Space Flight Mechanics

Prof. M.Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture no.# 28
Trajectory Transfer(Contd.)

We have been discussing about the trajectory transfer, so co planner maneuver we were working out, and one tangent bond we had started, so we will complete it today.

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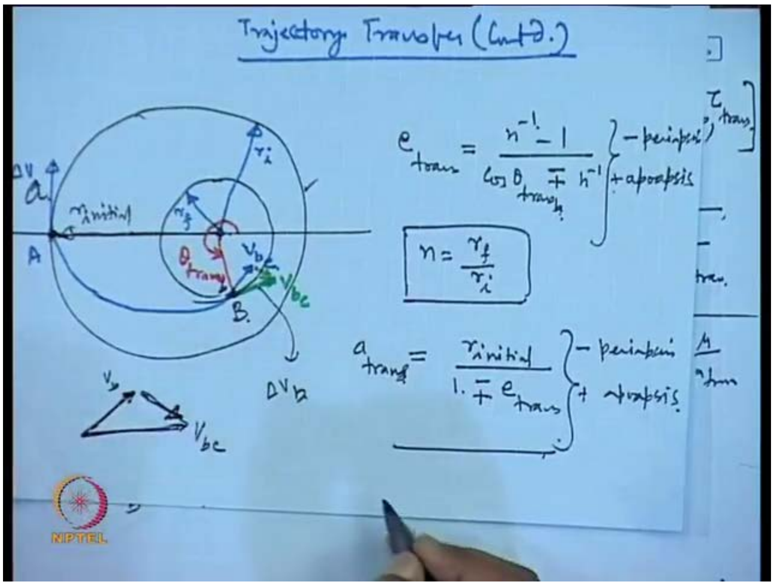

So, we had one outer orbit, in which the satellite was at this point, and there was an inner orbit, and then after giving one tangentbond at this place delta v A, it wasbrought to this place and then another bond was given here in this place, so this is v B in elliptical orbit, we show it by e. So, r i is the initial radius and r f is the final radius, and the true anomaly of this point was written as theta trans. And then we develop the equations which were, cos theta trans of the transfer orbit at the point B. So thissubscript b is appearing, so here the minus sign is for periapis, and plus sign for apoapsis. So, if you start here in this point then you take the plus sign. If you start here in this point then you take the negative sign, and n is written as r f by r i. So, using this information we calculated the a transfer orbit a eccentricity of the transfer orbit, and next we computed also, semi major of the transfer orbit, so this was given by r initial. So, in reality we will be given with two orbits, either it may be circular or it may be elliptical, and the true anomaly will be given. Initial position of the satellite will available to you. So, this is r initial position from here to here, this is r initial. And of course, this radius of this circle, if it is a circle the final orbit then this is given as r final.

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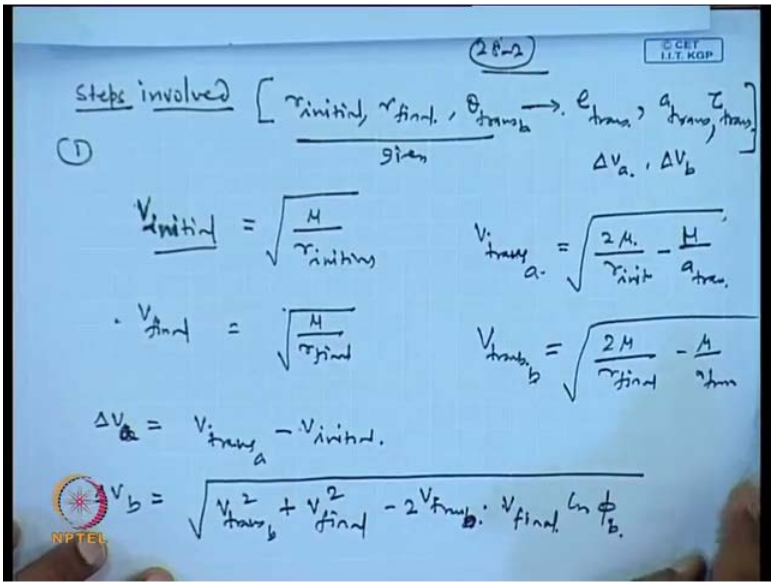

And thereafterafter doing this, we can work out rest of the things. So, the steps involved, so these are the quantities given, so from here you have to determine eccentricity of the transfer orbit, a transfer orbit and the time for transferring from a to b, and of course, delta v A and delta v B, the impulses required at the two points A and B, so this is your this is point B and this is point A. So here the how much impulses required, this will be denotedby delta v and this is delta v A. And we can remove the sign just we are, because we are showing it in this direction, so no need of showing the sign, so this basically we need to de boost in this place, and again here we have to bring it in the smaller orbit so we need to de boost it. So, here this is the v b c, the velocity at point b in the circular orbit if it is a circular orbit, and this is the impulse and required to bring it to this.So, let us compute this quantitiesone by one; v initial if the outer orbit is a circular orbit, so this simply written as mu by r initial under root.

The next you have v final you can write as mu by r final under root, and v in the transfer orbit at point a this will be 2 mu by r initial minus 1 mu by a transfer orbit square root.

Andat point b in the transfer orbit, this is mu by r final minus mu by a trans under root. So, first you have to need to compute all this quantitiesthen delta v b will be given as. Let us first compute delta v a, so v trans a minus v initial.Similarly, delta v will be given by. Now in the case of delta v b which is shown by this small green line, here it is difficult to see it, but it is appearing something like this is v b, and suppose this is v b c, so then you need to give. This, vector is larger so you need to give boast it here in this place. So, this will be quantity here delta v b. So, delta v b will be given by v trans b square and then v final square minus two, v trans b times, v final times cosphi at b, where phi a is the flight path angle at b.

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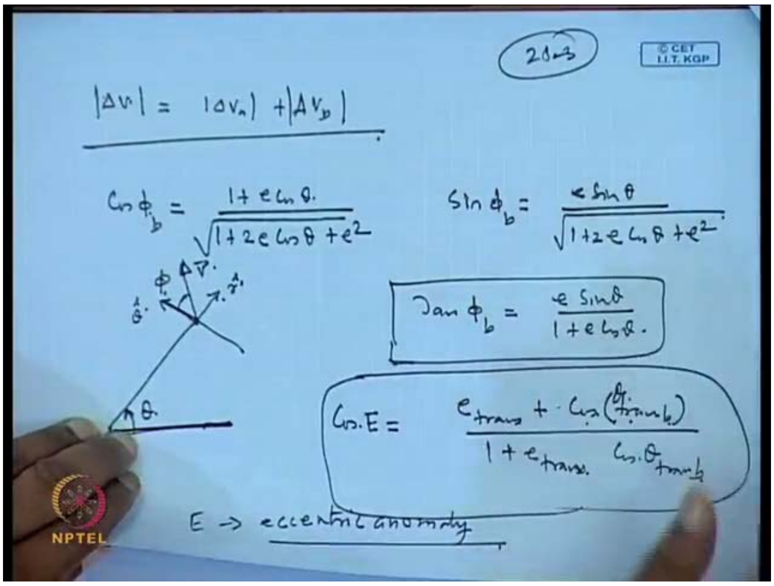

So the total velocity change required delta v will be delta v a plusdelta v b. The flight path angle cosphi b can be written as,1 plus e cos theta divided by 1 plus 2 e cos theta plus e square under root. And sinphi e can be written as e sin theta, where you can look thathow we define the angles again I am repeating. If this is the v direction, this is the radius direction, so this is the radius vector in this direction, and this is theta direction, then this is your phi, and this angle is your theta which gives you the true anomaly. So, we are measuring from the perigee position. And tan phi b this will be written as e sin theta by 1 plus e cos theta. So, these are the relationship that you can derive and all this things we have worked out earlier, so no need of repeating in this place again. Now, what you need to do here, find out the eccentric anomaly using this equation. So, look into earlier lectures we have developed all this things. So, e trans plus cos theta trans b divided by 1 plus e trans, eccentricity of the transfer orbit, times cos theta transfer orbit at b, and where E is the eccentric anomaly, you need this quantity I am going to explain.

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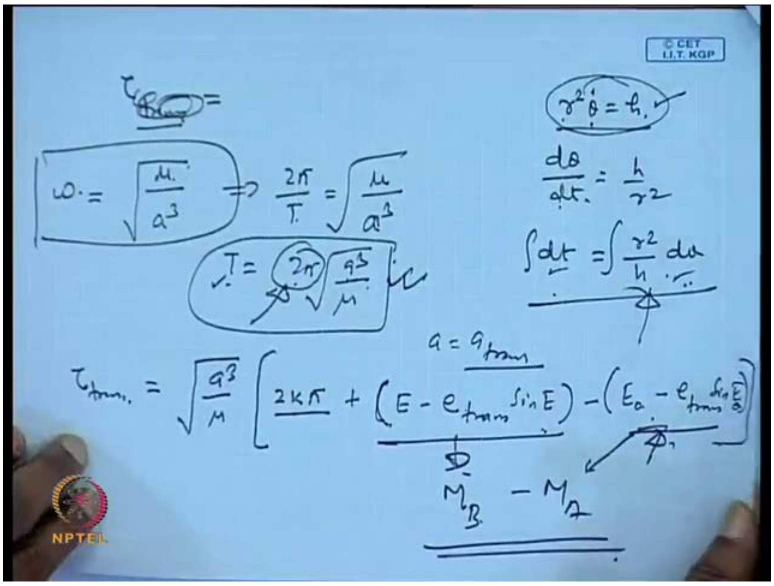

Say, if you want to find out the flight time from A to B, so you need to integrate the equation, which earlier we have written as r square theta dot equal to h, and this often this was written as d by d t equal to h by r square. And for finding out the d t, this was integrated and this is r square by h d theta, we integrated this andyou know that while working with the Kepler's equation, so we did all this things. So the same equation we have chosen here in this place, so while working for the Kepler’s equation, we derived this relationship and the same relationship we are using here. Now if you use the true anomaly, so finding out the time then you need to integrate the equation, so to avoid that we work in terms of the eccentric anomaly. So, the flight time will be given by, now the flight time taken, that will be obviously if we are expressing in terms of, this isan elliptical orbit and here the angular rate is obviously varying as the r varies.

And therefore the angular rate, you can see from this place h is a constant, and therefore as the r varies the angular rate, the theta dot which is, it keeps varying, therefore, instead of using a changing theta which needs the integration of this equation. So we use the mean angular rate,and mean angular rate obviously you know, that for mean angular rate this is given by mu by a cube under root and obviously from this place you can write 2 pi by t equal to mu by a cube under root, and t becomes 2 pi times a cube by mu under root.

So this is the time period of the satellite to complete one orbit. So if it starts from here and if you leave here, so it will come back after this much of period, where a is the a transfer orbit; semi major axis of the transfer orbit. Now, we do not want to use this equation particular equation given here. So, rather we will write the whole thing in the eccentric anomaly.

So tau transfer, this will be given as. Now, you can look from this place, this is for 2 pi, if you have to go through by 2 pi radiance then you need this much of time, which indicated by capital T and if you have to go by some smaller angle say theta, so you need a smaller amount of time.So, here the theta instead of working in terms of theta, we are just indicating using this mean angular rate, which is given by mu by a cube. So, here this will become a cube by mu under root times. Now, suppose you let the satellite go again and again in this orbit. It is going from here to here, passing here, coming here. So it may go get times and after that you are carrying out the maneuver. So, you need to include all those angles here, so this can be written as 2 k pi. This is for the repetition how many times you are repeating that orbit, plus E minus e trans sign E minus E 0.

So this is the eccentric anomaly of this point, and this is the eccentric anomaly from where you are starting, this point. So we are starting here in this point, and this is your elliptical orbit which is getting completed like this.So, eccentric anomaly of this point and what is the eccentric anomaly of this point, so you have to subtract like this. And this quantity is nothing, but we are writing in terms of the eccentric anomaly, so this is nothing, but your mean anomaly. So, mean anomaly of point B and this is mean anomaly of point A. So this gives you the transfer times which is required, if k is equal to 0 means, in the first shot only you are going to transfer it. So k will be equal to 0 and only these quantities are required.

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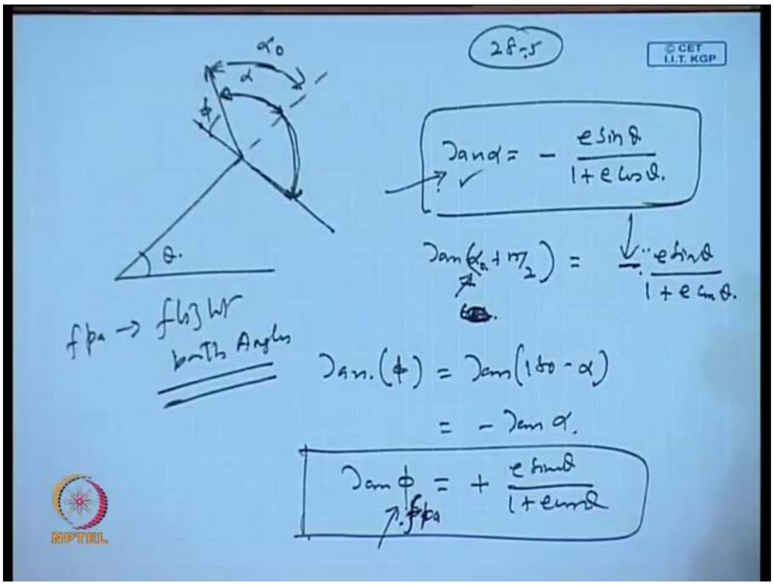

Now we have done this, so last time the problem which we left out. So what we need to do now, is to complete thatand before completing thatthe equations that I have used I will just work it out for you. This cosphi and sinphi,it isa easy to see that, if this angle is theta and this angle is phi, and earlier we defined this angle. If you look into earlier lecture we defined this angle as alpha. So, either we can define this angle as alpha or either I can define the angle from here to here as alpha, and depending on the how you are defining this angle alpha, the equations for tan alpha changes. So if you define tan alpha from here to here, so this becomes tan alpha equal to minus e sin theta by one plus e costheta, and this we have worked out earlier.

But we worked out by writing here alpha from this place to this place, now we have shown the alpha from here to here. If you want to do other way, so what you need to do. If you indicate alpha say from here to here, so let us say this quantity is alpha 0 you are writing. So tan alpha 0, you want to indicate it in terms of tan alpha 0, so the same here alpha will become alpha 0 plus pi by 2. And this is the quantity on the right hand side, so this quantity becomes cot. Soeither you express it in this way so from here you can get the value of alpha 0, or either you express it in this way, both are possible so. If we take this option then we have the working out for phi here. So we can directly write phi equal to tan phi equal to tan 180 minus alpha, and this becomes tan minus tan alpha. So this minus tan alpha you canput from this place, so this is esin theta by 1 plus e cos theta.And this is what we are interested in, and where phi is the flight path angle. This is f p a flight path angle.

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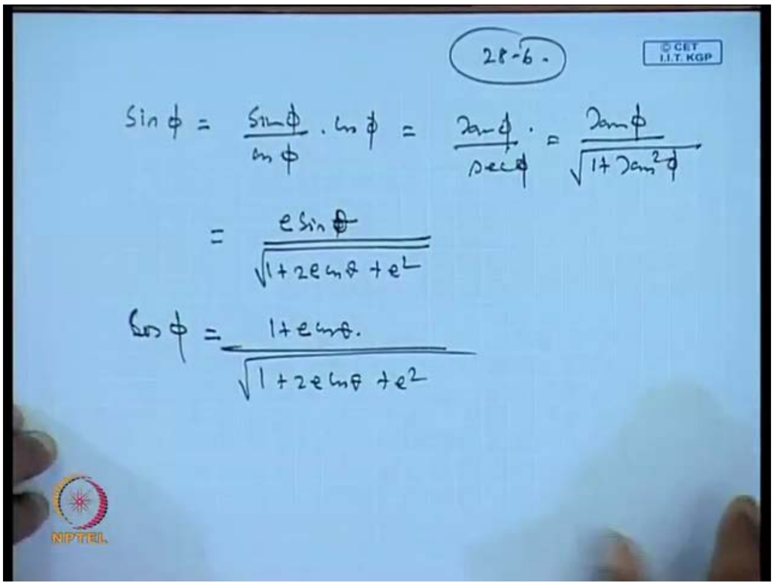

Once, this is known to you, then sinphi simply you can write as;sinphi by cosphi times cos phi. So this becomes tan phi divided by sec phi, 1 plus tan square phi under root. And insert the value for tan phi here and the value for the sinphi you get as, e sin theta divided by 1 plus 2 a cos theta plus c a square under root. And similarly, cosphi you can get as 1 plus e cos theta divided by 1 plus 2 e cos theta plus e square under root. It is easy to working out, you just look in to other lectures and you can complete it.

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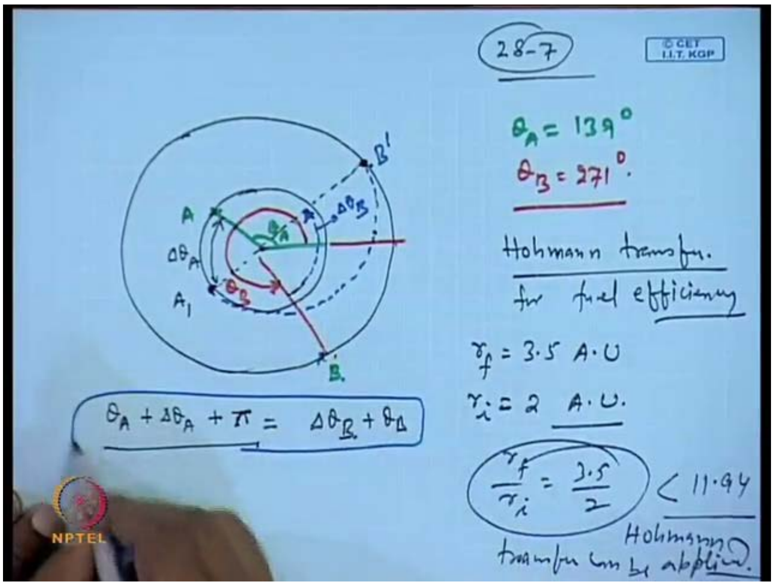

Now, we complete the last time problem. The last time we left it, because this problem was bit lengthy and I did not to cover it in this one go. So the problem given was the first part we were working out, that we havetwo heliocentric orbits, in which one asteroid was at the point A another asteroid was at point B, where this angle was say the theta A, so theta a was giving to be 139 degree, andthe angle fromhere to here, so this angle was theta B and theta Bwas given to be 271 degree. Now the problem was that, some satellite is to be launched from A, A is moving B is also moving and this satellite has to catch up B. So, the requirement was also that we have to do the Hohmann transfer, Hohmann transfer for fuel efficiency for saving the propellant.

And obviously this case r final was given to the 3.5 astronomical unit, and r initial was given to the 2 astronomical units. And therefore, r f by r i, this is 3.5 divided by 2. This is less than 11.94, and therefore Hohmann transfer can be applied, and it will be fuel efficient. But before catching this, so Hohmann transfer can be applied such that, if you are suppose you are applying the Hohmann transfer in this place, so at that time while the satellite is catching B, so B must come out to be here in this point. So exactly the pi degree, pi radiance the satellite has to move from this point to this point. So this point we termed as A 1 from where, and this angle we wrote as delta theta A. and then the satellites was launched from here to here, and it went and caught up the asteroid B at the point B prime.

So in the first go what we have worked out that, theta A plus delta theta A, plus the angle from here to here which is nothing but pi. This could be equal to the angle which B will move from here to here that it will cover, so that wasdelta theta B plus theta B. So this angle from here to here and then from here to this place. And from this place to this place, this is total angle here and from here to here this is angle your theta B. And from this place to this place this is, so we can show it here. This angle isfrom this place to this place, this angle is your delta theta B. So this must equalize, obviously it is apparent from our figure.

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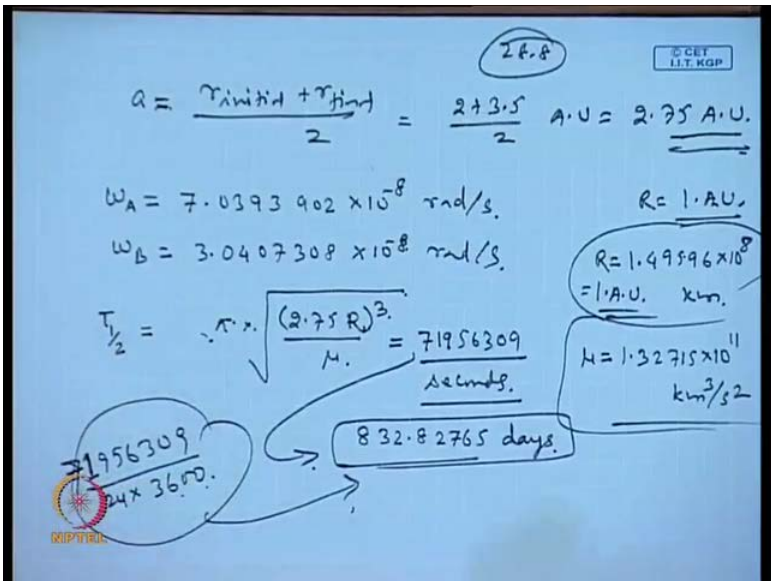

Because it is a Hohmann transfer and in this transfer orbit we are boasting at point A one. Sothis is basically you are doing the firing the rocket at the perigee of this transfer orbit. And this is the apogee point of the transfer orbit. So, the semi major axis x was given by r initial, plus r final divided by 2. So, this was 2 plus 3.5 divided by 2 astronomical units, so there 2.75 astronomical unit. This we calculated last time. And then using various relationshipthen we worked out omega A also, so omega A termed out to be 7.0393902 into 10 to the power minus 8 radiance per second. And omega Bsimilarly, we calculated to be 3.0407308 into 10 to the power minus 8 radiant per second. Now, the time of flight, for completing this orbit from here to here, so this is half the orbit. So we will say, this is T1 by 2. This is nothing, but pi times 2.75 astronomical unit. It is in astronomical unit, so this we write as R, so where R equal to 1 astronomical unitdivided bymu and this whole cube under root.

So, this is the half time period and if we put 2pi, means that will be time to a start from here to here for the satellite and come back again to this point. And putting the relevant values then we calculated to put the value for the R, so last time we have stated the value of R. So R was having a value of R equal to 1.49596 into 10 to the power 8 kilo meter, so this is one astronomical unit. And similarly, mu the value or the mu that we choose, it was 1.32715 into 10 to the power 11 kilo meter cubic per second square. So with this values inserting into this,this gives you 7195639 seconds. And if you convert this into day, so in day this will turn out to be 832.82765 days. Now, divided by 71956309, and divide by 24 times 36100, so this will give you this value, this is 719.

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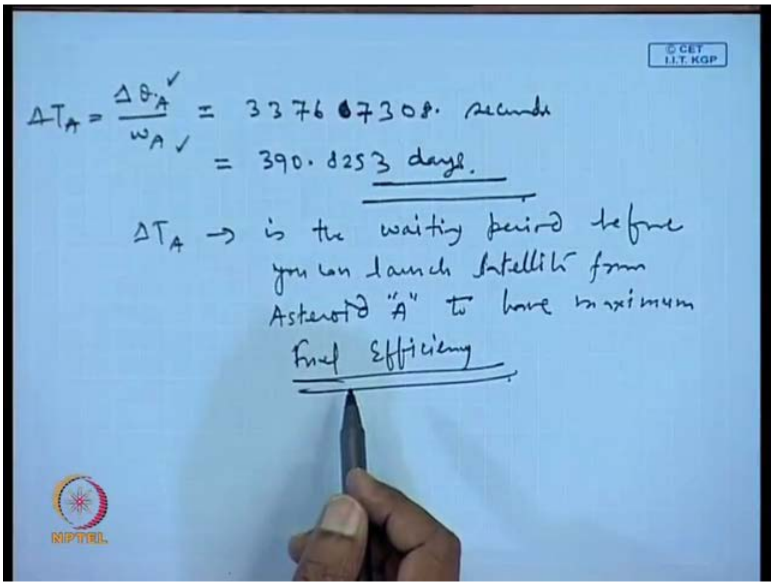

And then we also calculated delta theta A by omega A, which is equal to T A, delta T A. so delta T A is nothing, but the timetaken for the asteroid to move from this point to this point. And this is whatwas asked in the question that how much time is required, before the satellite can be launched from the asteroid A. So this is the time, this is the angle before you can launch, and if you divide by the angular rate of the asteroid, so that gives you the time, so this is what exactly has been done. So, this is the waiting period before you can launch the satellite. And this turned out to be 33767308 seconds, which was equal to 390.8253 days. So, this was the waiting period delta,TA is the waiting period beforeyou can launch satellite from asteroid A to have maximumfuel efficiency. So if you have to send your satellite from earth to mars, so you have to fallow exactly the same pattern.

Here you need to know the initial position of the earth, initial position of the mars, at a particular epoch from where you are counting, and then you have to compute when your window is available for launching the satellite. So that window you have to calculate exactly in the same way, and then you can repair the mission for sending your satellite to mars. Now, by how much angle the asteroid A will move from this epoch. So this is our starting epoch, and from here how much angle it is a going to describe. So the asteroid A is already here in this position,it is moving to this position, and then it is launching the satellite. So in the meanwhile asteroid A will move from position A to suppose,it is moving and reaching this position A prime, while the satellite catches up at B prime, so asteroid has moved to A prime, which is the position given here.

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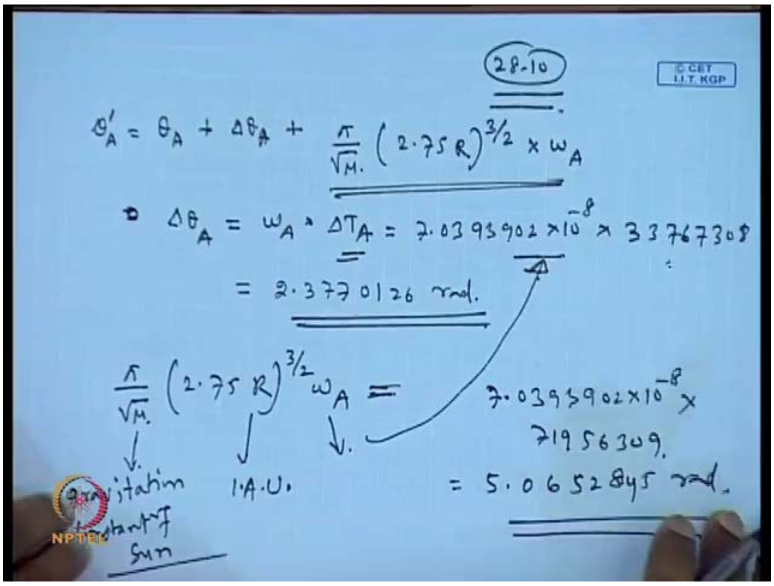

So,A prime will be the delta theta A prime, this will be equal to theta A plus delta theta A plus. So, you have the theta A from here to here, and this is the angle delta theta A, and the rest of the angle from here to here,here to this place. Now, this depends on the angular velocity of the asteroid. And the time taken in the meanwhile how much time the satellite takes from here to here. In the mean while, asteroid B is reaching from this place to this place,and that will be given bythe half period of the elliptical orbit. So, this is 2.57 times R to the power 3 by 2 times omega A. Now insert the values for all of them and you can calculate. So delta theta A we have already found out, delta theta Awas. We can write it here, omega A times delta T A. so delta T A is the time that we have just now calculated. This is the quantity here delta T A390 and second it is a given like this,and delta theta A also was available to us. Anyway we will writehere, this is omega A is 7.0393902 into 10 to the power of minus 8 times.

This is the angular velocity and times delta T, so delta T is your delta T A, this is 33767308 this is in seconds, this is what the quantity is shown. So we have taken here in this place, so this turns out to be 2.3770126 radiance. Similarly, this quantity you can compute and this quantity pi by mu under root, times 2.75 r to the power 3 by 2 times omega A. Inserting all the values R is the 1 astronomical unit, and omega is the angular speed which is shown here. So inserting this value pi you know, and mu earlier we have stated. This is thegravitation constant of sun. This is not the universal gravitation constant; this is the gravitational constant of sun, as in earlier lecture we have discussed. So, by inserting all the values, this will turn out to be. So, omega A is 7.0393902 into 10 to the power minus 8 and this quantity we can compute and write here. So, this quantity is 71956309, you can verify yourself, so this will turn out to be 5.0652845radiance.

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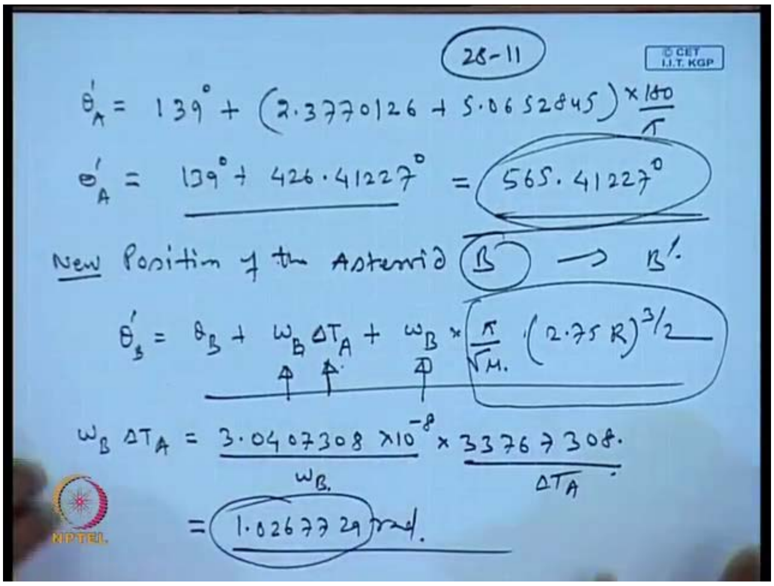

So therefore, theta prime A in degree this will be given by 139 degree plus,2.3770126 plus 5.062845 into 180 divided by pi,so139 degree plus 426.41227. Each of the competition you must verify yourself if you want to learn. And this is 56.41227degree this is what is coming, so around 56.41 degree. Similarly, we have to calculate the position of the asteroid B. So new positionof the asteroid B, which is B prime, so theta B prime; this will be theta B plus omega B times delta T A plus omega B times pi, where

102 mu 2.75R to the power 3 by 2. We can look what are the quantities these are. Look into this figure, so theta omega Btimes delta T A. So, this is the time in which the asteroid a moves from point A to A 1, this is the point A 1. So it is moving from this position to this position and that time we have written as delta T A, and this time we have earlier calculated this is known to us, omega B is known to us, this quantity is known to us, all this quantities are known. And it is basically your flight time of the satellite.

So the satellite the time it takesto move from here to here, so in the same time, so the time A takes to move to A1, so that time B will move from here to suppose, it moves to this place. And there after the satellite moves from here to here. So B will approach here in this point and both of them will met together here in this point B prime, so this is what the equation appears. Now each of the values you can compute separately, so omega B times delta T A, I am againstating that you must do it independently by computing and verifying all the results, and that will give you confidence to work out this example. So this is your delta T A and this is omega B. So this will turn out to be 1.0267729 radiant, this is the value. Now we next need to compute this, so this quantity earlier we have computed in the previous examplein the previous step. Here we computed this quantity, which is appearing here in this place 71956309. So we can utilize here and this has to be multiplied by omega B.

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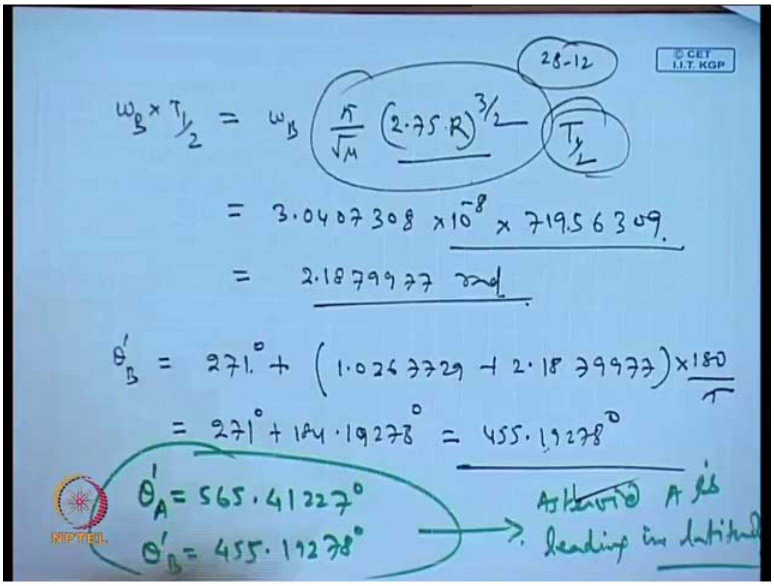

So next the quantity w B times T 1 by 2, this will be omega B times pi by mu under root times 2.75 this is the semi major axis of the transfer orbit what we are writing here. And this is the period for completing the half orbit, so this is your T 1 by 2. Now, inserting this values omega Bis30407308 into 10 to the power minus 8.And this quantity we have calculated earlier, this is nothing, but 33, no in this, not the quantity. Here we have done it 71956309. So if you multiply this,this turns out to be 2.1879977 radiant, and therefore theta prime B this can be written as 271 plus 1.0267729 plus 2.1879977 into 180 divided by pi. So 271 degree plus 184 point 1.9278 degree, so this turns out to be 455 and 19278 degree means point192 degree or point193 degree we can have approximate. So this is your theta prime b. So what ultimately we have got as the result, theta prime A this is equal to 565.41227 degree and theta prime B equal to 455.19278 degree. Therefore, you can see from this place that, asteroid A isleading a latitude.

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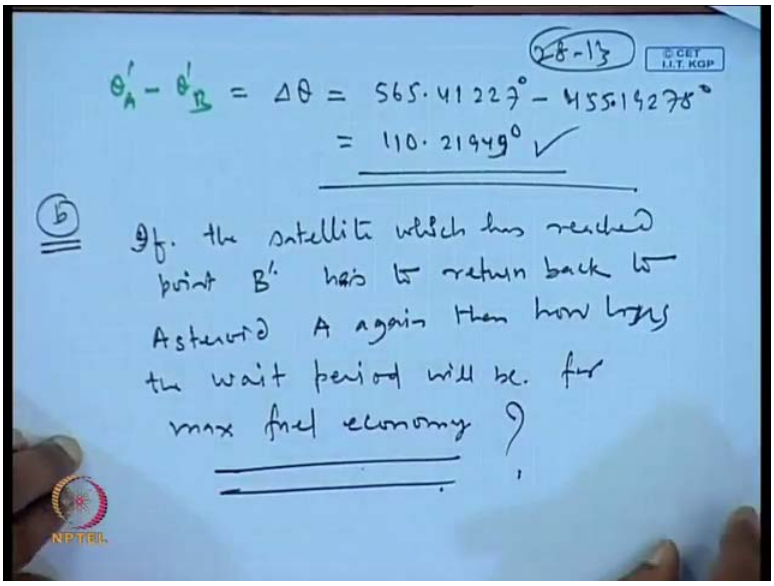

So this quantity theta A prime minus theta prime B, this will writesubtract and write this as delta theta, which will be 565.41227 degree minus 45519278 degree, and this turns out to be 110.21949 degree. So this is the lead available to asteroid A. Now the part B of the question is part A is over. So part B of the question is that, once the asteroid has reached into the point B, and the satellite has got this randhibo has taken place.So satellite has got the asteroid B at B prime. Now if the astronomer who is sitting in the satellite, if he wants to return back to A againhow long he has to wait? If he wants to minimize the fuel. Next question is; that if the satellitewhich hasfixed point B prime has to return backto asteroid A again, then how longthe wait period will be, for maximum fuel economy. So, this is the second part of the question.

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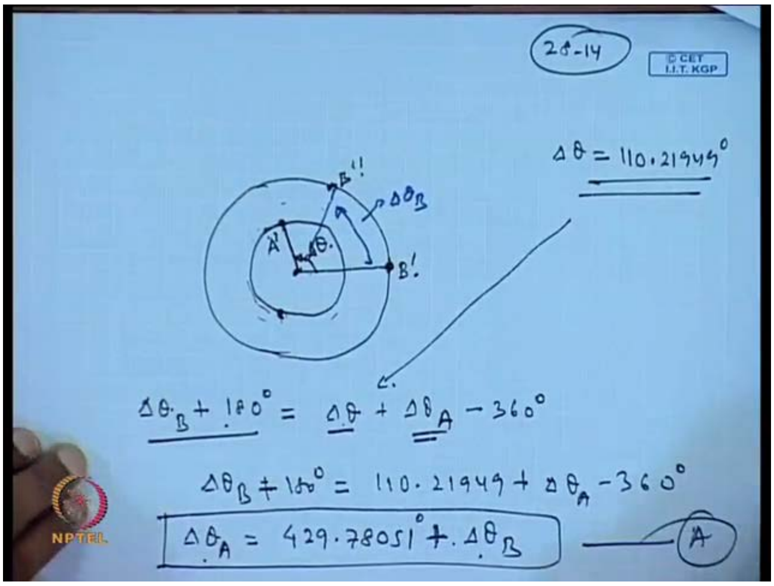

Again we draw the figure, so with respect to asteroid B we can show that this is the position B prime suppose. Andthe asteroid which is in a smaller orbit, so this is located somewhere in this point. So this is your A prime point, so this is the initial situation. And the lead is ofthe lead angle we will show it by delta theta and delta theta just now we calculated, it turned out to be 110.21949 degree. So, how long the waiting period will be before we can launch a satellite from the asteroid which is now at prime B prime.So suppose that, before the launch can be done, so this moves to some new position here B double prime. In the mean while it will move into A prime will also move. So A prime is moving thisasteroid A is also moving asteroid is also moving, but the thing is that we have to always launch in such a way. Now appear this will be the apogee position and then you have to catch up this at the perigee position of this transfer orbit. So, we need to work out further this quantity.

Now, let us see how to do it, so we will write delta theta B plus 180 degree. We are writing in terms of degree, later on we will convert into radiance.So you see that delta theta B this will be indicated by something like this line, where this is your angle delta theta B. And then suppose it catches A at this point, so this will be the angle from here to here. Now but, this satelliteasteroid A is in the smaller orbit so its angular rate is passed.So by the time it goes here, this will be covering a larger angle and it will cross it. So, it would not be possible to catch up here in this place, you will have to wait for one more period, once it returns back and comes into this position so we have to compute for that quantity. So, this will be given by delta theta plus say that delta theta A is the angle covered by the asteroid A.

So delta theta is already the lead available, and delta theta A is the another angle that it will be moving around, and coming into this position and then B will be coming and catching up here in, the satellite will start from here and catch up here in this position. So we need to subtract here 360 degree, because it has taken one round already hereto come to this point. So these things must be taken care off. If you write in this fashion then your delta theta B. Insert the value of delta theta from this place. So delta theta B can be written as, delta theta B plus 180 degree equal to delta theta 110.21949 plus delta theta A minus 360 degree, andworking it out delta theta A can be written as 429.78051 degree, plus delta theta B. So this time we have expressed delta theta A in terms of delta theta B. So let us say this is our equation number A.

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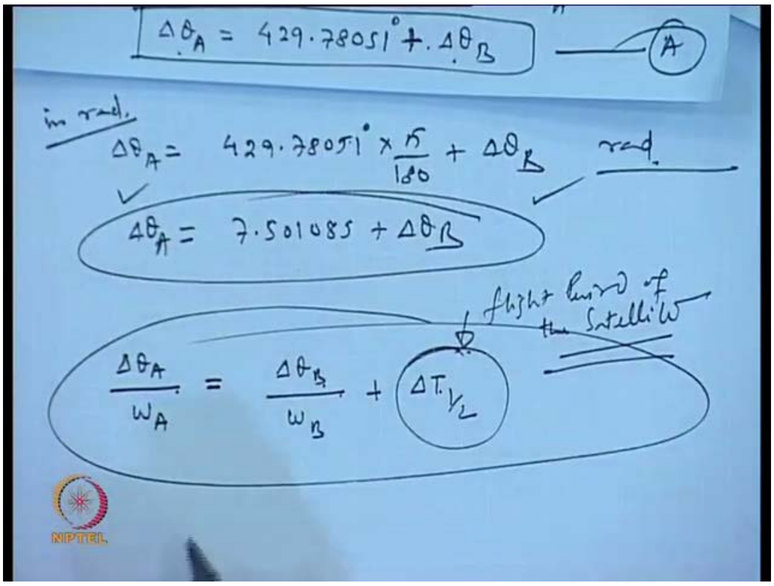

Now, the next step will be, to compute the time taken. So, to cover this angle the total time required and to cover this angle the total time required, they must be same. So, let us come convert this into the radiance. So, if we convert this into radiance, so delta theta A can be written as in radiance 429.78051 degree into pi by 180 plus delta theta B,where delta theta A and delta theta B are on now in radiance. So, this becomes 7.501085 plus delta theta B. Now, we can go for calculating the equating the time, sonow delta theta A divided by A will move from this place, and it will it goes once around this and whatever happens to this. So, we have the time delta theta A by omega A, this must be equal todelta T 1 by 2 is, is the flight period. So, we will continue with this, time is getting over. Sowe will complete in the next lecture. Thank you very much.