Space Flight Mechanics

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Lecture No. 30

Trajectory Transfer (Contd.)

Last time we have been working between rendezvous problem between two vehicles in circular orbit. So, we continue with that little bit was remaining, so I will cover it and then, we will go into the elliptical orbit.

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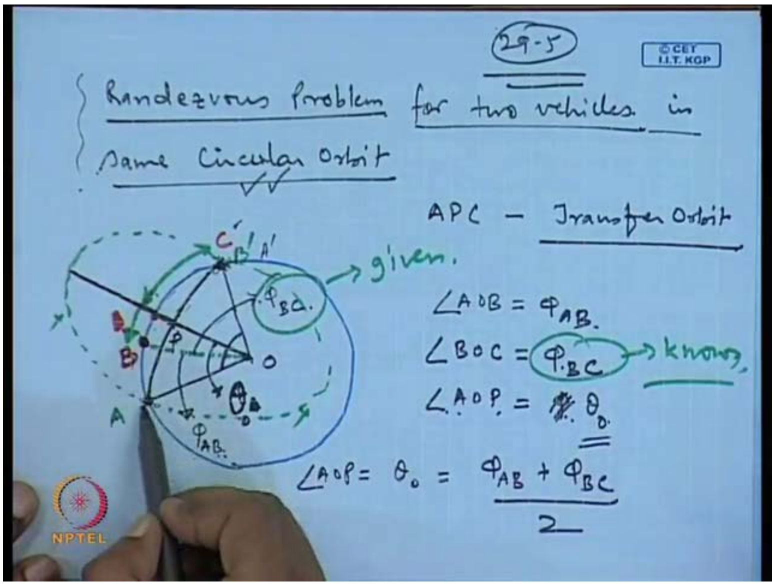

So, the problem has started with that we have two vehicles located at A and B, an angle between Aand B was given to pi A B, and what was required that while B moves along the orbit shown by the blue line. So, once it reaches the C, so the new position of the B is then shown by B prime which is coinciding with C, so at that time A has to reach at C and catch it up. So, for catching it up, it has to be sent in some faster orbit, because naturally here the time taken from going from this place to this place whatever is there.

So, if it matches with this then, only it is possible if it goes along the original orbit, then it is never possible it will always keep traveling by the same angular distance. So, if neither we saw that, if we also we saw that it is not possible to go along this, because this are going to take much longer time this is also not possible. So ultimately, we decided to send it along this orbit, and then we are started working in this direction.

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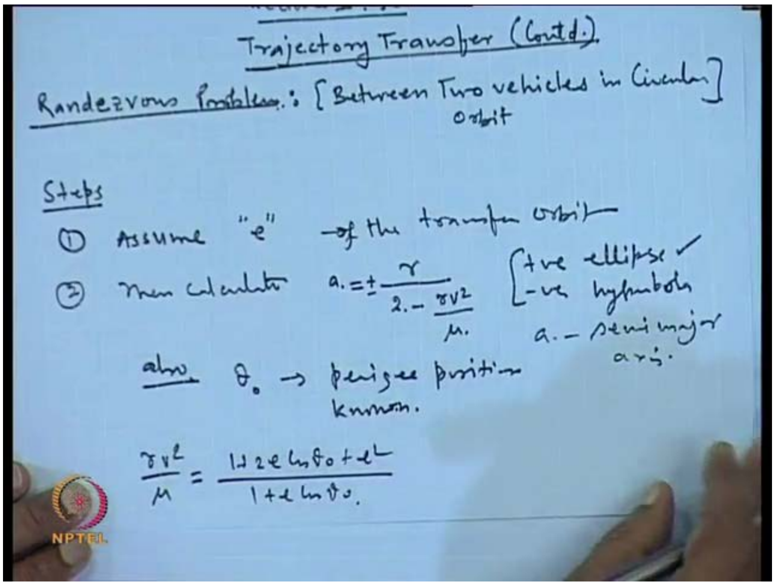

So, what we we wrote a number of steps, so in those steps involved here, the first one was assume eccentricity of the orbit, e of the transfer orbit then, calculate an equal to r byor a plus minus. We can write 2 minus r v square by mu which is plus for ellipse and minus for negative sign for hyperbola. So, if your trajectory is a ellipse, then the plus sign will be used, if your trajectory is hyperbola; then the negative sign will be used; where a is the semi major axis and also here the theta 0,which is the perigee position position known. So, if here in this step r v square by mu, this can be calculated using the relation that we developed earlier, so we can a state it here again and therefore, a will be available.

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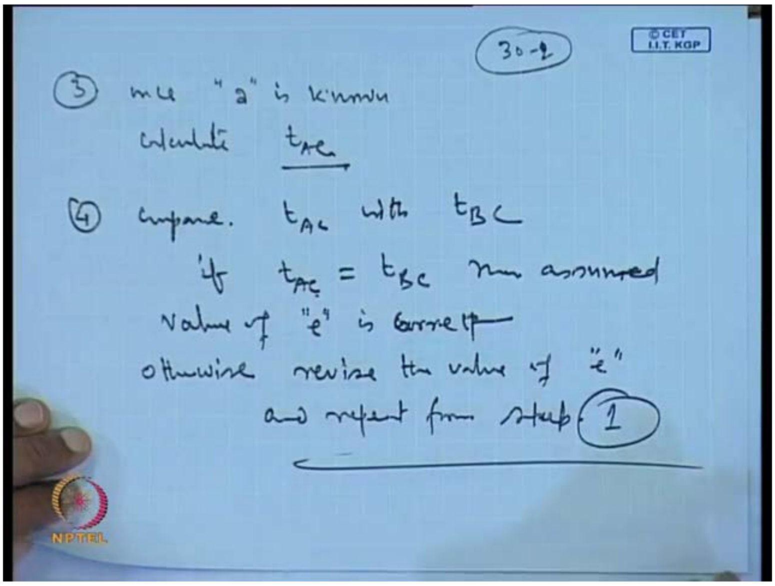

So, once a is known once a is known then, calculate t A C,compare t A C with t B Cif t A Cequal to t B C, then assumed value of e is correct, otherwise revise the value of e. And this process has to be hydrated until t A C equal become st A C equal becomes equals to t B C. So, if this ends the process for the otherwise revise the value of e and repeat from step 1.

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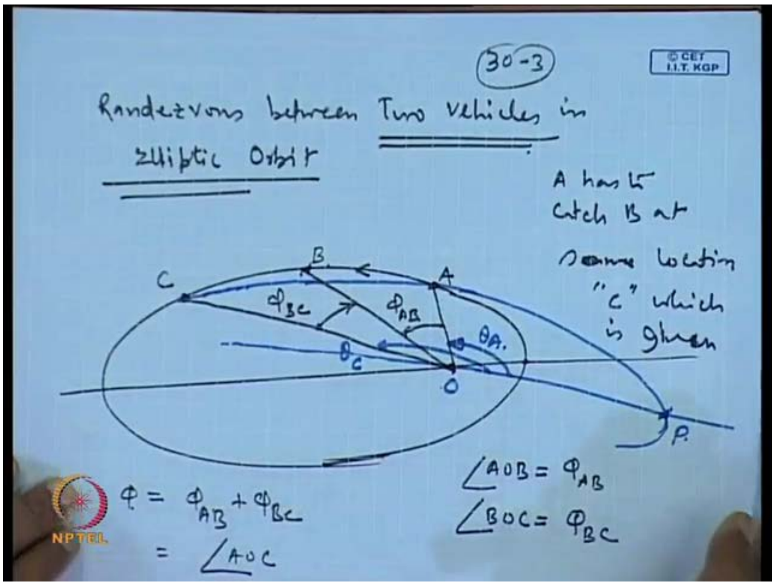

So, if this process is carried out repeatedly, then you get the desired solution, so the solution we already worked out last time. So, now, we go into the elliptic orbit, so rendezvous between two vehicles in two elliptic orbit. So, we have rendezvous between two vehicles in elliptic orbit. Now, suppose two vehicles are there which are A and another one is located at B, if focus is located here and A has to catch up B at some location say C.

So, A has to catch B at some location C which is given, so here we have this angle is pi A B and this angle we write as pi B C. Now to catch up B at C, so A must move faster than B and therefore, let us say that it has to be send along but different orbit in perigee position for the transfer orbit is shown by this blue lines. So, the position from here to here, the angular position of the O A, it can be shown by theta a similarly, though angular position of C can be shown as theta C. So, we have angle A O B this is equal to pi A B angle B O C is equal to pi B C angle write pi as pi A B plus pi B C and this will be this nothing but angle A O C.

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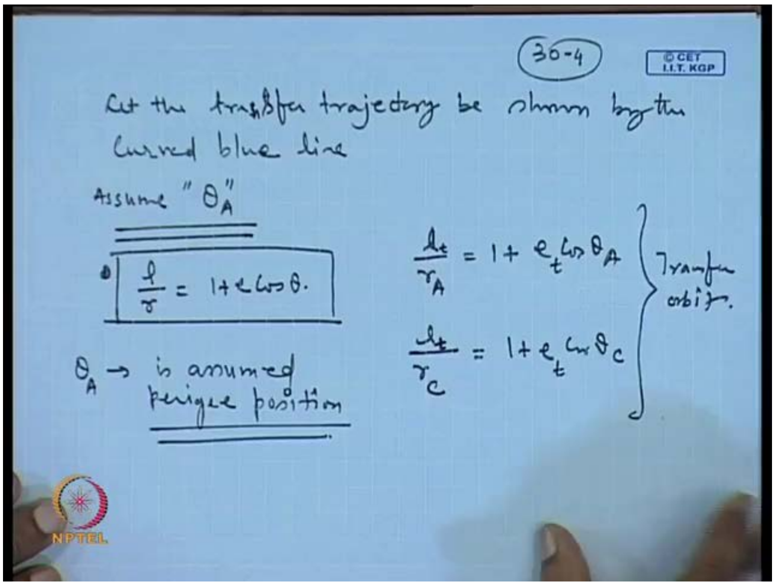

So, let the transfer trajectory this shown by the curved blue line as shown in this figure this is the called blue line, so this is the transfer trajectory. So, we now need to proceed in the same way as earlier we have done but the earlier that case was little more easier, in that the theta 0 that we compute it if you look into this the earlier figure in the circular orbit.

So, this theta 0 became known, because this whole angle was known and therefore, we divided and got the perigee position here in this place but here in this case it is not easy to do that, if it is not possible using working this policy will not work actually here, in this place, because it is elliptical or may be the hyperbolic orbit in which you are sending it and it is a faster orbit.

So, if here the theta A need to be assumed ,so assume theta A and then we can use equationl by r is equal to 1 plus e cos theta. So, for the once the satellite is here, so for this position we can write l by r A equal to 1 plus e cos theta A. Similarly we can write l by r Cequal to 1 plus e cos theta c and this is in the transfer orbit. And you will put the subscript here t to indicate that this is the transfer orbit and may be you can put here t also to indicate this is the semi, let us rectum of the transfer orbit. Here, theta A this is the assumed perigee position, so from this two equations let us say this is equation number 1 and this is equation number 2.

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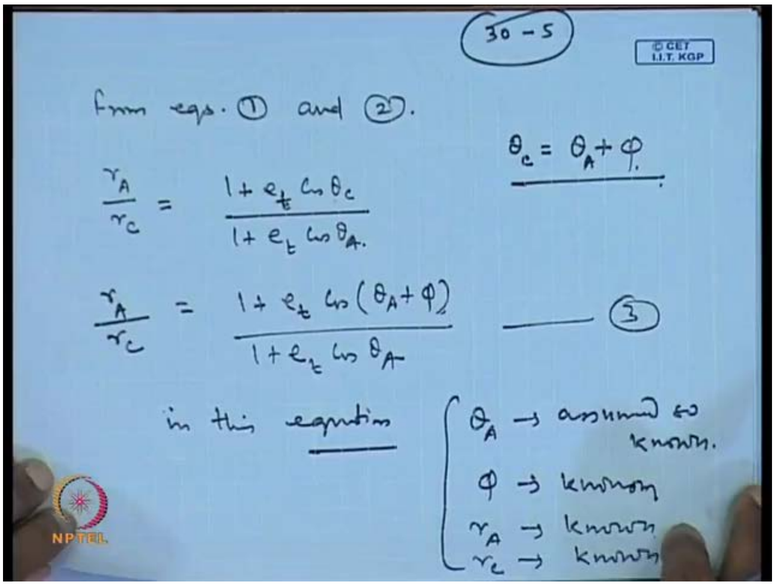

From equation 1 and 2, we can write r A by r C 1 plus e t cos theta C divided by 1 plus e t cos theta A. Now theta C this can be written as theta A plus the angle A O C, so A O C we have written as pi. So, we will add up here in this place and insert into this equation, so this becomes 1 by e t cos theta A plus pi by 1 plus e t cos theta A. So, this is our equation number 3,in this equation theta A is assumed.

So, known therefore, this is known or but maybe it is a wrong value but it is a known. Now, once you have assumed pi is known, r A is known and r C is known, because of elliptic orbit is given to you and the position of r A and r C may be given in terms of the true anomaly of the original elliptical orbit, this is shown here by the black black ink. So, from here it is a possible that we can compute the value of e t.

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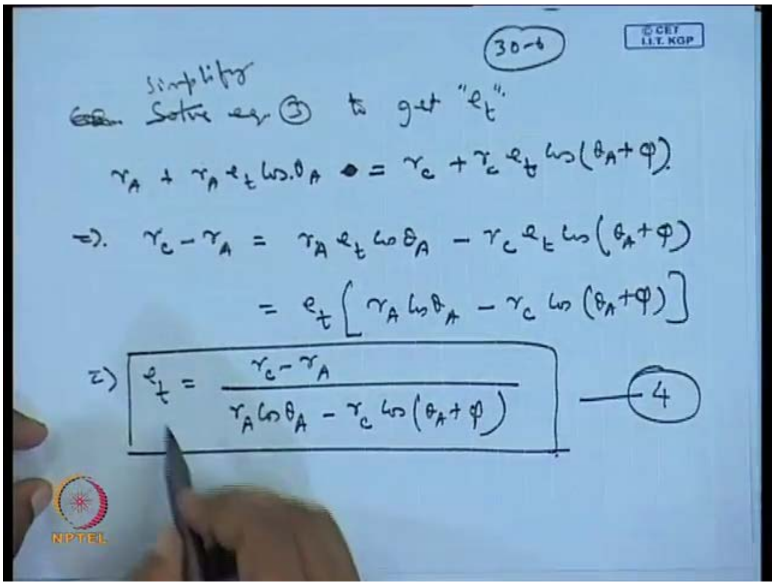

So, from equation 3, so solve equation 3 to or simplify equation 3, that simplify get e t. So, if we do that, so this will be r A plus r A e t cos theta A, this is equal to r C plus therefore, this implies r C minus r A,r C e t cos theta A plus pie t we can take as common. And this implies e t equal to and this is our equation number 4, which gives the value of e t. So, here r A r C theta A and pi all these are known and therefore, e t can be computed.

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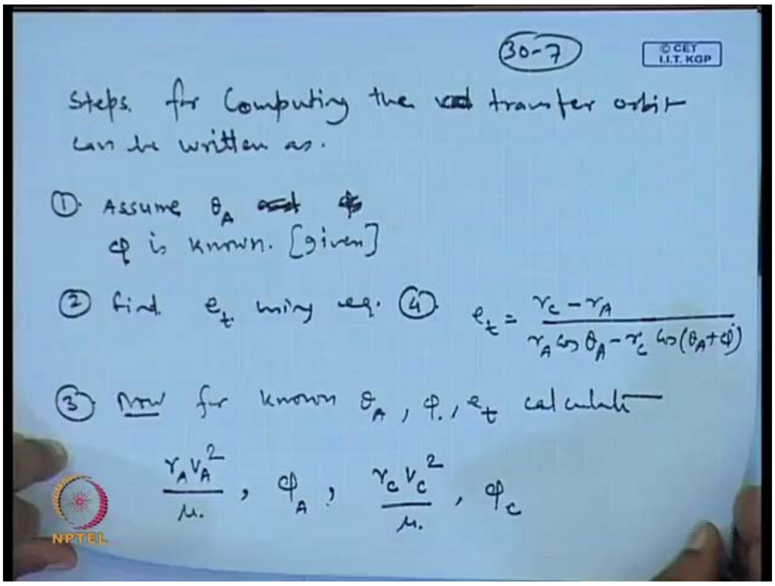

So, now, the steps for computing the transfer orbit can be written as assume theta A. And assume theta A, pi is known, this is given; point e t the eccentricity of transfer orbit using equation 4 that is you need to equate using r C minus for known theta A pi e t calculator A v square by mupi a the flight angle at a then r C V C square by mu and pi C.

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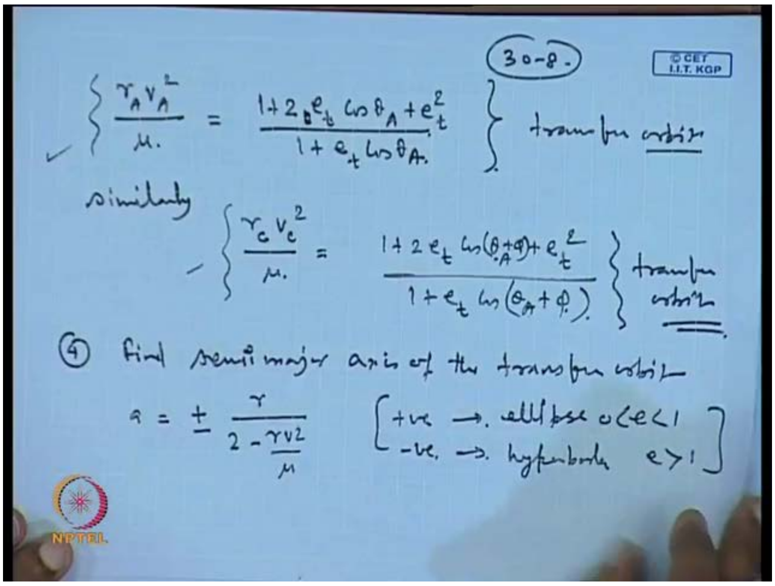

So, r A v A square by mu, this can be written as 1 by 2e t cos theta A; similarly r C V C square by mu. So, this you are writing for transfer orbit, A plus pi cos theta A plus pi find semi major axis of the transfer orbit.

So, this will be given by a plus minus r by 2 minus r v square by mu here plus sign for ellipse. So, if your e is turning out to be less than 1,then you need to choose the plus sign and if your e is turning to be the e which you have calculated in the second step. So, if it turns out to be greater than 1 means hyperbola, then you need to choose negative sign in this equation. Now, if the pi A and here whatever we have written pi A.

And so, we will write this as flight path angle at A flight path angle at C. So, we will see that how the flight path angle to be indicated in this case,we will do at the end after writing all the steps.

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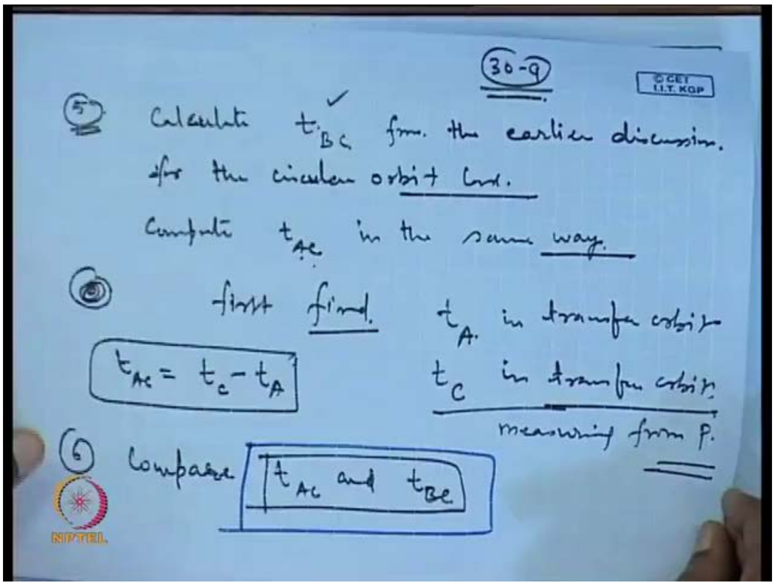

So, step number 5, we have now calculate t B C, from the earlier discussion for the circular orbit. And it is the easy doing calculating t B Candin the original orbit how much time B is going to spin to go from B to C. So, this we are writing as t B C and then also you need to calculate along the trajectory transfer trajectory this t A C. So, compute t A C also; now because the eccentricity is known to use semi major axis you have calculated for the transfer orbit also, this position already you have assumed. And therefore, from this position to this position going from this position to this position in the transfer orbit, how much time will be required that is indicated by t A C can be computed exactly in the same way.

So, for calculating t B C is easy but for computing t A Cwhat you need to do, so first point t A in transfer orbit and then find t C in transfer orbit and measuring from P. So, this implies that you find out how much time it will be required to move from this place to this place. And how much time is we required to move from this place to this place in this transfer orbit, and then subtract. So, once you subtract,so t A C will be equal to t C minus t A. So, once you have done this compare t A C and t B C, this step is important; once t A and t B Cyou are comparing.

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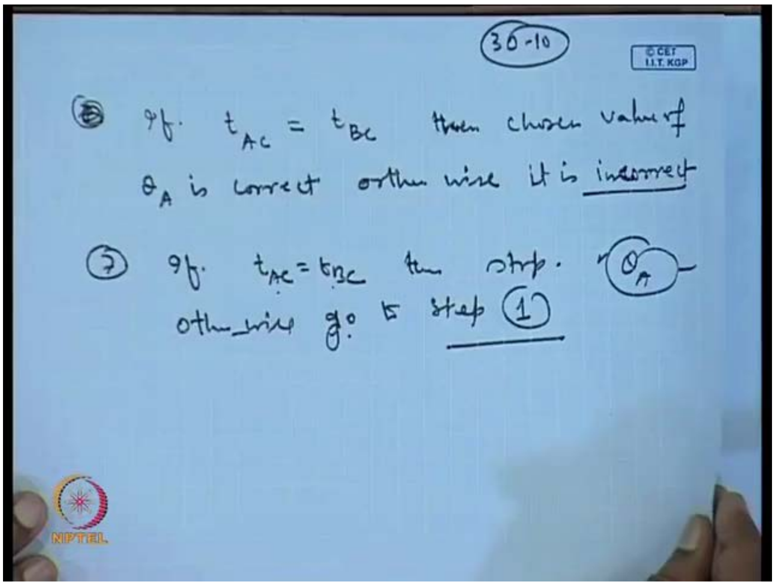

So, next step we can write if in the same step let us write if t A C equal to t B C then chosen value of theta A is correct, otherwise it is incorrect. So, if it is incorrect,soas a next step if t A Cequal to t B C, then I stop, otherwise go to step one. So, now, you can see that if t A C equal to t B C, then your assumed value of theta A is correctand once theta A is known. Then obviously, e t can be calculated the semi major axis can be calculated if it is not correct, then go to the step one assume theta A again and at rate in the same manner and therefore, you will be getting the required value of theta A after some migration.

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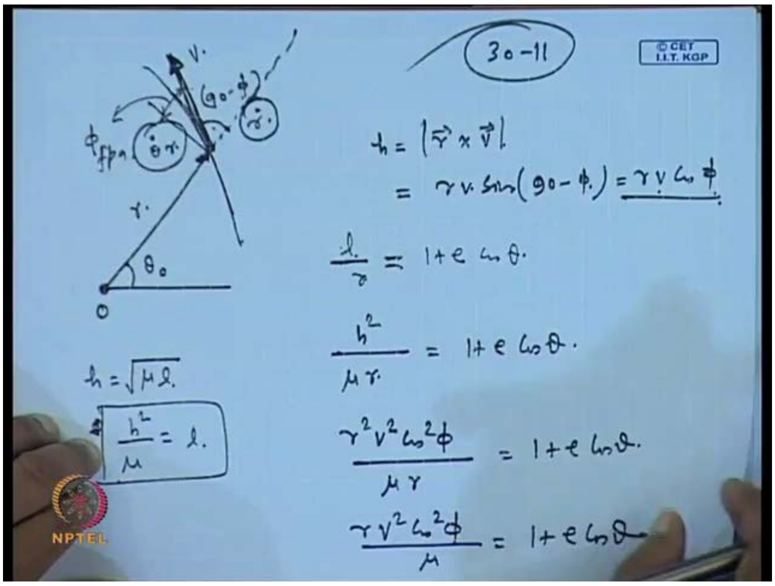

If now let us finish, we have done this earlier also but I will revise it here say this angle is theta 0 and this is the center of attraction and we are measuring r along this direction. So, this is our r, suppose this is the velocity vector at this position, and this is the trajectory and velocity vector is tangent to this, so this is called the flight path angle a which we write as f p a. So, V can be broken into two components r dot in this direction and theta dot times r in this direction and h we can write as r cross v magnitude, where the angle between this and this is 90 minus pi, and also earlier we have written in terms of alpha.

So, it is multitudes of way are there ways are there in which you can write. So, this is r times v sine 90 minus pi is equal to r v cos pi. So, ifhere we know that h is equal to mu times l under root and therefore, h square by mu we can write as l. So, here in we can replace this as h squareby mu r equal to 1plus e cos theta and h square from here again we can insert r square v square cos square pi divided by mu r equal to 1 plus e cos theta. And therefore, r v square cos square pi by mu this becomes 1 plus e cos theta.

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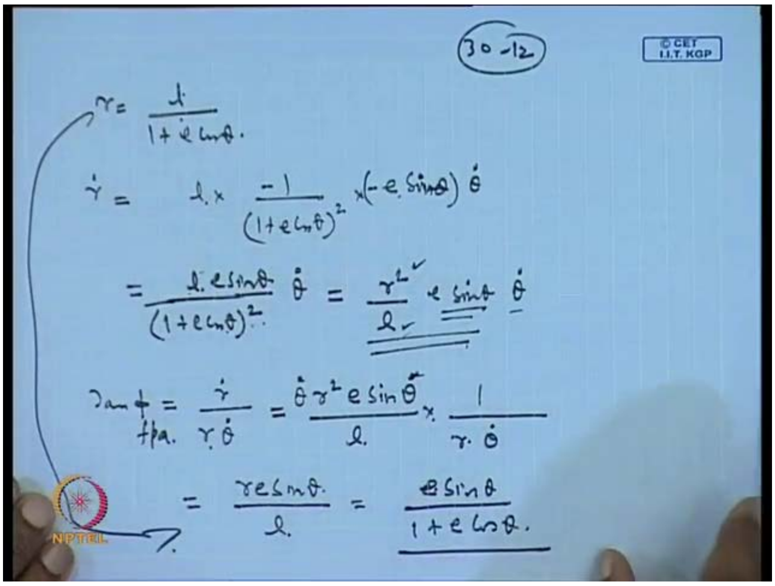

Now, taking again this equation r is equal to l by e plus cos theta, so we have r dot equal to l by l times e cos theta whole square times e time sine theta into theta dot within minus sign here. Now, here in this equation we can do little bit simplification. So, this can be written r square by l times e sin theta and theta dot. So, what we have done here multiply the numerator and denominator by l1. So, this becomes l square and the below you have 1 plus e cos theta square and you will have l turn here.

So, from here we can see that l by 1 plus e cos theta is equal to r and therefore, that becomes r square and denominator whatever the l is there and that comes and then; obviously, we have this terms here. Now, we can see that, tan pi this is given as r dot by r times theta dot here this is the angle pi, so tan pi will be this is f p a we can write this f p a. Sothe, this distance divided by this distance, so in this direction we have theta dot r and here this is r dot component of B.

So, r dot just now we have got r square e sine theta times theta dot divided by l and then sine theta times theta dot. So, theta dot we write here and when, we have 1 by r times theta dot. So, this we will cancel out theta dot term and what we get r e sine theta by l and from here you can see, that r by l is nothing but 1 plus e cos theta. So, this becomes e sine theta divided by 1 plus e cos theta this we are using here in this place.

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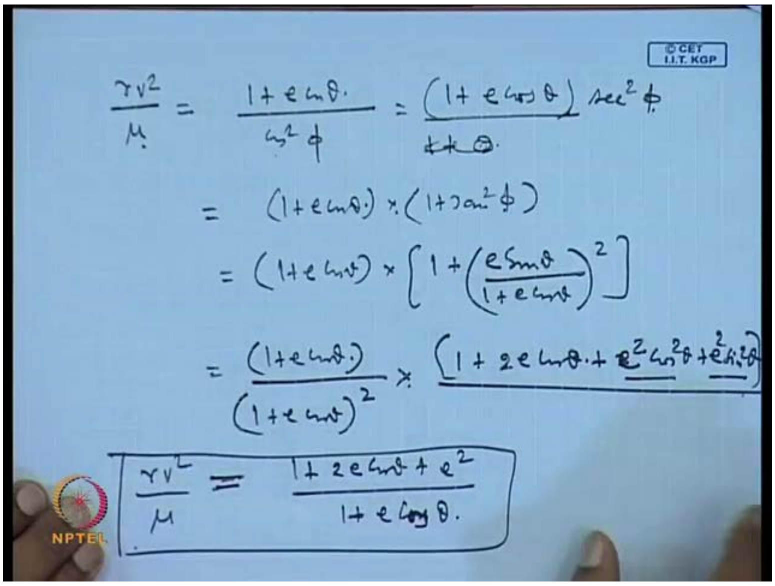

So, this flight path angle this is necessary to calculate and it is always required and therefore, r v square by mu e cos theta and 1plus this we can write as r v square by mu is equal to 1 plus e cos theta divided by cos square pi and this we can take our sec square pi, 1 plus tan square pi.

Here this term will cancel out and here these two terms can be added to give v square. So, this becomes 2 e cos theta plus e square by 1 plus e cos theta. So, this equation plays an major role in finding out the value of r v square by mu given the true anomaly theta ,this is nothing but the theta which is appearing here this is nothing but your true anomaly. So, once the true anomaly is known e is known and therefore, r v square by mu can be calculated and this r v square by mu.

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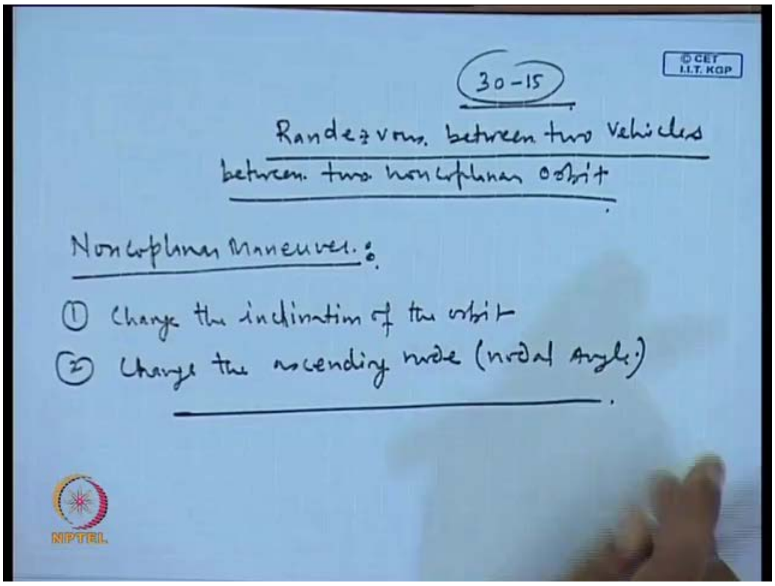

This is required in the in calculating the semi major axis, which we have always written as plus minus 2 minus r v square by mu. So, once you get the r v square by mu. So, therefore, a can calculate. So, in all the process just we see that, either we have to assume just some eccentricity has we have done in the circular case or either we have to assume the perigee position and then utilize the eleventh equation to get the value of the eccentricity.

So, if by this process we are able to work out the desired transfer orbit in which the satellite should be sent to do the rendezvous. And half coat from the satellite which is near the, so if this is the precisely theoretically what we do but in the actual orbit except once the satellite reaches near the another satellite, then your precise maneuver using the camera is done and then the rockets are fired and there are camera.

So, exactly whatever the requirement is here that is used to (( )) the two satellites, so mini missiles you might have heard that,the one satellite is (( )) to the other. So, this is done using this technique. So, thus we have computed all the relevant information's for if rendezvous in two orbits either B at the circular or B at the elliptical orbit but till now whatever we have discussed this is all about the coplanar transfer. Now the question arises what if we have the non coplanar transfer.

So, and so, non coplanar transfer we have little more complicated steps involved but the procedure will remain same. So, we will go through the non co planar maneuver to do the rendezvous between two vehicles, which are moving in two different orbits between two non coplanar orbits.

So, if the question now arises why it is, so important to do this non coplanar maneuver there are various reasons for that. So, let us first go into that non coplanar maneuver, so as the meaning implies the two orbits which are not in the same plane. So, this will change the inclination of the orbit and it will change the ascending node or what we have written as the nodal angle.

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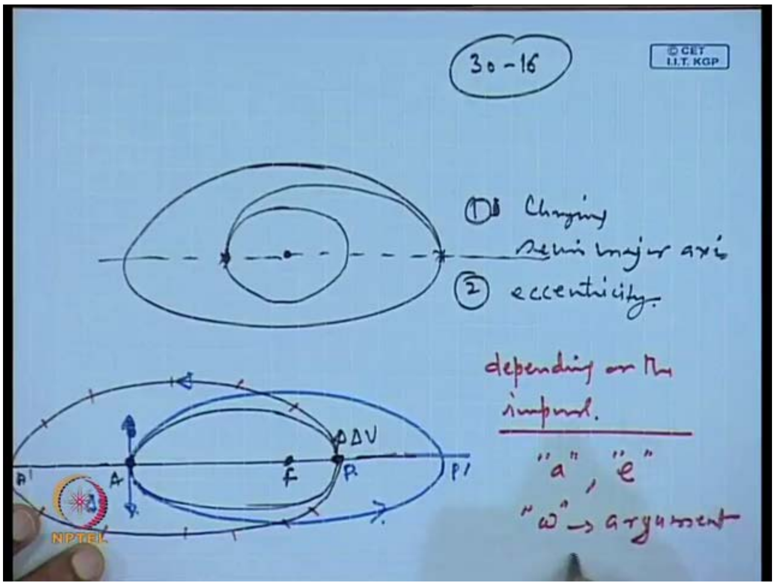

Now, if we look into if this the coplanar maneuver, so in the coplanar maneuver what we did either we had the elliptical orbit or the circular orbit. So, in the case where we are inside may be the circular orbit and then, we have go into the outer orbit which the elliptical one or both are elliptical.

So, in such cases if you give impulse here, in this place if you give here impulse in this place. So, what exactly you are doing we are just changing semi major axis one the second thing what we are changing eccentricity. So, this will be always the case that, if we are applying the impulse either at the apogee or the perigee, so if we are say this is our focus f, so if we apply the impulse here delta v.

So, when we are going to boost up the apogee position, so apogee will A will move from here to here, on the other hand if we apply the boost here, in this place that you say let us apply in this direction, so if we apply her in this direction.So, then we are going to boost it the perigee position, so perigee this is the perigee position which will move to P prime.

On the other hand, you can apply the impulses at different points in the orbit other than apogee and perigee, if you do that then what will happen. So, depending on the impulse impulse your semi major axis a the eccentricity e and the argument of perigee of small omega, this is argument of perigee. Thus the coplanar maneuver,we can see that it is able to change the semi major axis, the eccentricity and the argument of perigee and all these quantities they lie in the same plane and that is why it is called the coplanar maneuver.

So, coplanar maneuver only can change the coplanar quantities, it cannot change the non coplanar quantities, which are characterized by the angle of inclination I and the nodal angle.

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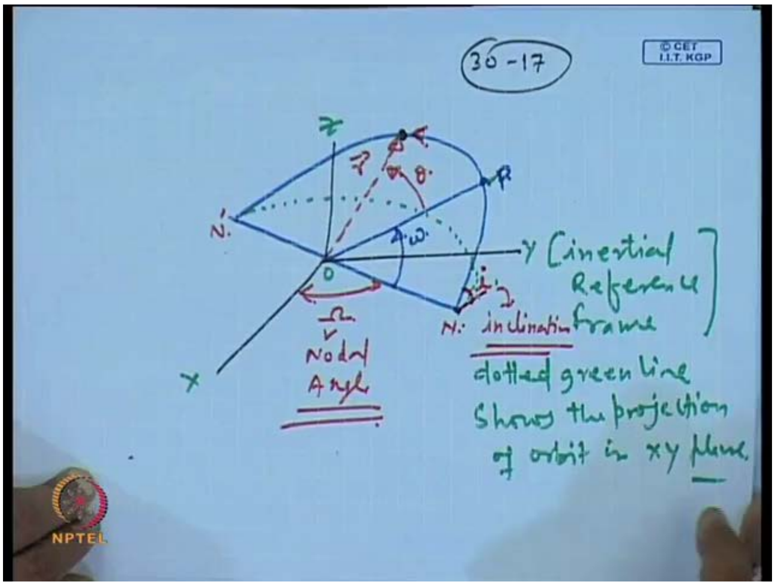

Say this is x, y and z this is inertial reference frame, this is the center of the force here; we can show by this green line, the projection of this orbit on the x y plane. So, the dotted green line projection of orbit in x y plane. So, angle between this projected line are the x y plane and the orbit this is your angle of inclination and this we have written as the ascending node and this is the descending node o prime if the satellite is moving in this direction.

So, you have the angle of inclination and the nodal angle then we showed as angle from here to here, argument of perigee we showed as p here in this place. So, this was the perigee position, so arguments of perigee was shown like this omega, and then have the satellite in the orbit in some place here, whose radius vector is r. So, this angle was shown to be the true anomaly. So, what exactly we are doing in the coplanar maneuver,we are just doing the maneuver in the plane of the orbit,while the known coplanarnon coplanar maneuver it is a concerned with changing the capital omega or the nodal angle this is your nodal angle and this is I this is your inclination.

So, the our next topic of interest is as we stated earlier rendezvous in the rendezvous in the two orbit which are non coplanar and this two orbits can be either circular or both can be circular or both can be non circular. Either one can be circular another can be elliptic but the thing is that the whole philosophy is that whatever we have discussed earlier the same analogy will apply here,only thing we have to causes about that out of plane.

Now, in whichever orbit the actually the satellite is moving and it has to catch up another satellite in a different orbit which is non coplanar with it. So, out of plane impulse now need to be given. So, those how much will be the value of our work quantity of the out of plane impulse need to be given that must be computed in a proper way.

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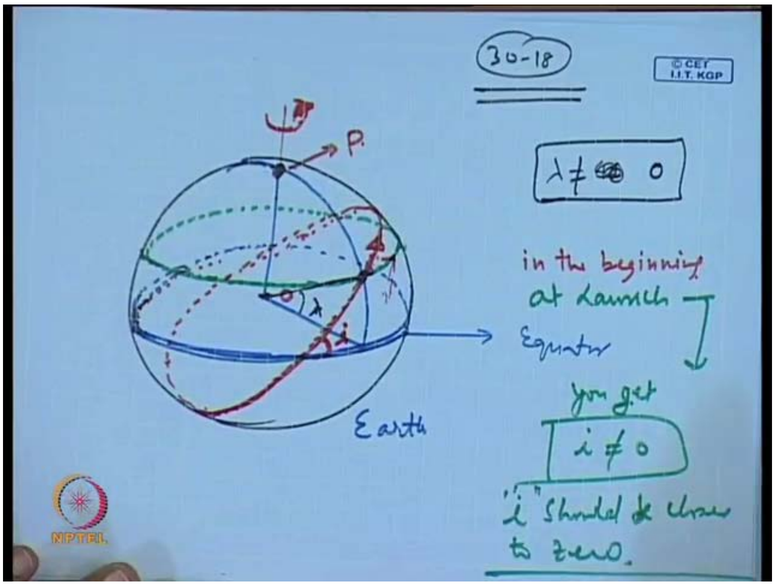

So, the now the question arises why there is a need for the non coplanar maneuver. So, the need arises,because a we can see from a diagramis the equator of the earth, this is the pole P this is the pole p this is the center of attraction or center of the orbit of, this is your equatorial plane.

Now suppose this is your earth and take another latitude, so we will indicate this by this line from here to here and let us say thisthe latitude of this point, this is lambda. Now, suppose your in this point,soif are in this point and this lambda value may be say 40 degree, 45 degree or 30 degree or even it may be towards the pole. So, lambda is not equal to 0, if lambda is not equal to 0.

So, in that you can see that the satellite launch can be done only in the along the diametric along the diameter of the earth. So, we have number of diameters here, so if this is the equatorial circumference. So, we will have the non equatorial circumference which can be shown by say, it can be shown by this line, this is the non equatorial circumference for non equatorial circle around the earth. So, if figure is not very good but they are with me, so here we can see if here if you're here in this place you cannot launch this satellite along this direction, it is not possible because once you launch in this direction.

So, there will be a component of gravitational force which will be acting vertically down and it will re orient from this place to this place, so if this is not a proper direction. So, which is a proper direction this is our proper direction of launching. So, we will launch along this direction and let us say that earth moves from west to east in this direction. So, it is a rotating anti clockwise here in this direction. So, if you launch it in this direction, so you can see that this direction is actually making here the angle I this is the angle that it makes with the equatorial plane.

So, if in the beginning itself at launch you get I not equal to not equal to 0, while your requirement may be that I should be close to 0 or may be 0. So, this requirement you know it very well that for the geo stationary satellite or for the geosynchronous satellite, this angle of I either it should be 0 or it should be small. So, that the north south drift is less, now if such condition arises, then naturally you need to transfer it from this orbit to in a orbit.

So, if this is just the projection of the orbit on the earth, similarly we can have a equatorial orbit, so we take the projection ofthat orbit on the earth and which is showing here as the equatorial circumference, so this is also called the ground track. So, if your situation is like this, then you need to give impulse in this orbit if we can give impulse once it reaches here. So, we can give impulse here in this place to transfer it from this orbit to this orbit.

So, various based, so here the non coplanar maneuver is there and this maneuver is very simple, because both the both of them are here in it is in one orbit another one is in another orbit, which is just rotated what we will have a multitudes of complicated condition rather than this one and we need to tackle that. So, we will do rest of the work, we will start with of rest work for the non coplanar maneuver in the next class. So, thank you very much for listening.