**Space Flight Mechanics**

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**Lecture No. # 29**

**Trajectory Transfer (Contd.)**

We have been working with our Trajectory Transfer problem. And last time, we were solving one problem for the (()) which are moving in two different circular orbits; and then one satellite has to move from astride A to astride B and back from astride B to astride A. So, in that context, the part b of the problem we were continuing with.

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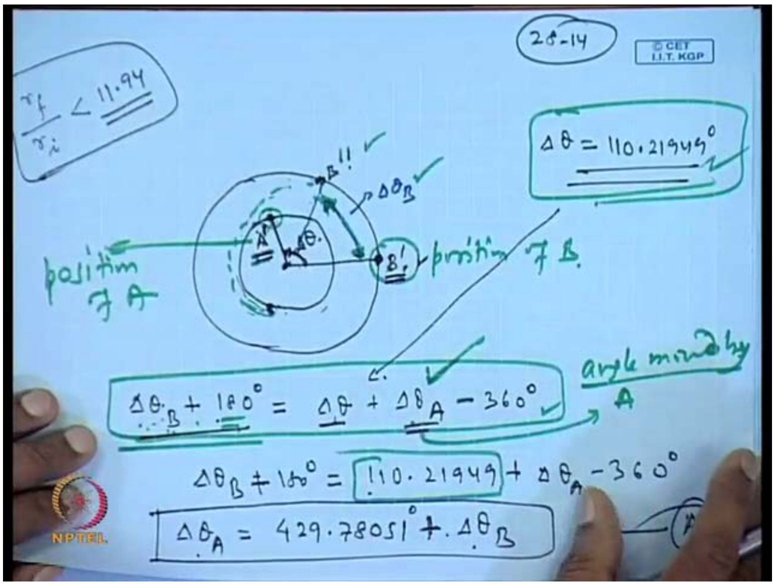


So, in the part B we saw that this is astride A and astride B is here in this place. So, initially there is some (( )) in the, once the problem was that, first the satellite has moved from astride A to astride B and now again after certain wait period, the satellite has to be returned from the astride B to astride A. Now, the wait period has to be such that, the fuel should be minimized.

It is a one possibility is that, immediately the satellite can be launched and the say and the astride A can be can be reached after some maneuver, so that is always possible. But if the wait period has to be such that the fuel economy is to be minimized, then we need to apply the Hofmann transfer; because as we have seen that, the r f the final radius by the initial radius in this case the earlier once the astride was going from the satellite was going from astride A to astride B. So, this was turning out to be less than 11.94, and therefore the Hofmann transfer is the best possible best possibility.

So, once the astride, the satellite from astride A it moved to astride B, so at that time, the position of the astride A was given by A prime and the position of the astride B was given by B prime. So, the astride A was leading, astride B by certain angle which we showed to be139 degree (( )) this, this we calculated to be 110 degree.

So, delta theta has we can see, we calculated it to be 110 degree, which we have shown here in this place, and we have written here in this place also. So, now the problem starts with that a satellite has to be launched from B again, which should reach A with maximum fuel economy. So then, we started writing the equation and balanced the angle first.

So, delta theta B which is the angle moved by the astride B, before the satellite can be launched it was written by delta theta B this quantity. So, this is the angle before the satellite can be launched from the astride B. So, this is the position of the B, which has been written by B double prime.

Now, the astride will be launched from, the satellite will be launched from B. So, which is now at the B double prime position and it should catch up the astride A which is now at A prime, which is at the position A prime at the beginning of the motion from B prime. So, once the astride was at B prime, so at that time astride A is at A prime. And this is the position of B position of B and here this is position of A and this is the gap delta theta, which we have earlier stated.

Now, before the launch astride B will be move from this place to this place, which is given by B double prime in the mean while astride A prime, because it has the higher angular rate. So, it will move from this place to may be somewhere here suppose, or it may move to some other place depending on the angular rate and the radius of depending on the angular rate and the time available, so that time will be decided by how much time the astride B takes to go from this place to this place.

So, obviously if the satellite is to be launched from this position, so by that time astride A must reach in this position; so, that it can come and catch up here in this position, but because it is a moving faster. So, therefore, it will go one round and it will (( )) another around then only, it can be catch. So, that situation by was described by this condition; delta theta which is the delta theta B, which is the distance moved by the astride B from here to here plus 180 degree, which is the angle from here to here again and this must be equal to delta theta, which is the lead initial lead plus delta theta A, this is the distance and this is the angle moved by A this is the angle moved by A which we will start from this position from A prime. So, it will start it will pass over this place, it will go and again once it will come here, then only it will be able to catch up this.

So, this delta theta A it will include the time for the passage of B from here to here and then, the passage of the satellite from this place to this place. So, whatever the time involved in this process is the same time will be involved in delta theta A. And then we have subtracted minus 360 degree, because A is making one complete round and therefore, this 360 should be subtracted, and we got this equation.

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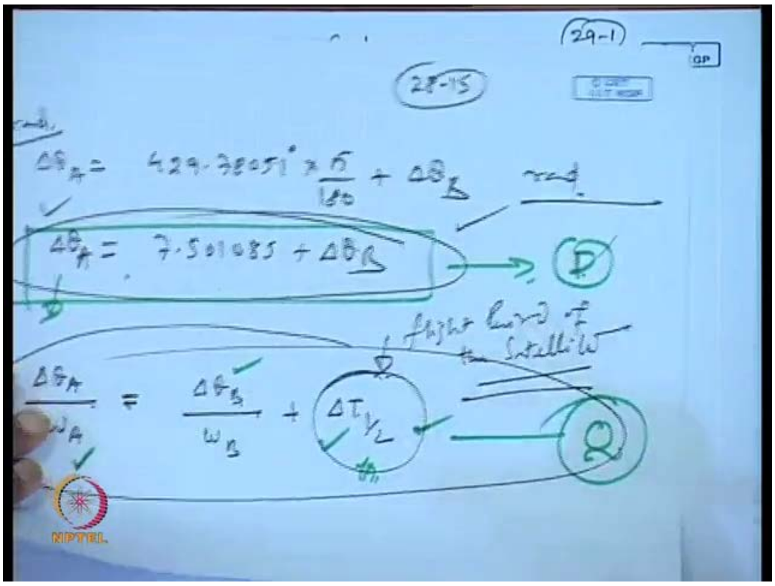

Next, so this was this is the final reduced equation. Next, we wrote the equation that, delta theta A by w A which is the time taken by A to move from this place and it will come back and again it will come here. So, this is the angle (( )) delta theta A, it will start from here pass over this place, come here and again it will come here; so, that will constitute the angle delta theta A.

So, delta theta A by omega A, which is the angular velocity of the astride A, this must be equal to delta theta B divided by omega B. So, this is the angle from delta theta B divided by omega B, which is the angular rate of the astride B. So, the time taken took over this angle and there after time taken to cover this angle from here to here. So, that is satellite will move from this position to this position. So, how much time it is going to take? So, that time we are indicating as delta T 1 by 2.So, this quantity is already we have computed.

Delta theta B we need to work it out. So, we have one relationship available, but this quantity is available to us. So, this is the half period of the satellite for moving in that electric orbit and that is the same as earlier we have done, either it goes from this place to this place or either comes back from here to here; because the semi (( )) axis,axis is not going to vary it will not differ either, if it goes from here to here or either if it comes from here to here, it is not going to change. And therefore, the time period half time period or the full time period to go in this electric orbit it will remain same; so, which has been added up here in this place.

Now, we can insert this equation here in this place to work out for delta theta B. So, we can name this equation as say1 and this equation or maybe we will write this as P and this equation as Q.

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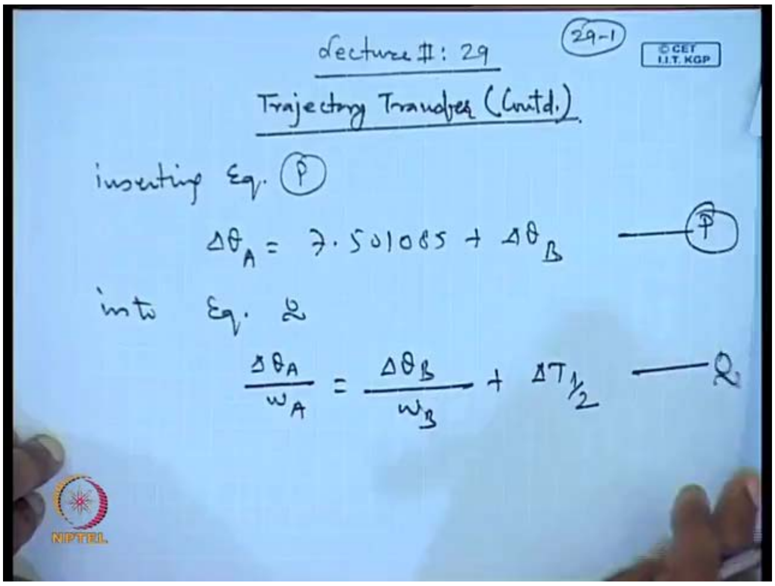


So, inserting equation P, which is delta theta A equal to 7.501085 plus delta theta B this is our equation P into equation Q, which we are writing as delta theta A divided by omega A equal to delta T 1 by 2(( )).

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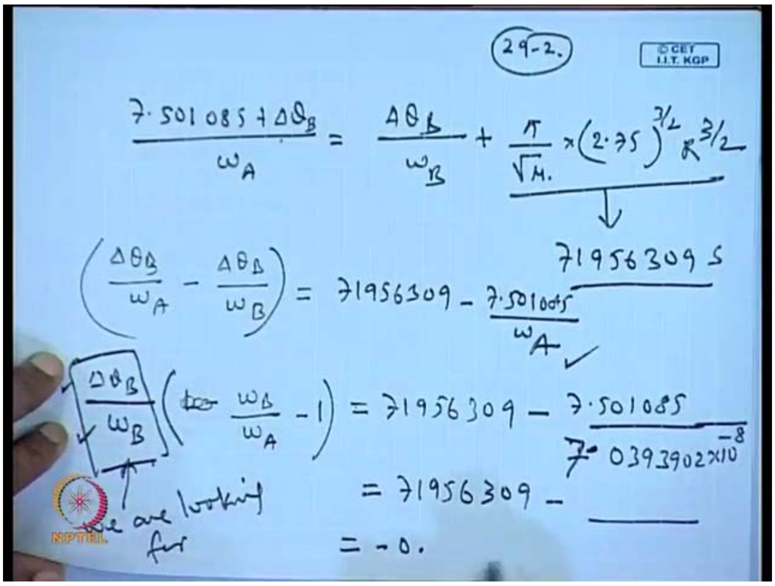


So, the resulting equation after we inserted for delta theta A we have7.501085 plus delta theta B divided by omega A is equal to and T half we have written as pi by mu under root times 2.75 to the power 3 by 2 times r to the power 3 by 2. And (( )), this quantity we have calculated it to be 71956309 seconds, this we have done in the part A.

Therefore, this is delta theta B here in this place, now we can write delta theta B by omega A minus delta theta B by omega B; this is equal to 71956309 minus 7.501085 divided by omega A, omega A is also known to us. Now, here what we can do? Delta theta B we can take common and also omega B; so, this we will make us, this will give us omega B by omega A minus 1.

And omega A is the quantity which is known to us, this is (( )) this is larger (( )) something then it have this; so, omega A is 7.0393902 into 10 to the power minus 8 the radiant's per second. So, once we work it out, so this will turn out to be71956309 and after dividing this once we compute this quantity. So, this will turn out to be minus 0;so, here delta theta B and omega B these are to be worked out, this is the quantity we are looking for. So, we need to compute this quantity and also we need to compute this quantity.

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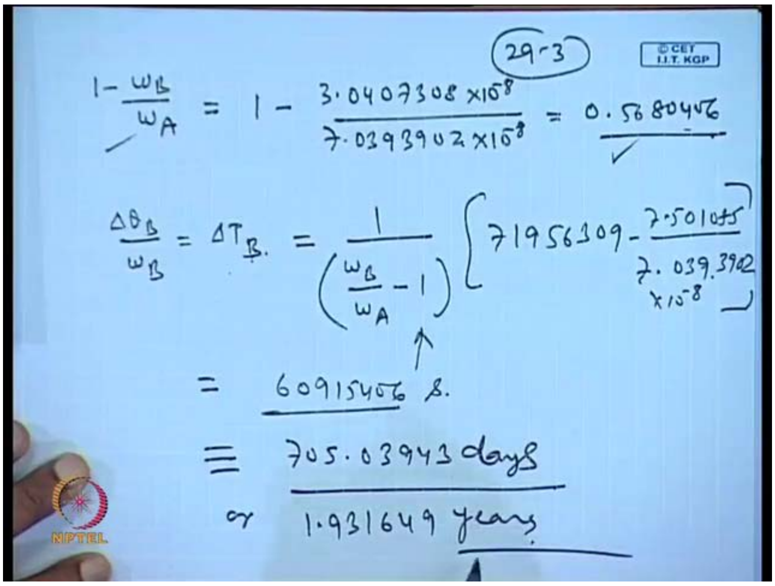

So, 1 minus omega B by omega A this will be 1 minus3.0407308 into 10 to the power minus 8 divided by. So, this turns out to be 0.5680406 therefore, delta theta B by omega B will be equal to delta TB; and this we can write as 1 by omega B by omega A minus 1 times 71956309 minus 7.501085 divided by7.0393902bracket close. So, these quantities will obviously, this is positive here, so this quantity will turn out to be negative. So, we will see that, this quantity will also turn out to be negative.

So, after computing this, this will turn out to be around 60915406 seconds. And this is equivalent to 705.03943 days or 1.931649 years. So, this is what we wanted to compute; that how much will be the wait period that is how much time is going to take to move from the astride B is going to take to move from B prime to B double prime. So, that at this point we are launching, so this is our wait period from here to here or this you can say here, this is the rate angle (( )) term saying in terms of angles and this can be converted into time like this.

So, delta theta B this angle divided by omega B which is angular rate of the astride B, that gives you the wait period. So, finally we have got the results, this can be summarized as, so part B is complete.

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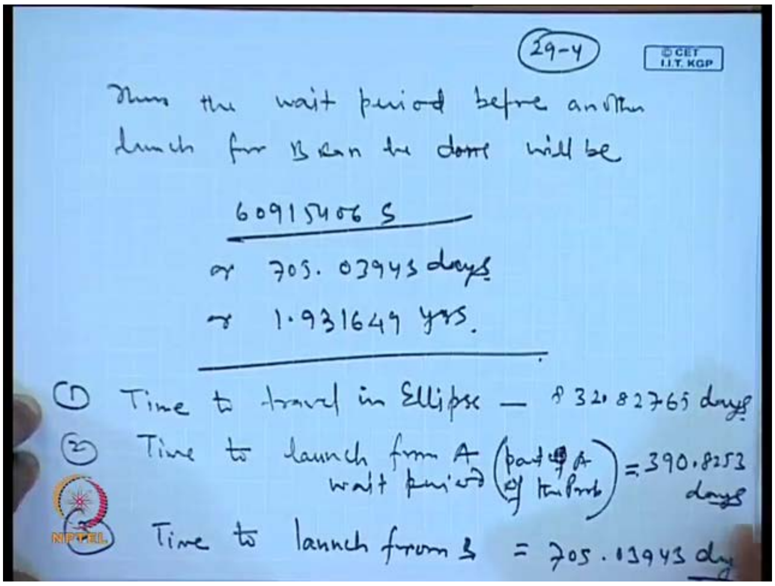


So, summarizing the results, thus the wait period before another launch from B can be done will be 6091540 seconds or 705.03943 days or 1.931649 years. So, what we have got, time to travel in ellipse this is832.82765 days. And time to launch from A, so this is the wait period; so, part A of the problem part A of the problem, so this is 390.8253 days. And third now we have got, time to launch from B this is 705.03943 days.

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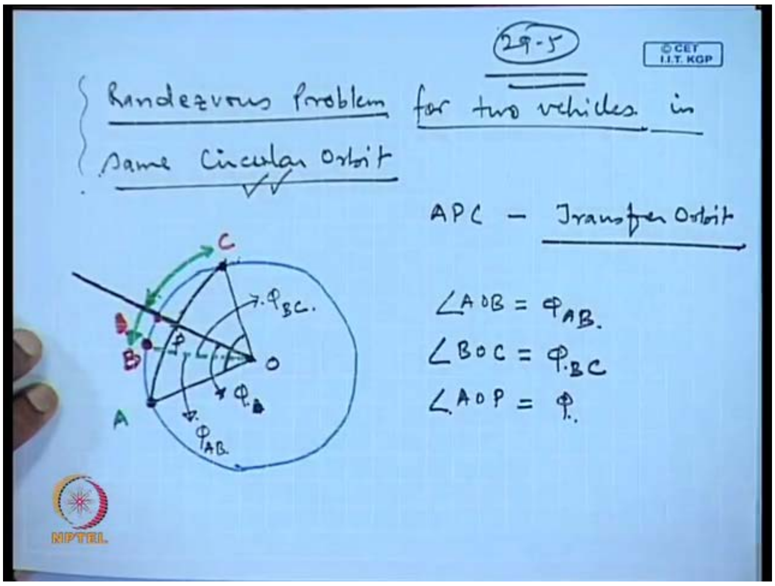


Now, once we have completed this, so whatever the work we have done till now, it is all about transfer in the coplanar orbits. But if we see the work till now it involves transfer from one orbit to other orbit or if we consider that, the transfer is to be done in the same orbit. So, if we have to do the transferring the same orbit then, it is called the rendezvous problem.

So, for the rendezvous problem we have, rendezvous problem for two vehicles in same circular orbit. So, this is the last discussion for the coplanar last (( )) one for the coplanar case. Next, we will take in the circular orbit; here in the electrical orbit right now we are considering the circular orbit. And therefore, thereafter we will go for the non coplanar maneuver. So, the non coplanar maneuver will be the last case to be considered.

So, here in this case, suppose this is the orbit and we have two satellites here, satellite A and satellite B now since both are moving in the same circular orbit; so, in the same circular orbit we cannot catch it. So, satellite A cannot catch satellite B, because if you try to thrust it, so its velocity will increase and it will go in the different orbit. So, what we need to do that, once the satellite this B is moving this is already the gap, the lead the B is having. So, once the satellite B moves from B to suppose another point which is C.

So, whatever the time it takes to move from here to herein the same time, A will move in a different orbit and it will go and catch this (( )) here in this place. And here given the o is the center of this circle. So, we join here in these two places and we will move B to little here in this place let us say that, this is the position of the B. So, the satellite has to be, so satellite B will move from this position to this position.

Now, this is the orbit in in which we will send the satellite A. So, we can divide this angle, the whole angle into two parts by using a by sector. And this give the, this point will give the perigee position of the satellite A, while it is moving in this transfer orbit A P C. So, A P C this is your transfer orbit, this is point B, this is your point B here. Let us name this angles this angle is phi, this let us write as the (( )) let us write this as phi.

And angle from here to here, this angle we write as phi A B. So, angle A o B this is your phi A B; an angle B o C we will write as phi B C, so angle B o C means angle from here to here, so this is your phi B C. An angle A o P this can be written as phi. So, we have to (( )) our plan how to work out this problem; such that satellite A can catch satellite B, while it reaches position C. So, the new position of the satellite B will be here in B prime, which is coinciding with C. And in the same place A is going to catch up, so this we are showing this as the A prime also.

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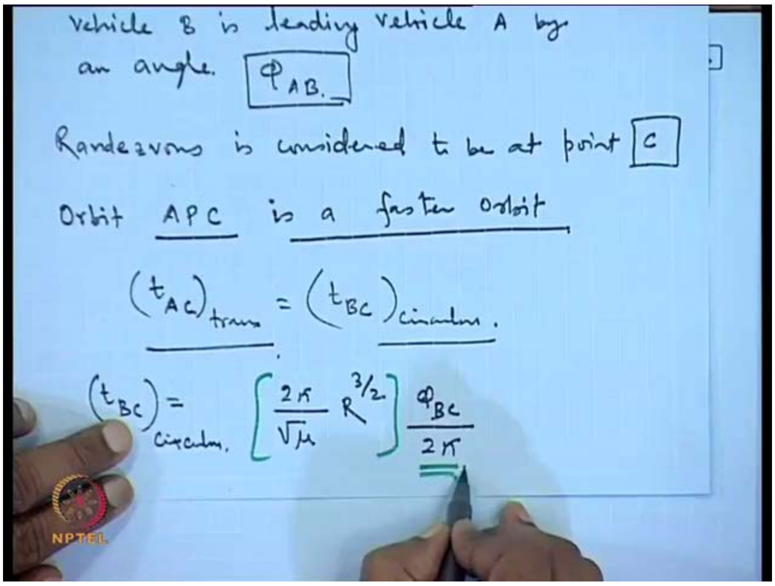


So, we have vehicle B is leading vehicle A by an angle phi A B. Rendezvous is considered to be at point C. So, your orbit A P C, the orbit A P C is a faster orbit, because it is a covering larger angle in the same time as the satellite the vehicle (( )) or the satellite will be covering a smaller angle.

So, therefore, we can write here as t A C for the transfer orbit equal to t B C for the circular orbit, this both time will be same. Now, t B C it is easy to write how much it will be t B C circular this should be 2 phi by mu under root R to the power 3 by 2 times phi B C divided by 2 phi. So, you can see from this place that, this is the time to cover the complete orbit means 360; and therefore, this has been divided by 2 phi and in which is 360 converted to radians and then multiplied by the angle phi B C. So, phi B C is the angle from here to here.

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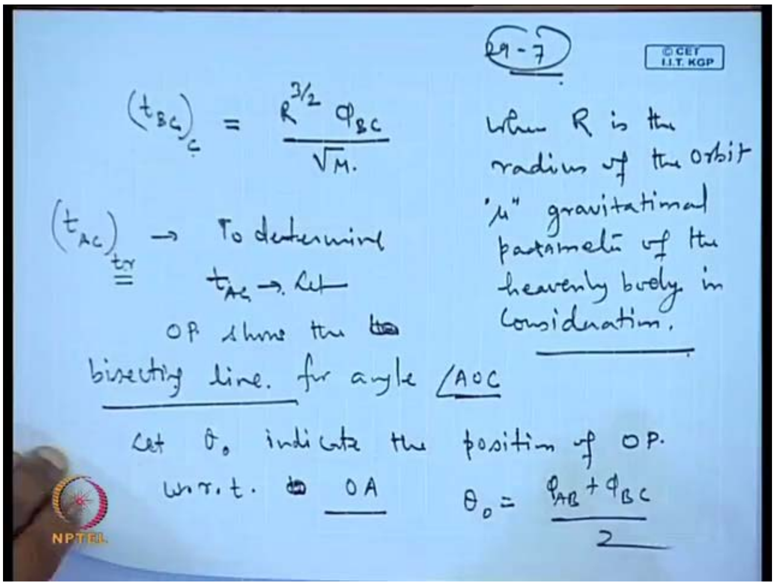


So, t B C it gets reduced to now instead of writing the full circular we will just denote this with a subscribe C to indicate it circular; so, B C circular will become R to the power 3 by 2 times phi B C divided by mu under root, where R is the radius of the orbit, and mu as usually is the gravitational parameter of the heavenly body in consideration.

Now, we need to work out t A C transfer, so we will just indicate this with the subscribe t r to be short. So, to determine t A C let, O P show the bisecting a line, so bisecting a line for angle A o C. Now, one modification we will do here, so from the to make it different sort of angle A o P we have written as phi. So, we will write this as theta 0.To make it different from the phi, which is for (( )) A o B and B o C. So, it will make little (( )) to this and you can see that, this angle theta 0.

So, A o P which equal to theta 0.This can be written as phi A B plus phi B C divided by 2.It is very clear, because this is the O P is the bisecting it; the angle A o C, A o C is nothing but phi A B plus phi B C; so, theta 0 known for the bisecting. So, determine t A C let O P show the bisecting line for angle A o C.

Let theta 0position of O P with respect to O A. So, theta 0 we have written as, phi A B plus phi B C divided by 2.Now, we have to find out, how to here two objectives are there, now you can look into this, if we can some how rather calculate the eccentricity of this orbit. So, our case will get fairly convenient because here, at what time it is going to catch here that is not known.

So, if this angle is not known to at what angle it is going to catch. So, that will that is going to be decided by what will be the eccentricity of the transfer orbit; because, depending on the eccentricity of the transfer of the orbit, the point C will decided. So, what you have do we do that, we assume the eccentricity of the orbit transfer orbit.

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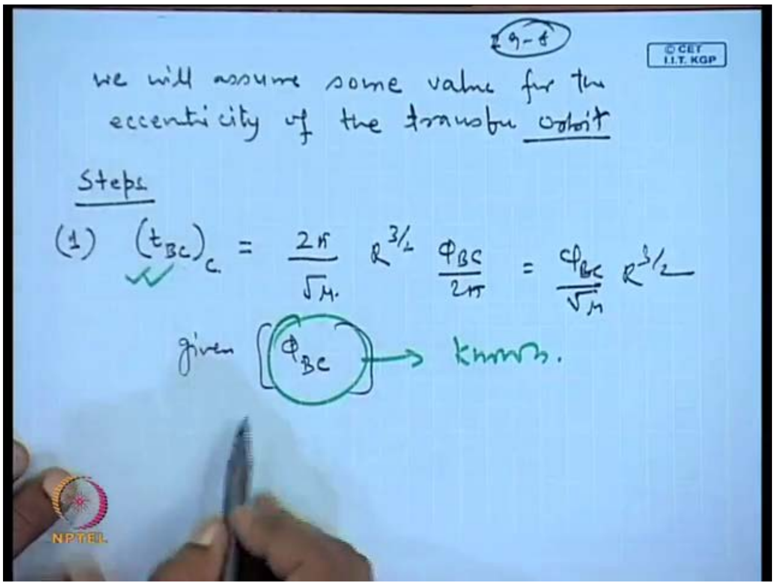


So, we will assume eccentricity of the transfer orbit and then, this case need to be (( )) iteratively because, you are not going to get the solution in one short. Here, information are not complete and therefore, you assume the eccentricity it try to find out the time involved in the transfer. So, that whatever the time B takes to reach C and from A,A takes to reach to again C; so, this two times would match out. So, this has to be done by the iterative process here.

So, we assume the values of the eccentricity calculate the time of transfer from here to here and whatever the time of transfer from here to here is there, both of them should match. So, we list the steps to be worked out. So, here your angle is given, so your t B C calculates t B C, which is 2phi by mu under root given phi B C.

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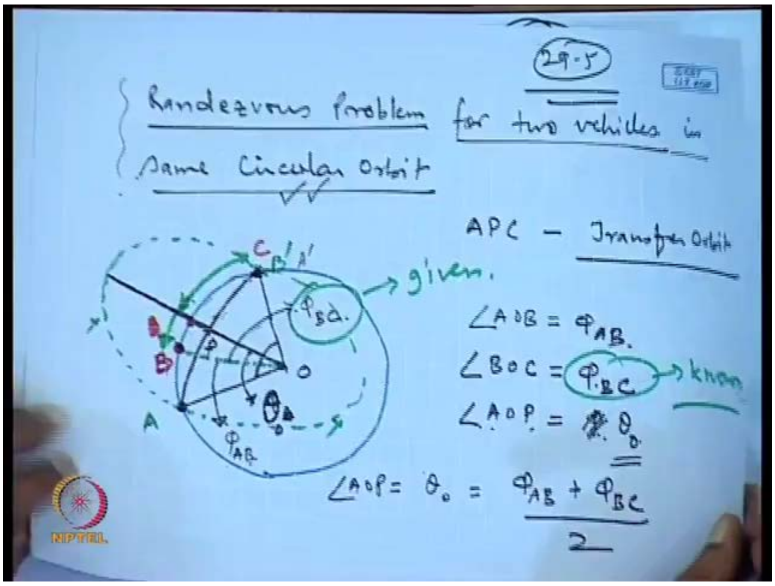


So, phi B C this is known quantity. But going from this place to this place, this can be done in a number of ways, either it is possible that I go along this orbit rather than going through this path; or it may be possible that, I throw it along this line and go along this line; but the thing is that, if it is moving from here to here if it goes in this orbit, this orbit will be taking much more time. And therefore, it will not be able to reach in this place. So, may be it passes over this place and comes again here in this place then, you go in a much larger orbit and then, you are able to catch up in this place. So, those are discarded.

So, these are the possibilities, where these are the faster orbits and you can come from this place and catch up here in this place. So, this angle is given to you, phi B C is given, this is the known quantity.

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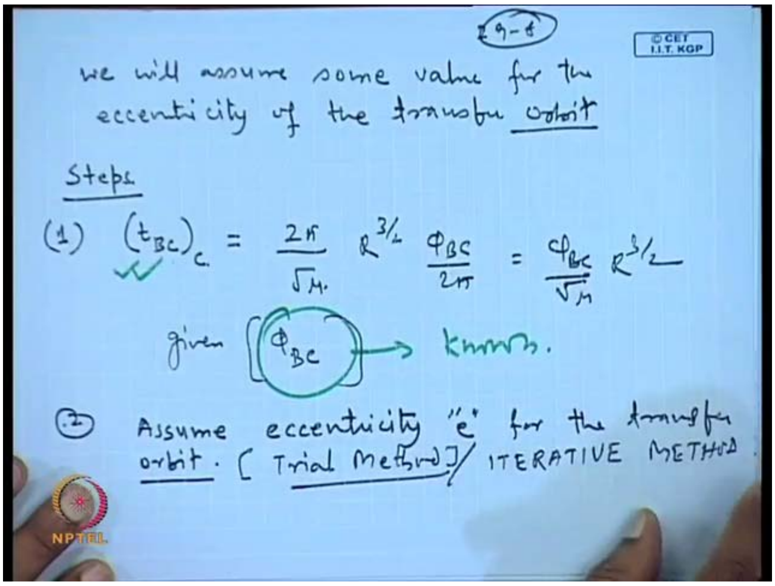


So, once phi B C is known then therefore, t B C can be calculated. And your t A C must to be equal to t B C, this is the requirement. So, the second step is, assume eccentricity for the transfer orbit. So, therefore, this becomes a trial procedure or Iterative Method.

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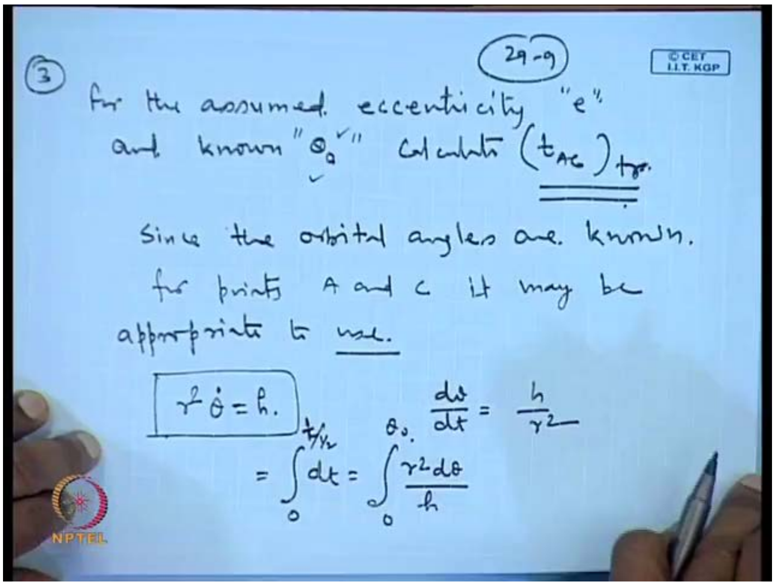


Step three is for the assumed the eccentricity and known theta 0, theta 0 we are finding it from the phi A B in a phi A B and phi B C. So, for known theta is 0, calculate et A C transfer. So, known for known theta 0, we will just calculate it for the half, half of the angle. So, how much time it is going to take from half of the angle and therefore, the time to go from A to B will be just double of that. So, by equations for the angular motion is a known to us, since the orbital angles are known, equation of motion is known. So, this is the equation of motion we will utilized. Integrated between 0 to T 1 by 2, 0 to theta 0.

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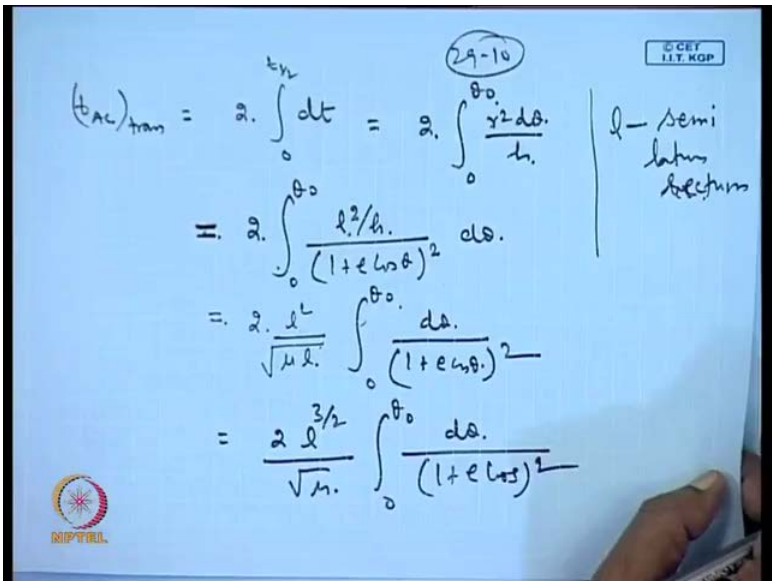


So, we can write from this place t A C transfer 2 times d t 0 to T 1 by 2 and so, this will 2 times r square d theta divided by h 0 to theta 0;or we can write in terms of semi latus rectum and then in terms of eccentricity, we can express it eccentricity we have already assumed. So, this becomes r equal to l by 1 plus e Cos theta square. So, will become l square divided by h, this is we are taking 1 plus e Cos theta whole square and d theta; h is nothing but mu times l under root, here l is the0 to theta 0.

So, this gets reduced to 2 times (( )) to the power 3 by 2 divided by mu under root. Now, here onwards we have already solve this kind of case for (( )) equation. So, we can apply the results obtain there, so this we have already done in our previous lectures.

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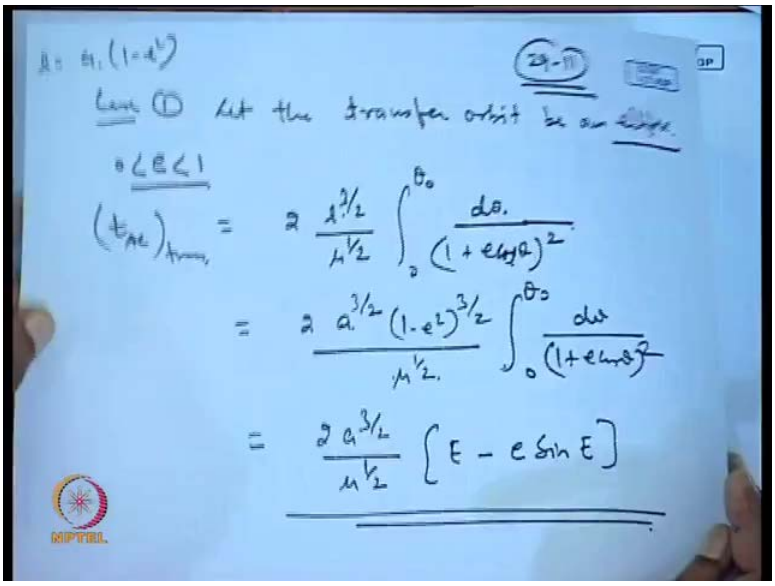


So, case 1 let the transfer orbit bean ellipse. So, if the transfer orbit is an ellipse, so in that case e will be lying between 0 and 1. And therefore, t A C transfer this can be written as 2 to the power l to the power 3 by 2, this already we have developed e Cos theta. So, utilize the previous results and if you utilize it, l you can write as l is nothing but a times l you can write as a times 1 minus e square.

So this will get reduced to a to the power 3 by 2 times 1 minus e square to the power 3 by 2 0 to theta 0 d theta by 1 plus e Cos theta whole square. If you integrate it, so integration already we have done in the earlier discussion; so, from there we can directly write the results herein terms of eccentricity anomaly.

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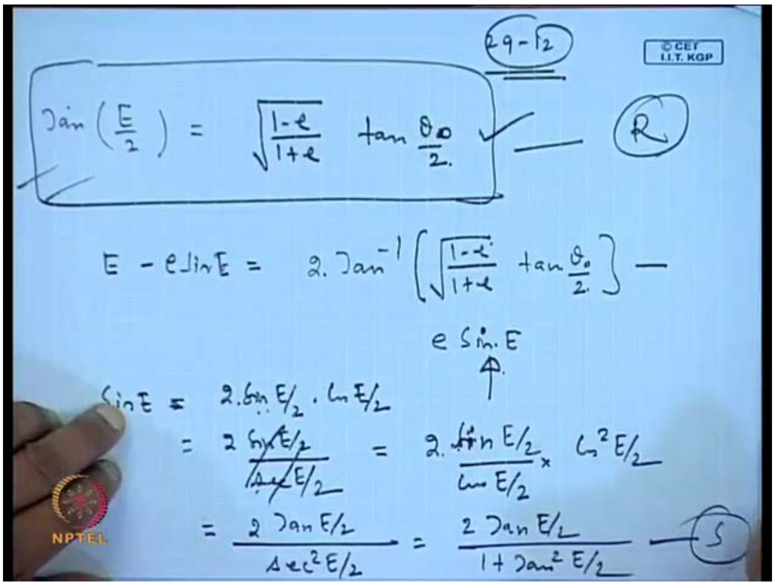


Now, tan E by 2this we have seen in the earlier derivations in the earlier lectures, this is theta 0, so we utilize this result here. So, we will have E minus e sine E 2 tan inverse we can get from this place 1 minus e by 1 plus e tan theta 0 divided by 2minus e sine E. Now, here the sine E we need to replace in terms of the theta 0, which is the actually (( )) of true anomaly.

So, sin E can be written as, 2 sin E by 2 times Cos E by 2equal to 2 sin E by 2 divided by (( )) E by 2.What we are trying to do here? Convert this sin into Cos of rather I am writing like this, we can write as we have to convert sin into tan; so, sin E by 2 and we divided by Cos E by 2. So, here it gets multiplied by cos square E by 2, this is the simple trigonometric relationship. So, this is2 tan E by 2 divided by sec square E by 2, tan square E by 2. Now, we can (( )) insert the results from here into this equation.

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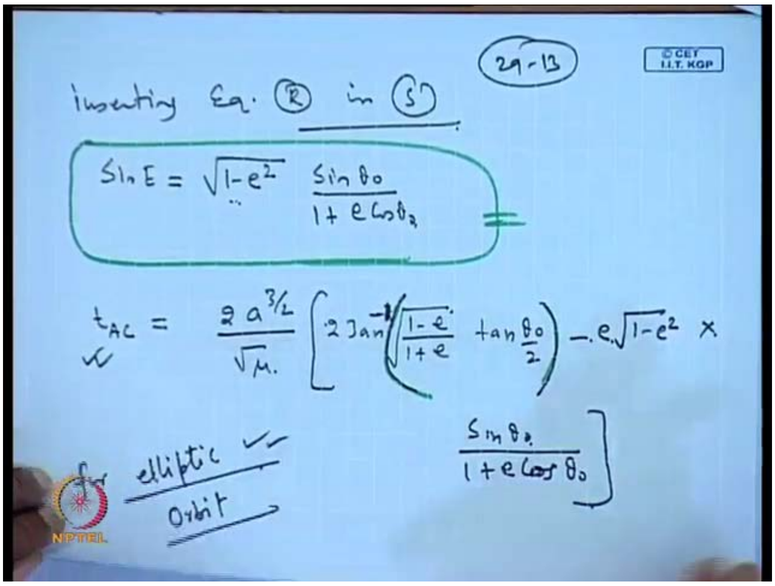


So, we will write this as the equation number R and this is equation number S, inserting equation R into S. So, sinE, E then will get reduced to, so you can do this exercise yourself. This will gets reduced to 1 minus e square under root times sin theta 0 divided by 1 plus e cos theta 0, this is easy checking.

So, once we have done this, then our t A C it will become 2 times a to the power 3 by 2 divided by mu under root; and if we look into our own equation that we have written this was the quantity here. So, now we know E, we also known sin E in terms of the true anomaly.

And therefore, writing those back into this place, this is 2 tan inverse 1 minus e divided by 1 plus e under root times, this we are writing from this place;  so, this we can show it in bracket rather tan inverse tan theta 0 by 2 bracket closed, this is minus 1. And then obviously, e times this quantity. So, e times 1 minus e square times sin theta 0 divided by 1 plus e Cos theta 0. So, this gives you the time of transfer in the electric orbit. So, we have assumed that, its electric orbit then, this is the result.

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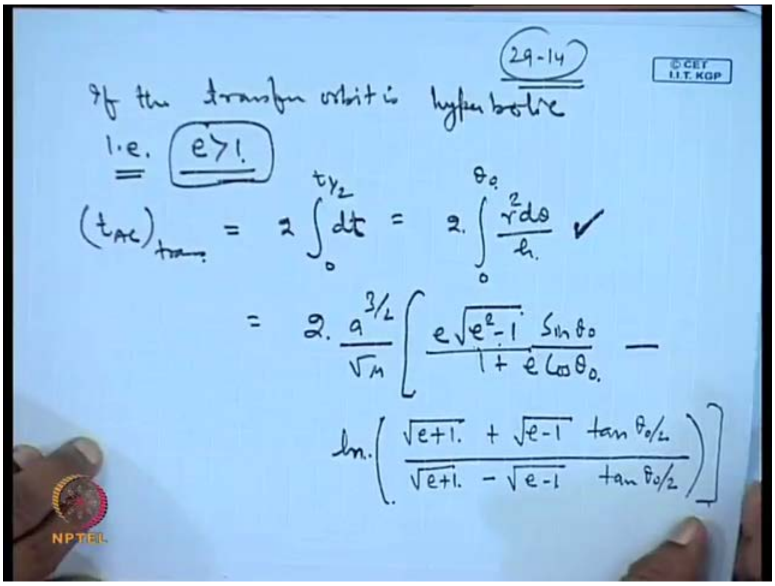


So far, now from t A Ct A C calculated if the transfer orbit is hyperbolic, this implies e is greater than 1. So, in that case t A C transfer 2 d t 0 to T 1 by 2 equal to 2 times r square d theta by h, theta equal to 0 to theta 0in the previous equation, basically we are writing here. So, for the hyperbolic orbit, you can reduce this earlier we have done it for the (( )) equation. So, here the way we developed here you can follow that method for the case of e greater than 1and you can reduce it to this equation.

See here, the difference where it is appearing, here we have written e times 1 minus e square, because e is here less than 1;in case of hyperbolic orbit, e is greater than 1 and this is appearing as e square minus 1.And moreover, here that converse which a negative sign, here it has gone into the, with a positive sign, this times sine theta 0 1 plus e Cos theta 0minus l n this is the lower (( )), so e plus 1 under root plus e minus 1 under root proving this equation can be taken has (( )) exercise.

So, the earlier case for the elliptic case we have here, the a which is appearing this a can be written as, the a here is r divided by 2 minus r v square by mu; and this we have also derived earlier in our lectures. And for the case of hyperbola, a will be r divided by r v square by mu minus 2.Once we have done this, so here the quantity r v square by 2, we can work it out in the next lecture, because time is getting short.

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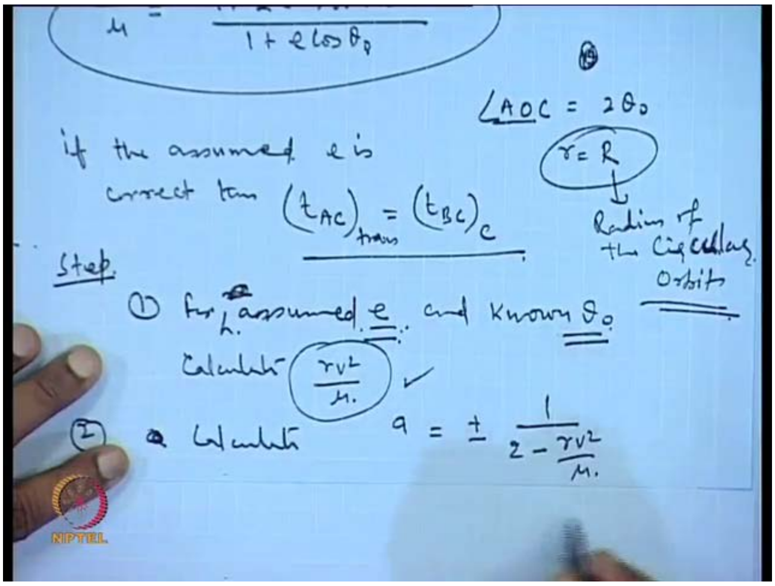


So, here r v square by mu at least we can write its (( )) this expression how much this will be, this will be 1 plus 2 e cos theta 0 plus e square divided by 1 plus e cos theta 0. And here theta or the angle A o C, this is equal to 2 theta 0; and r equal to r, which is the radius of the circle. So, this is the radius of the circle or radius of the circular orbit radius of the circular.

So, now if we assumed e is correct then what you will see that, t A C transfer this is equal to t B C circular; if it is not then you need to reassume the you have to reassume the value of the eccentricity. So, the again the step will be for reassume the value of e for reassumed and known theta 0, theta 0 is known to us and assume the again assume the (( )) e, calculate r v square by mu.

For the assumed value of e reassume, what we need to do that, here if this is not matching the t A C and t B C then, you repeat this again; once you find that, these are matching then for the assumed value of e and known theta 0, calculate r v square by mu; and then obviously, you can calculate a. So, the next step will be calculate a, which is given by equation plus minus 1 by 2 minus r v square by mu; so, minus sign for hyperbola plus for ellipse. And obviously, then t s e can be calculated.

So, we will continue in the next lecture few more steps to be written, but time is getting over. So, let us leave it for the next lecture, thank you very much.