Space Flight Mechanics

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Lecture No. # 03

Particle Kinematics (Contd..)

In the last lecture, we have started particle kinematics. So, we continue with that lecture. So, if you remember, in the last lecture, we derived this expression for the acceleration of a particle as d square s by d t square e t cap plus d s by d t square times en cap divided by R, so, where R or perhaps, R or rho, whatever we have taken, this is nothing, but the radius of the curvature. And, this radius of curvature, from the geometry, from your coordinate geometry, you can recall that, this can be written as 1 plus d y by d x square, 3 to the power 2, divided by d square y by d x square.

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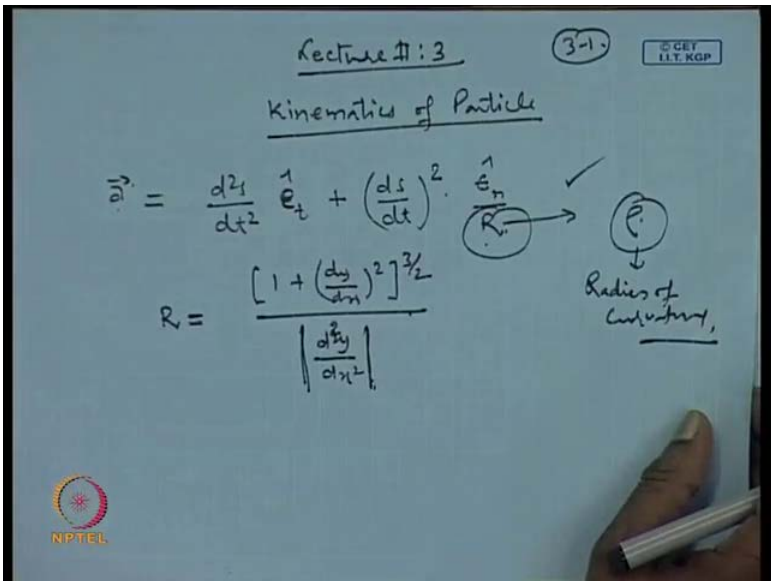

And this, you can, this is a modulus and you can prove this using calculus also. Then, this is a very useful result. So, in that context, we proceed with our further derivation, which will serve us while we work on altitude dynamics, or if you see, one says, once you are putting a satellite, say (( )), you have gone into a, an space craft which has a robotic manipulator. And now, you want to study the movement of the robotic arms in this place. So, what kind of equations you are going to use, you have to study such kind of motion. So, those can be, studied using very simple kinematics. (( )) if the moment of inertia, or the other things, they come into picture, then, you have to naturally include the dynamics part also. But the kinematics part, you can obviously, study from this place. What we are going to study now, say, it is a part of our particle kinematics. So, first of all, we will find a derivation, last time as I was stating that, the rate of change of a vector as observed from an inertial reference frame E 1, E 2, E 3, that is, non rotating reference frame, is equal to the rate of change of this vector, as observed from the e 1, e 2,e 3, from a moving reference frame, or rotating reference frame, plus omega cross A. So, in that context, we made a reference frame here, E 1, E 2,E 3, where the unit vectors along this directions are, E 1, E 2 cap and E 3 cap and then, we had a rigid body here.

We fixed a triad of axis, termed as e 1,e 2 and e 3; it is for the body x; body axes fixed in the body; so, it is rotating along with the body. So, let us say, omega is the angular velocity of this body, with respect to the inertial reference frame, or,or small omega, or capital omega, whichever notation we want, we can keep. And then, we can point a vector here, in this body, we can take a vector. So, will make a separate figure here, in this place for that; let us say this is our e 1,e 2 and e 3. And, you have a vector A, whose coordinates can be determined in this reference frame; instead of putting here e 3, we make it little vertical. So, the coordinate of, you can break it here. So, the coordinate here, of this, can be written as A 1, A 2,A 3.

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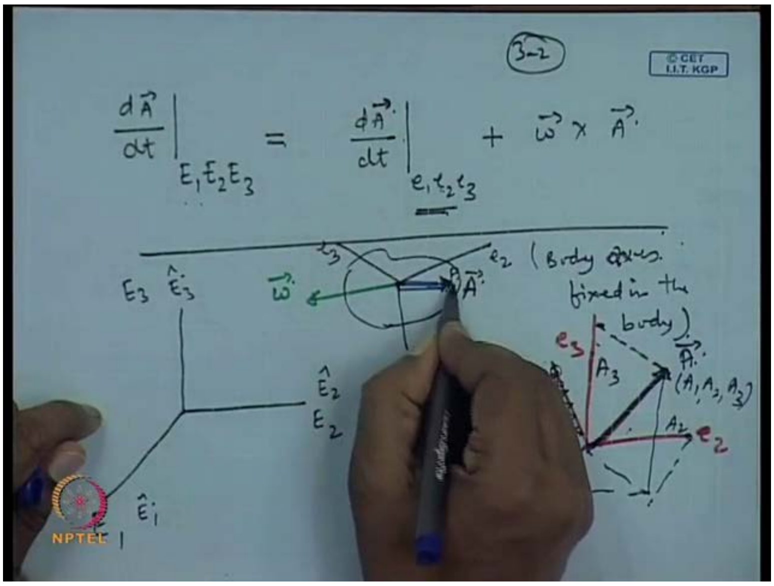

Now, these are the components along A 1 direction, A 2 direction; here, this will be your A 2 and this vertical component, A 3. So, here, some point is there, whose radius vector is A, and let us write it as p; if this is a rigid body, so, naturally, this vector gets fixed, with respect to e 1,e 2 and e 3. Now, we can start writing the equation. Let us say the vector A, we define as A 1,e 1 cap, A 2,e 2 cap, plus A 3,e 3 cap, where the magnitude of A can be written as A 1 square, A 2 square and A 3 square under root. Now, this vector has been written in terms of the components along the body axis, A 1, A 2 and A 3; equally, this vector, if you see from the, this inertial reference frame, or the non rotating reference frame, this is the center of mass here. If you see this vector from this place…So, you can see, it is a component lying along the A 1 direction, A 2 direction and A 3 direction; naturally, this components will be different. They will not be same as A 1, A 2 and A 3 written here, in this place. And, we are using capital notation, capital A to indicate, differ it from the small a, which is often used for acceleration. So, our objective is to find that, this d A by d t is a rate of change of d A by d t as seen from the E 1,E 2,E 3 reference frame, how does it appear.

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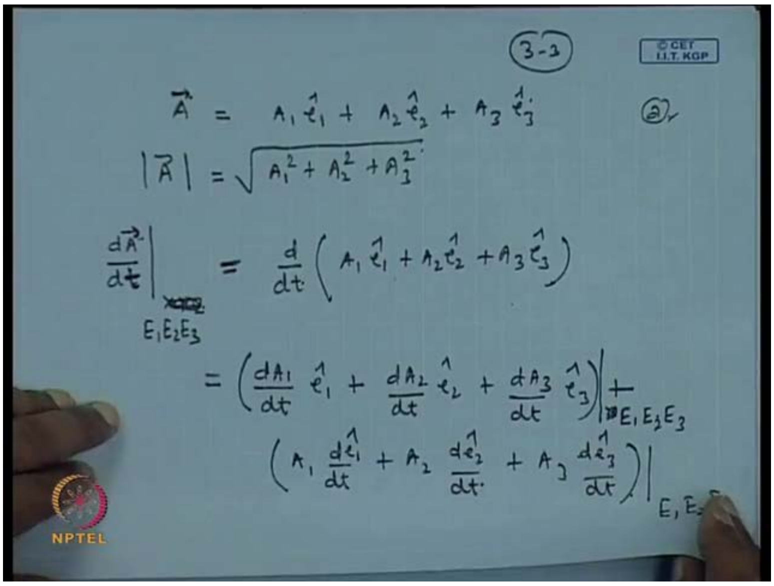

So, to start with, we can write this equation as d by d t and A we can write as A 1,e 1 cap… Now, let us take the derivative of this. So, this is its derivative and this derivation we are doing with respect to the A 1, A 2,A 3 reference frame. Now, the quantity which are appearing d A 1 by d t, d A 2 by d t and d A 3 by 3 by d t, so, these are the rate of change of the components A 1, A 2 and A 3, as seen from the reference frame E 1,E 2 and E 3. So, as we were discussing in the last class that, the rate of change of a scalar, as seen from a non return rotating reference frame, here, we have a rotating reference frame and this scalar is present here. If the, this is a fixed point in the rigid body, so, naturally, we can write…So, we take the case 1 here as case 1, point P is fixed in the rigid body. So, if the point P is fixed in the rigid body, so, the vector A which is fixed here, so, the components of this in the, this, the body reference frame, it will remain constant; it will not be changing.

So, what about the rate of change of this components, as seen from the, this non rotating reference frame E 1,E 2 and E 3. So, the last time, as per our discussion that, the scalar, as we see from a rotating reference frame, or from a non rotating reference frame, they, their rate of change will remain same, so, irrespective of their temperature. So, if the temperature raises, if temperature is your, is your scalar, so, if your, you have kept something, some, say some crucible in the, in a rotating reference frame, and you were looking at it, so, the temperature that will be measured there and the rate of change of temperature, it is basically a scalar. So, it will be same, as we see it from the non rotating reference frame. So, what, here we want to state, I want to state here that, the rate of change of a scalar, or the time derivative of a scalar, it is the same, irrespective of whether it is, say, the moving reference frame, or is a rotating reference frame, or non rotating reference frame. So, the rotation of the reference frame, it does not affect the scalar and its derivative.

So, the length of a, length of a fan blade, it is remain same. And, if you contract the length of the blade in the rotating reference frame, which is the fan itself, sitting on the fan, if it is contracted, so, whatever the rate is, of the contraction is there, the same rate will appear also from the non rotating reference frame. So, from that point of view, what we have written here, this d A 1 by d t, d A 2 by d t and d A 3 by d t, with respect to the E 1,E 2,E 3, so, these are basically the derivatives of the, time derivatives of the scalar components and therefore, they will appear same, as you look from the, either E 1,E 2,E three reference frame, or either if you look from the small e 1,e 2,e 3, that is the body reference frame. So, in both of them, it will same, it remains same.

But if the point P is fixed in the rigid body, so, naturally this component, d A 1 by d t, even in the body reference frame, naturally, it is 0. And therefore, this will appear to be the same also, with respect to the e 1, e 2, e 3. So, this will also appear to be the 0, from the capital E 1,E 2,E 3, which is n non rotating reference frame. So, in that case, you can write, d A by d t as A 1 times d e 1 cap by d t plus A 2 times d e 2 cap by d t plus A 3 times d e 3 cap by d t. Now, you need to determine this quantity d e 1 by d t ,d e 2 by d t, d e 3 by d t. So, we have done earlier this exercise also, if this is a unit vector at time t and this is the unit vector at time t plus d t, this angle is d theta. So, you look from this figure; here, this is a vector e 1; this is the vector e 2 and this is a vector e 3. So, rotation of this vector, it will depend on rotation of this vector, in this plane. And, in this plane, it will depend on the rotation, rotation of this vector will depend on rotation around this x and rotation around this x will affect in this plane. So, in this plane, this rotation will affect; in this plane, in this plane, we have this rotation is affecting; in this plane, this rotation is affecting.

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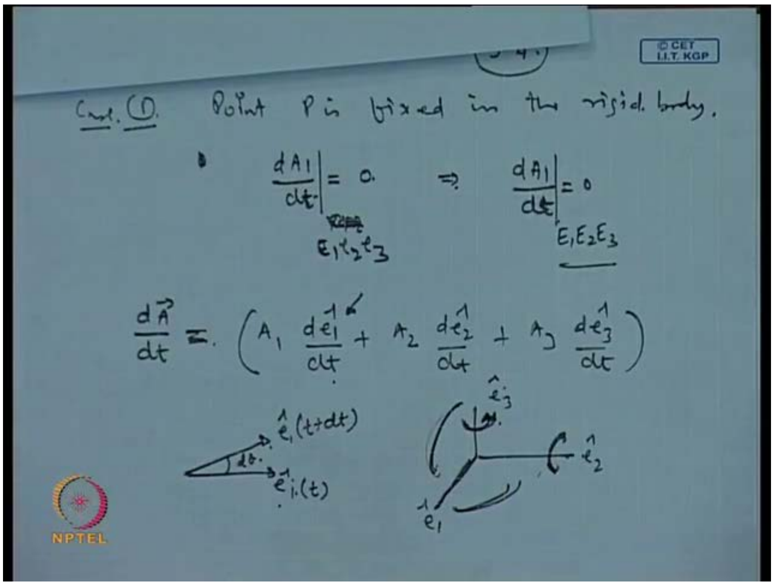

So, this two rotations can be corroborated. Now, we need to work out all of them. So, if we say that, this is the rotating, the rotation about this x, the component of omega vector about this is omega 1; about this axis, this is omega 2 and about this axis, this is omega 3. So, the rotational rate of this, now, we start and let us put this as the, something, we write it as d theta only.

So, the change is delta, delta e t and this change you can write as, e 1 cap magnitude times d theta. And therefore, delta e by delta t in the limit, we can write here, limit delta t tends to 0, delta e by delta t; this we can write as, let us first take the magnitude, and thereafter we will take the direction part. So, d by d t will be equal to magnitude of e 1 cap times d theta or delta theta by delta t; so, here we can show this as the delta theta, instead of delta t, to be consistent, and this quantity is nothing, but equal to 1. So, this gets reduced to theta dot. So, theta dot is nothing, but your omega 3 here, in this place, and omega 3 because, this is rotating like this; if this vector is rotating like this, so, we will write this as the omega 3. Now, in which direction this vector will change? So, the changed vector will be, as this delta theta becomes smaller and smaller, this will be perpendicular to this e 1 vector. So, this is pointing in this omega 3 direction. Now, similarly, e 1 can, because this axis is also rotating, so, e 1 is also rotating about this axis and for the positive rotation, we are giving the, taking the right hand rule. So, this is rotating anti clock wise here, and about the x 1 axis also, e 1 x, we are giving the anti clock wise rotation. Similarly, about the e 2 axis, we give the anti clock wise rotation. So, if the rotation is given about this, so, e 1 will shift from this place to this place, downward direction. So, similarly, we need to work out for this and the net change then, will be combination of this two. So, over all, this d A by d t, this can be written as omega cross A.

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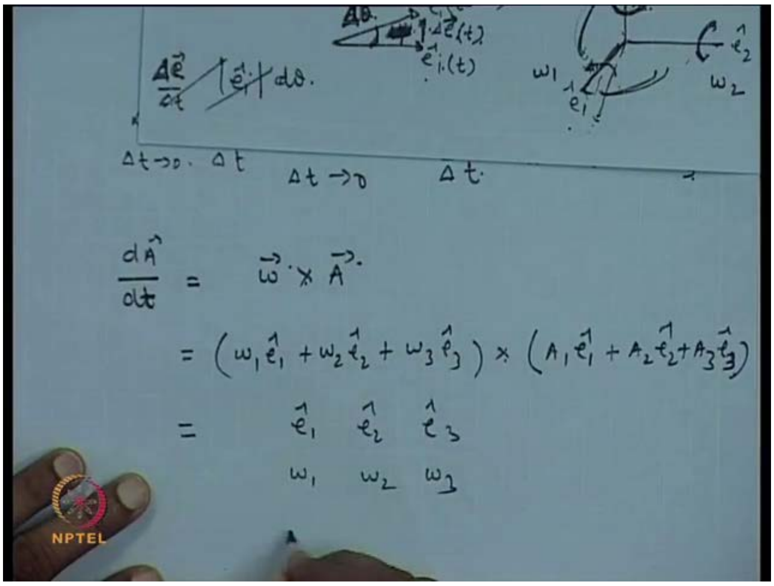

We write, break up this part and then, we will come to this to show you what exactly has happened. Omega 1 times e 1 cap omega 2 times e 2 cap plus omega 3 times e 3 cap and cross A 3 times e 3 cap. So, if you break it up, taking the cross product, so, you can write it in this way, e 1,e 2,e 3 and omega 1, omega 2, omega 3, A 1, A 2, A 3; take the determinant of this and this will yield you, e 1 cap times omega 2 A 3 minus A 2 times omega 3 plus e 2 cap times A 1 omega 3 minus omega 1 A 3 plus e 3 cap times omega 1 A 2 minus A 1 times omega 2. So, what exactly has happened…We had started with this. So, this term, we are writing as 0; this term, if the vector is fixed, the case 1, we were taking, where the point P is fixed. So, in that case, this part is becoming 0 and only this part we are considering. So, in this part, we need to insert the expression for d e 1 cap, d e 2 cap and d e 3 by d t cap. And, procedure for doing the same thing, so, d e 1 by d t cap we were trying to find out. So, due to one rotational change, we saw that, this is, omega 3 is coming into picture here. So, the angle delta e by, delta e 1 by delta t, because of the rotation about the e 3 axis, this will be omega 3 and it points in the direction of omega 3 itself. So, this vector becomes d e 1 cap by d t; this you can write as omega 3 times e 3 cap. But here, because this is only due to, only due to rotation about e 3 axis; then, we have to take care of your rotation about the e 2 axis.

So, these things we will be doing comprehensively in our attitude dynamics part. So, if we continue with this…So, with A 1, we can see that, with A 1, ultimately the e 3 vector is manifesting. So, sorry, the vector region, the direction of the e 3 and omega 3 is manifesting; sorry, this we write as, where it has gone; let me make another figure here. This is your e 1 cap and this is your e 2 cap at time t and this is basically due to the rotation of omega 3 here, that it is changing in the e 1 e 2 plane and this we have written as delta theta. So, that change, once e 2 approaches this, this you are writing as, e 1 t plus delta t and if this is e 1, so, this your delta e 1.

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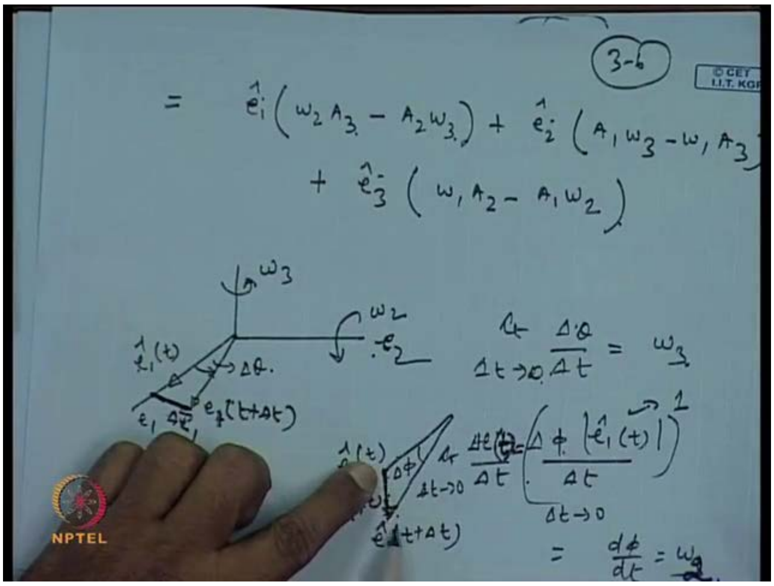

So, as this approaches this, this vector points in the direction of the e 2, not in the direction of the e 3; but the rotation, rotation which is talking place, delta theta by, delta theta by delta t, this is in the direction of this. This is basically nothing, but your omega 3. So, in the limit delta t tends to 0, this you can write as omega 3, in this place. Therefore, the one part of d A 1 by…We had the first part d A 1 by d t times d E 1 cap by, sorry; this is A 1 times d E 1 cap by d t. This is the first part, plus the second part is A 2 times d e 2 cap by d t; the third part is A 3 times d e 3 cap by d t. Now, naturally from this place, what we see, this part gets reduced to A 1 times omega 3 and the vector, now, it is directed along the e 2 direction. So, e 2 cap will manifest here. Similarly, if you give rotation about the e 2 axis, we are giving positive rotation, using the right hand rule. So, this is omega 2 here, and then, this vector instead of now taking it up, we can show it going down. And, let us say, this angle, we show it as delta phi. So, you consider the rate of change of this delta phi; again, you can write this as e 1 t and this is e 1 t plus delta t. We are considering each rotation separately. So, if it rotates, this is your delta e 1 t, putting cap here, so that, this vector which I changed, as we have discussed in the last lecture, this is nothing, but your delta phi times e 1 cap t magnitude. So, this is the magnitude of this vector delta e t.

And, if we divide both sides by delta t and let delta t tends to 0…So, limit delta t tends to 0 and here also, take the limit delta t tends to 0. So, this will give you, this quantity is basically 1. So, this gives you d phi by d t and this is nothing, but here, serving as the omega 3; in this direction, this is serving as omega 2, not as omega 3. And, what is the direction in this case is, you have this vector, the rotation is taking place like this. So, this is, the rate of change of this angle will be pointing in this direction, and this vector, the vector which has to change. So, this vector is associated with the direction, as this approaches this place. So, this is in the negative direction of omega 3. So, we write here, with the same thing here, you can write as omega A 1 times and then, multiplied by omega 2 times omega 3, but with a negative sign. Now, you search for the expression that we have written for, expanded expression we have written here, here, in A 1, where it appears. So, look for that sign; A 1 with omega 3, this is A 1 with omega 3, this is, and e 2 is here.

So, this part is, you consider this is positive sign and then, you look for the A 1 omega 2 and omega 3; A 1 omega 2, A 1 times omega 2 times omega 3; this is your omega 3; this is with a negative sign. So, this way, you can, assemble component-wise, each of them, and then, combine the terms with e 1 cap, e 2 cap and e 3 cap together, and you will get this expression. And, the same expression, I wrote this in a one sort as, d A by d t as omega cross A. So, here finally…

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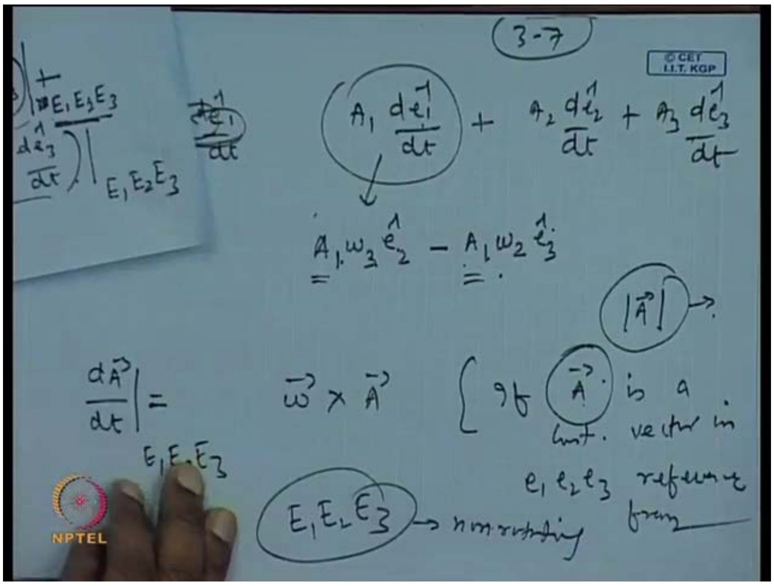

So, finally, this expression, once we are reducing it...So, we are writing as d A by d t in E 1,E 2,E 3, this will be equal to omega cross A, if A is a constant vector in e 1, e 2,e 3 reference frame. So, the magnitude of the vector, this vector, the magnitude of this vector, also becomes a constant in the capital E 1, E 2,E 3 reference frame, which is non rotating. So, you see, if this, how important this conclusion is. So, from here, what we get that, if the vector is fixed in magnitude, then, simply the first part here vanishes and we get the, only the second part as omega cross A. So, this gets reduced to omega cross A. But if this magnitude, the component of the A vector, it is a changing in the body reference frame itself, so, it will also change in the x y z reference frame, and I, as I have stated, the scalar change, the rate of the scalar change, as in, as it appears in a non rotating reference frame, it is the same as from the rotating reference frame. And therefore, in that case, we can write d A by d t, the case 2, A is varying in, and this is case 2, varying in body reference frame, that is in e 1,e 2,e 3. So, here, in this case, you can write, this is d A by d t with respect to E 1, E 2, E 3, small e 1,e 2,e 3 plus omega cross A, in this case, as the rate of change of this appears from the e 1,e 2 reference frame.

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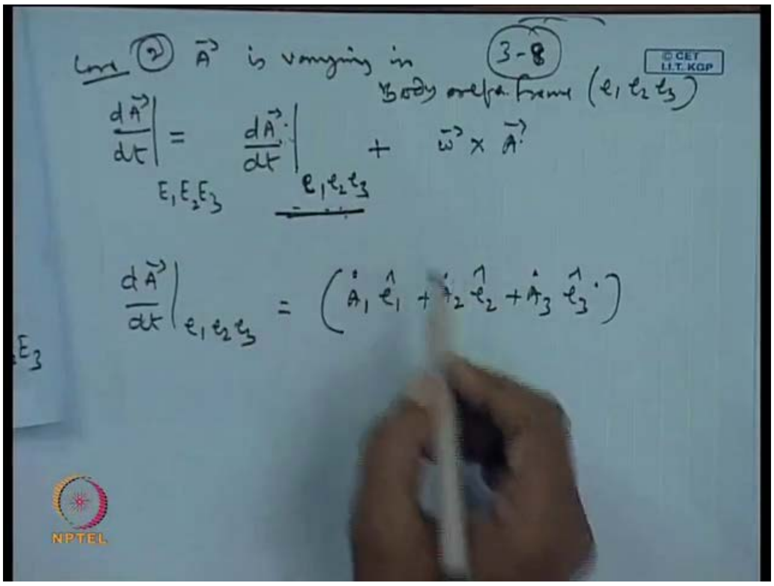

So, here, you have, these are the, in the case, the vector A is changing in the magnitude; its components are changing. You just have to take the components of this and the same thing has been written here, in this format. So, here, basically, your d A by d t e 1,e 2,e 3, this we are writing as A 1 dot e 1 cap plus A 2 dot e 2 cap plus A 3 dot e 3 cap. So, this is going to be the same, as seen from the E 1, E 2,E 3 reference frame, or either from the small e 1,e 2,e 3 body reference frame. So, we have done a part, which was very important for understanding further, our derivations. So, next, we go into, sorry. So, here, P is a point and say, this point O is the center of mass of this body; this is the point we have taken inside a rigid body and e 1,e 2,e 3 reference frame is fixed in this. And, this reference frame is rotating with an angular velocity of say, omega. So, and this vector, we have written as A. Now, instead of writing this as A, we write this as rho, and the vector from this place to this place, we write this as R.

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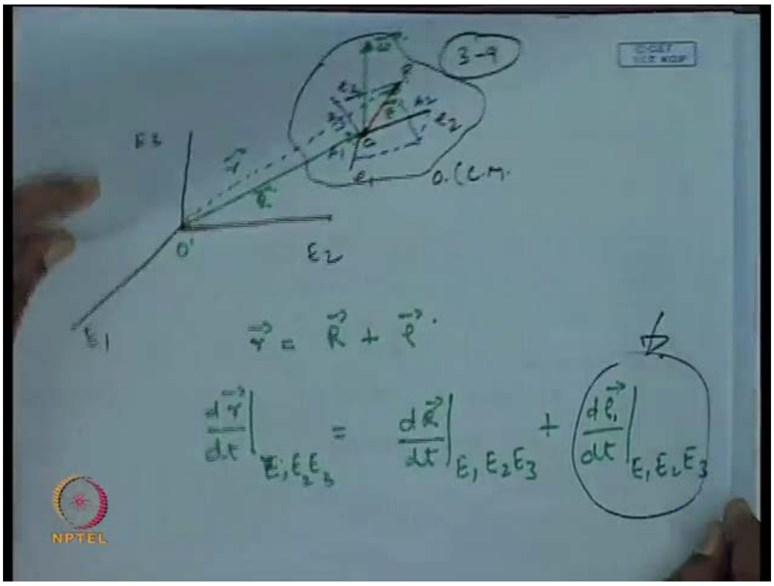

And, the vector joining the point rho prime and the point P, this we write as r. So, obviously, we can write r equal to R plus rho and therefore, the rate of change of this, in the E 1, E 2, E 3 reference frame, this can be written as d R by d t E 1, E 2, E 3 plus d rho by d t E 1,E 2, E 3. Now, whatever the conclusion we have made in the last part, so, that applies into here, this place. So, this is a vector, which is been referred from this point to this point in the rigid body. So, this vector can be either expressed in the components of the E 1, E 2, E 3, or in the body components. But here, we will describe this vector in terms of the body components. So, r double dot, we write this as, left hand side, E 1, E 2, E 3, this will be equal to R double dot E 1, E 2, E 3 plus d rho by d t; so, that we have written as, we can write as, d rho by d t as seen from e 1,e 2,e 3 reference frame and plus omega cross rho. This is the first derivative that we have taken. So, the, this quantity has been, the quantity, this quantity has been expanded and written in this format. This is, sorry, this is the first derivative; only one point, dot will come; this is the first derivative. Next, we take the second derivative. So, this, we write as d square r by d t square and this is with respect to E 1,E 2,E 3reference frame and here, this, we will write as r double dot E 1,E 2,E 3.

So, naturally, we need to differentiate this quantity here, R dot E 1, E 2, E 3, plus, this we will write as rho dot E 1, E 2, E 3 plus omega cross rho. And, this, we have to do with respect to E 1, E 2, E 3 reference frame. So, expanding this part, because this is a vector, again r dot is a vector, which is in the E 1, E 2 reference frame and we are taking the derivative in that itself. So, it does not make any difference. We can directly write this as R double dot E 1, E 2, E 3 and plus d by d t rho dot. Now, this is rho dot is a vector, which was in e 1, e 2, e 3. So, we need to drop out this symbol here; this symbol, may be, this symbol we can take as e and this symbol we can keep as capital E, to abbreviate here, in every place.

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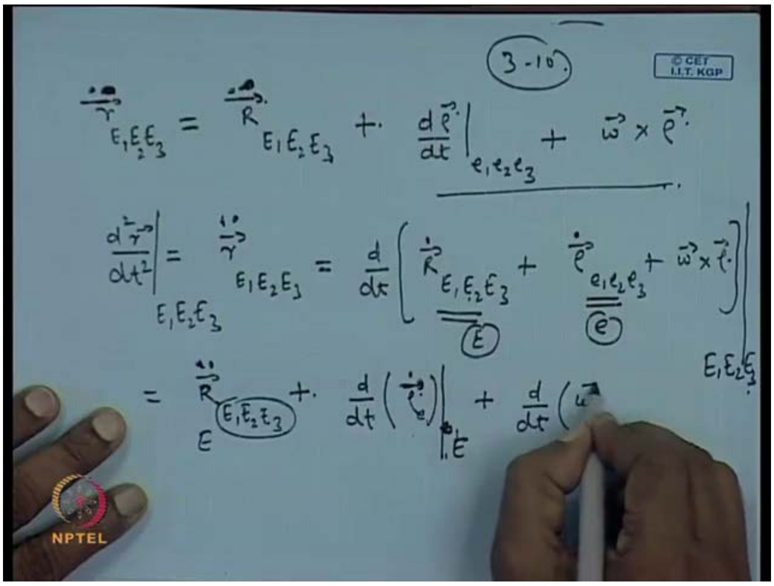

So, here, we can put as e and outside we can keep as E. So, this, we can replace by E. And then, of course, we have to write d by d t omega cross rho and this is with respect to this E. So, we have r double dot E, this becomes…Now, differentiate the quantity which is written inside; rho dot, as it appears here, if you remember from the last part, as we have written, so, this can be written as rho 1 dot e 1 cap plus rho 2 dot e 2 cap plus rho 3 dot e 3 cap and plus the next one again, we have to write. So, this term we can write as, d omega by d t cross rho; this is with respect o E and plus omega cross d rho by d t; this is with respect to E. Now, we have to take care of this particular part. As proceeding in the last part, so, we can write here directly as, rho 1 double dot e 1 cap plus rho 2 double dot e 2 cap and rho 3 double dot e 3 cap and this we are writing with respect to E. But this is also equivalent to writing it with respect to the small e, as we have argued just before; exactly we have done, as for the A vector, we are doing the same operation here. And, the other part of this can be written as rho 1 dot times d e 1 cap by d t plus rho 2 dot times d e 2 cap by d t plus rho 3 dot d e 3 cap by d t and this is for, with respect to this capital E reference frame and of course, this, this quantity we can write as omega dot and this is rho, and the quantity which is present here, this is omega cross. Now, d rho by d t with respect to E we have already written as d rho by d t with respect to the small e, that is, with respect to the body reference frame plus omega cross rho.

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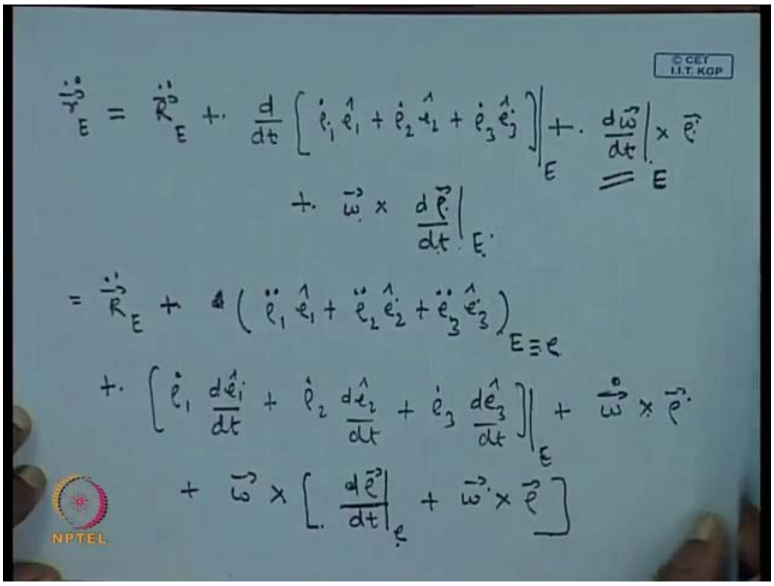

Now, writing this equation here, on the next page, so, we have r E double dot, which is the acceleration in the inertial frame, this can be written as R E double dot; and the term which is present here, which we have writing as ae, so, this term is basically, the acceleration with respect to the body reference frame; because, as we have discussed, the time derivative of a scalar, it does not depend on the reference frame. So, like the temperature in a rotating reference frame and so, we have the temperature in a rotating reference frame. So, its time derivative, it will be same from anon rotating reference frame also; the time derivative does not get affected. So, we are taking here, the time derivative of rho 1, rho 2 dot, rho 3 dot. The, here the, E notation, which is for the inertial reference frame, we replace it by a small E , to indicate this is with respect to the body reference frame, and therefore, we will use a e for this. So, we write here a e for this acceleration. Now, this acceleration, this you can write as omega cross rho dot; for as earlier described, this, this is e 1 cap, e 2 cap and you are taking the derivative of this. So, we can, earlier I have proved this, how to work out this. So, here, you can replace this in terms of omega cross rho dot. So, once this is done, then the last, this term is remaining omega dot cross rho and then finally, we have his term here. So, this is your, basically rho dot with respect to the body reference frame. So, we can write here omega cross rho dot plus omega cross rho. And now, accumulating the terms together wherever possible and this rho dot, we can replace as v, and I can put a e, to indicate this is with respect to the body reference frame, and this also, we replace by v and here, omega cross omega cross rho.

So, here, this one and this, this can be combined to give 2 omega cross v and plus omega dot cross rho plus…So, this term is your Coriollis term; this term, you can identify as the centripetal acceleration. For very simple cases, you can check it; say, your omega, you can write as omega times e 3. If you are writing this very simple case, so, the omega cross omega cross rho and the rho vector, you can write as, say again, rho vector, we described as rho times e 1 cap. So, this will become omega times e 3 cap cross omega times e 3 cap cross rho times e 1 cap. Now, check from this place; this is omega square and rho 3 times rho 1, this is nothing, but rho two. So, this becomes e 3 cap times e 2 cap and e 3 times e 2 cap and here, we put the rho which is present here. So, this becomes omega square times rho and e 2 times e 3 times e 2; this is minus e 1. So, this is minus e 1 cap. So, basically, what it shows that, your rho vector, this is along the even direction; you have taken from here that, this is your rho vector. So, this indicates that, this acceleration is directed towards opposite to the rho vector; that is, it is directed towards the center of the point about which the rotation is taking place. So, this is basically your centripetal acceleration in inertial reference frame and this is the Coriolis force, that we call, the Coriolis force; these are basically the Coriolis force…

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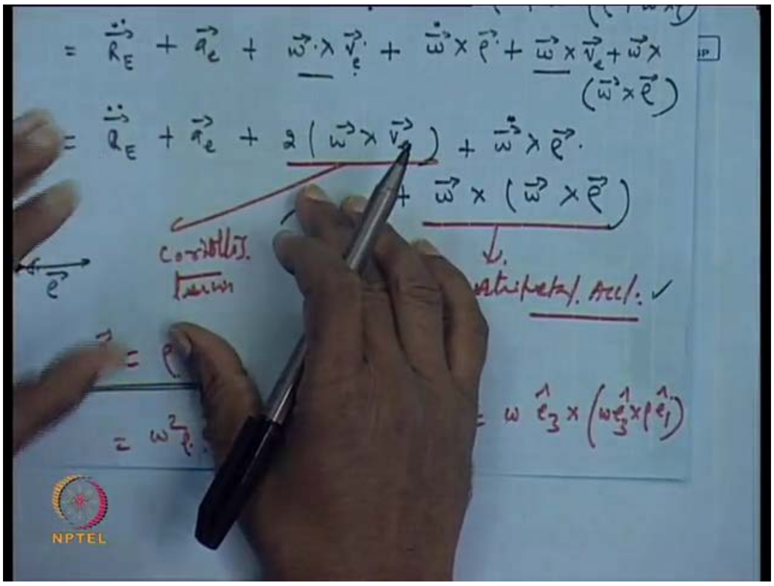

You will be able to see it in the, if you are in a rotating body reference frame and this is related to the angular acceleration. This term is nothing, but your angular acceleration and this multiplied by rho. So, acceleration related to the angular acceleration, this term becomes. So, here we can…So, to show the effect of the Coriolis acceleration, we take the case of, of the earth, and say, this is the R vector and here, we have the body reference frame which is e 1 directed radically outward, e 3 cap; e 1 is in the East direction and e 2 is in the North direction. So, earth is rotating about this axis, with capital omega.

So, here we can show that, this is the angular velocity of the, this body frame. Now, I will take this quadrant only and show this picture here, separately. e 3 cap, e 2 cap; this is your lambda. So, this will be lambda; this is angular acceleration vector; this angle will be lambda. So, in the e 2 direction, this capital omega vector, this you can write in the body reference frame as capital omega cos lambda times e 2, taking component along this direction and plus sin lambda is component along this direction multiplied by e 3. Now, suppose the wind is coming from the North Pole; this is your North Pole and this is, below the South Pole is there. So, wind is coming from this direction. So, here, we have, I can show that, the wind is in that direction; v I can write as minus v times e 2 cap. So, the Coriollis term, that becomes 2 cross omega cross v. So, this is minus 2 times omega, from here we can insert…So, this is capital omega cos lambda e 2 cap plus capital lambda e 3 cap cross v, which is minus v times e 2 cap. So, this implies, this quantity becomes minus minus, this gets plus, and we can write 2 v, e 2 e 2 cross product, this will vanish here; we can put the cross product here, in this place.

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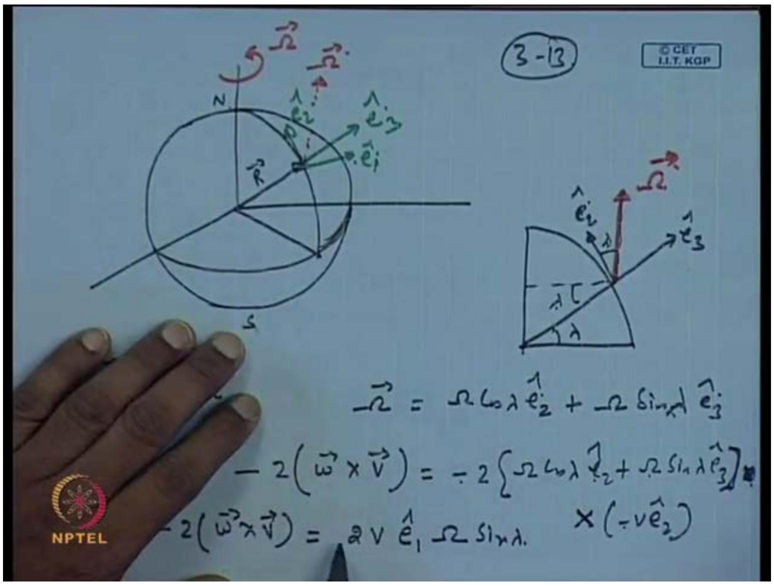

So, e 2 e 2 vanishes and we have to take the cross product of this e 3 cap cross e 2 cap, this is e 1 cap minus. And then, we have to take care of this capital lambda. So, capital lambda we can put here and then, we have the sin lambda and this is within minus sign.

So, what it gives that, minus 2 times omega cross v; this is your 2 v capital omega sin lambda times e 1 cap. So, this Coriolis acceleration in the body reference frame…So, your body reference frame is shown by this green line. So, it is acting in the direction of negative e 1 direction. So, it is acting in this direction. So, this is the East direction; this is your East. This is local vertical and here, this is your North direction. At any point, if we identify like this, this is your North direction. So, East, North and this is vertically up. So, this is the West direction. So, this, we can shown here as the West.

So, if the wind starts from this direction, so, it will turn towards the West; and that is what happens. So, in the northern hemisphere, we will see that, the, if you read your geography book, you will see that, the wind which starts from the North Pole, so, they turn towards the West. So, this is your North Pole and this is your South Pole. This is equator. So, they will turn toward the left hand, and this happens, because of the presence of this Coriollis acceleration. So, I have shown the Coriollis term here; then, I have shown the centripetal term here; and this is basically, due to the increase in the angular velocity; remember that, in the body reference frame, once we take it to the body reference frame, by writing, I can write this as a e equal to r E double dot minus R E double dot minus 2 times omega cross v e minus omega dot cross rho minus omega cross…

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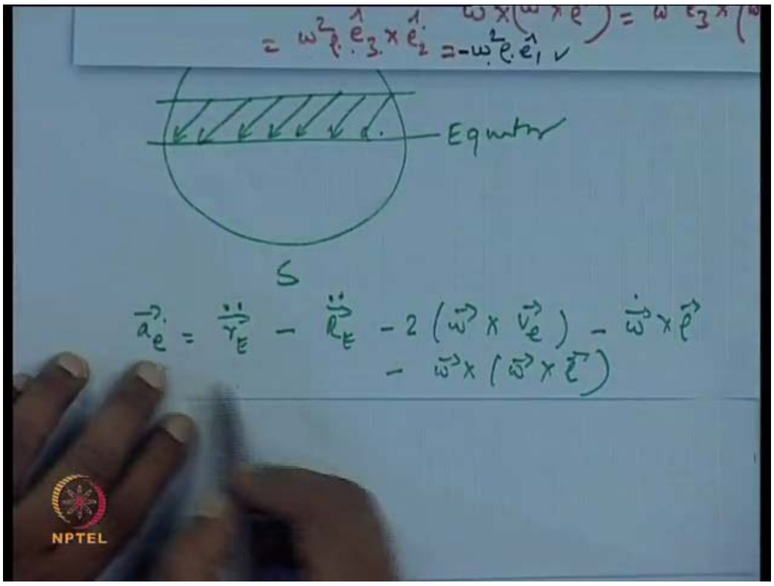

So, once you are looking in the body reference frame, so, this term which is present here, this is basically, your, the negative term that I was showing, the Coriollis term, and this is your (( )) centripetal term; we will write this as, showed this as the centrifugal force, because in the body reference frame, what you will fill, is the centrifugal force. So, this is gets a centripetal term, once we change the side, once we are looking from the, relative to the body reference frame; so, centripetal will appear as the centrifugal one. And, this is basically related to your tangential acceleration, because of the increase in the, increase in the angular speed. So, we have to determine this omega dot cross rho term.

So, we can take body reference frame e 1, e 2, e 1 cap, e 2 cap, e 3 cap. Now, omega is a vector, which is directed along this direction. And, this vector is, because we, here, we will take a simple case that, omega is directed along this. So, we can write omega equal to omega times e 3 cap. So, we can write d omega by d t, this is equal to d omega by d t times e 3 cap plus omega times d e 3 cap by d t. Now, we can look from this place that, here d e 3 by d t, because the rotation is taking place about the, this body axis only. So, there is no change in the direction of the e 3 cap, unlike the earlier one. So, this, we can drop it out; this becomes 0 and here, we can write d omega by d t equal to omega dot times e 3 cap. So, this term gets reduced to omega dot cross rho; this will become omega dot, and rho is basically, we are taking here, in this direction.

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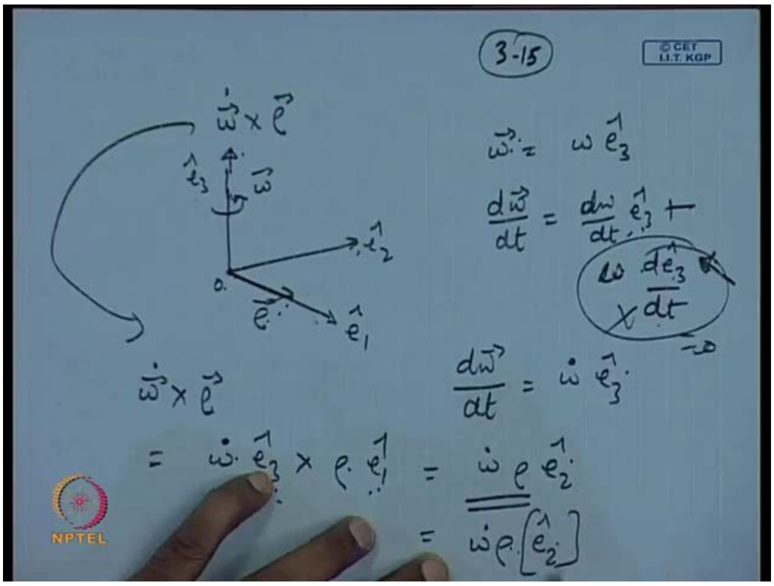

So, this is your rho vector; so, omega times e 3 cap and rho vector is rho times e 1 cap. So, this becomes omega dot rho and e 3 cross e 1, this is nothing, but your e 2 cap. So, now, you can see that, omega dot rho times e 2, this is representing a vector in the direction of e 2. So, e 2 is in this direction. So, if your, this is the vector and if this is rotating, so, this is pointing that, this acceleration is directed in this direction. And, the same thing, if you look from the body reference frame, so, the negative sign will come into picture and you just have to put a negative sign before that, and this gets into the opposite direction. So, in the body reference frame, it will appear in the opposite direction.

So, thus, we conclude this part on the kinematics of particle. In the next lecture, so, we will start with the particle dynamics. So, in the particle dynamics, in the space dynamics, we are concerned with the heavenly bodies. So, we will look into the dynamics of the, basically, we are going to work with the dynamics of the heavenly bodies, or the satellites. So, when the earth is here, around the earth a satellite is moving, and there is some other planet. So, we will explore all this things in the future lecture. So, we stop here, thank you very much.