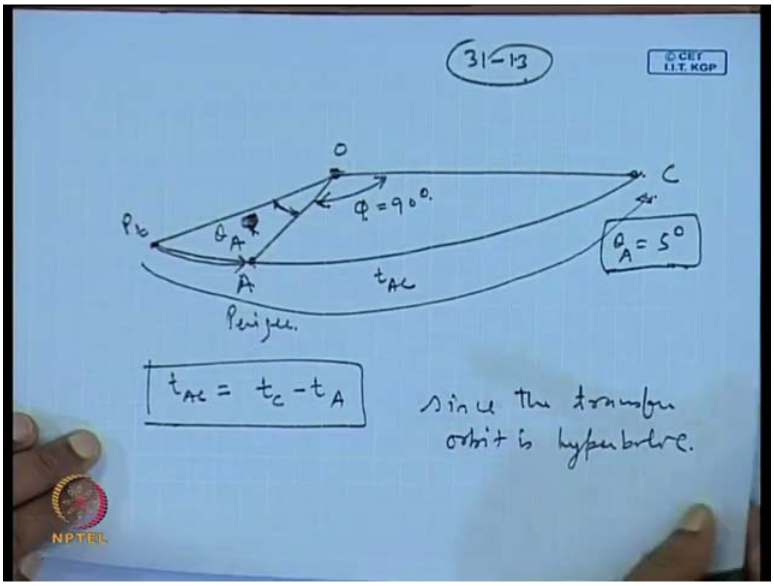

So, you have this point two and this is the point P t this is the point c. And somewhere your point P or point a is lying here, which is the perigee position perigee instead of writing here p we can write here as perigee. And this we have done just for convenience equally, this could have been a non perigee position, but then becomes more complicated to do a as example in a lecture class. So, this angle is theta a and the angle from here to here this we are writing as phi, which is nothing, but 90 degree and this theta a you assume that 5 degree. So, to find out the time t A C we need to find out the time to go from this place to this place and from this place to this place and subtract it.

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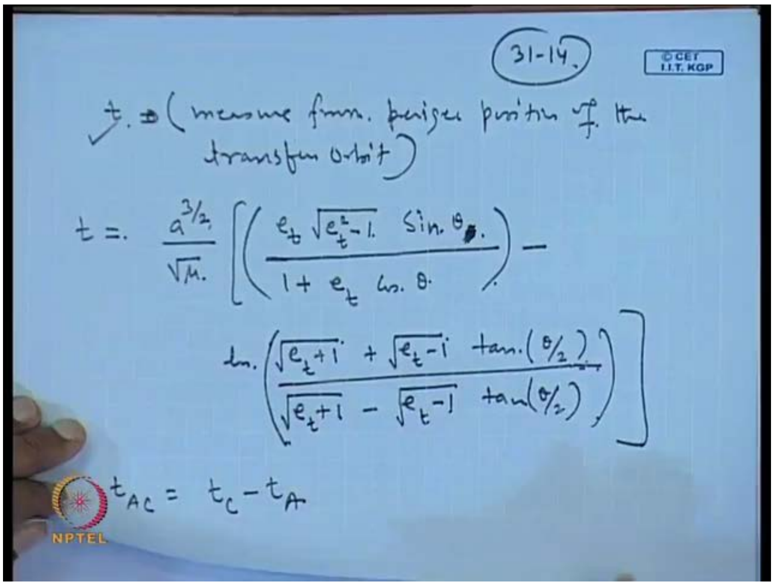

So, t A C will e equal to t C minus t A. And since the transfer orbit is hyperbolic therefore, we need to use the equation that we have developed for hyperbola earlier. So, e for hyperbola t being measured from perigee position of a is the perigee here you remember a is the perigee position of original orbit. While P t is the perigee position of transfer orbit so, we are concerned with right now, with the perigee position of the transfer orbit. So, this t Can be written as a k by 2 a a to the power 3 by 2 at by mu times e t times e t A square minus one times one times sign theta let us say this is theta 0 is the position of the satellite in the orbit at, which we are calculating the time or zero we can drop just we can write theta here. So, this equation we have written earlier so, using this equation so we can write t s e equal to t C minus t A.

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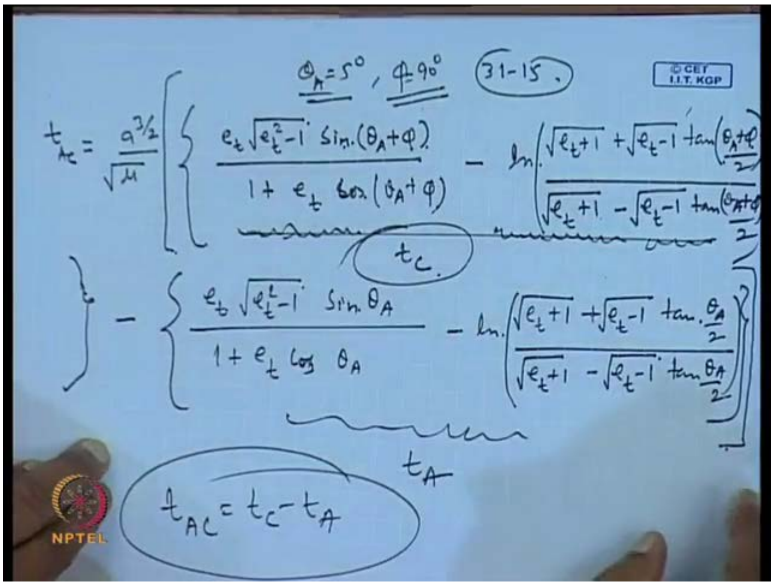

So, therefore, t A C will become a to the power 3 by 2 divided by mu under root in bracket they will have here e t times. E t square minus one under root sine theta a plus phi divided by 1 plus e t Cos theta a plus phi here theta a we have taken as 5 degree assume and 5 is given 90 degrees, which is nothing, but the position of the this angle is your five from P this is A o C angle, A o C here we can change this to is P t op or this we can change it to a. Because the p we have change to notation the p notation we have written as perigee. So, instead of this we are using this a.

So, a o P t is theta a this angle and A o C angle, A o C is 5 wherever we have written earlier this needs to be modified little bit. So, this earlier we have written as the P O c. So, we can modify this and write this as A o C equal to 90 degree equal to phi and when this is P O c is equal to theta a plus one this is one. So, little bit modification here and there rest are fine say only under little bit in notations. So, here this rest of the terms we an complete l n e t plus one under root plus e t minus one under root tan theta. Now, the total angle is from here to here from this line to this line this total angle is theta a plus phi.

So, that is entering here in this place. So, here you will have theta a plus phi divided by 2 and this divided by e t plus 1 under root e t minus one under root tan theta a plus phi divided by 2 and this has to be closed in other curly bracket. So, we are closing here in this place and this minus. So, this quantity is here t C and it same way we have to write here for t A. So, for t A only sine theta will appear here in this place a t Cos theta a minus l n this is not the dividing line. So, e t plus one under root plus e t minus one under root minus tan theta a divided by two and this divided by tan theta a divided by two then this bracket gets closed and then the last brackets gets closed. So, this is your quantity t A. So, t A C equal to t C minus t A which we have written here in this place and you need to insert all the values in this equation.

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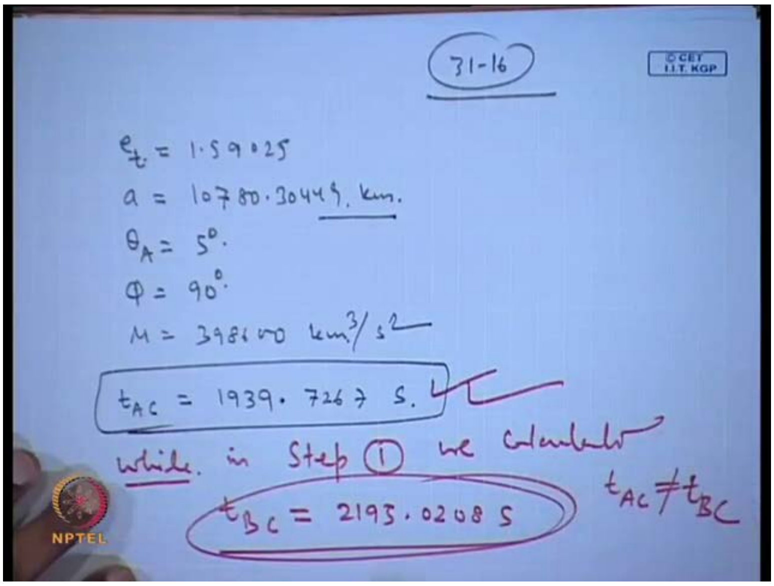

So, in this equation now you can see e t we have already calculated. So, that value turned out to be 1.59025. And then a also is available to us 10780.30419 kilo meter. So, therefore, and theta a is you have assumed to b 5 degree and 5 is given to be 90 degree rest all the quantities are known there inside this equation.

This is known mu already known to us this is 3986000 kilo meter cubic per second square e t is known theta a five is known therefore, we can compute this quantity. And if you work it out. So, t A C will turn out to be one nine three nine point seven two six seven. So, this is the value of the t A C now, while in step one we calculated t v c equal to 2195.0208 second. So, we can see from this place that t A C is not equal to t B C and therefore, we need to repeat again from step two where we assume the value of theta two. So, the next step will be this was our step number 5.

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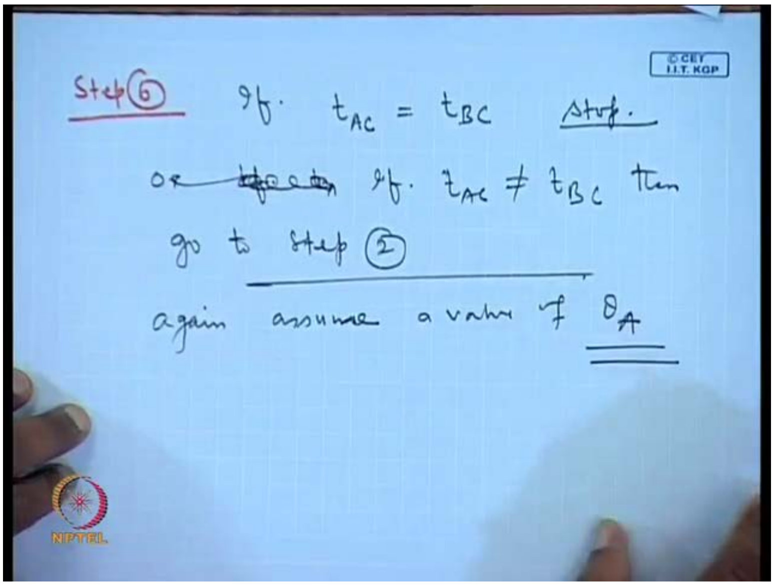

So, the step number 6 if t A C equal to t B C stop or if t A C and we can say, if t A C not equal to t B C then go to step 2.

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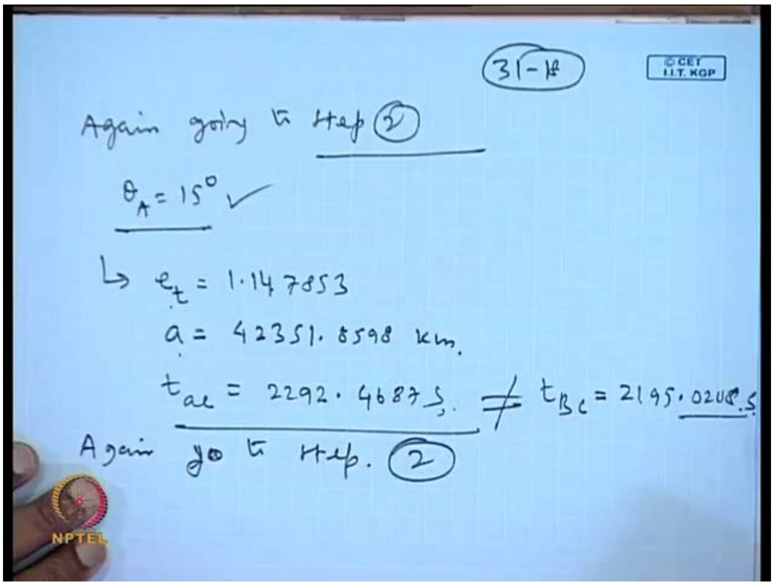

And again assume a value of theta a. So, if we assume the value of theta a so; obviously, we need to hydrate. So, again we need to go to the step two and assuming theta a to be 15 degree. So, then you can calculate e t equal to 1.147853, if you assume theta a. And follow all the steps and will have a equal to 42351.8598 kilo meters this is the semi major axis of transfer orbit. And t A C will turn out to be 2292. 4687 second. So, still you can see that this is not equal to t B C, which is equal to 2195 this is 5 this quantity is 5 here 2195.0208 second.

So, this is not equal to then again go to a step 2. So, we need to hydrate once more, because know this is closer to this. So, time is getting over. So, we will continue in the next lecture, but we have raised quite closed to the value, which we are taken here this t A C is coming 2292.467 seconds. And while the original value is 2195102 seconds. So, little bit difference is there so we will be able to plot this difference in the next lecture. So, we will continue in the next lecture Thank you very much.