Space Flight Mechanics

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Lecture No.# 27

Trajectory Transfer (Contd)

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Last time we have been discussing about trajectory transfer, so we discussed about the Hohmann and the bi elliptical transfer. So, Hohmann transfer we will take one more example to show how does it work.

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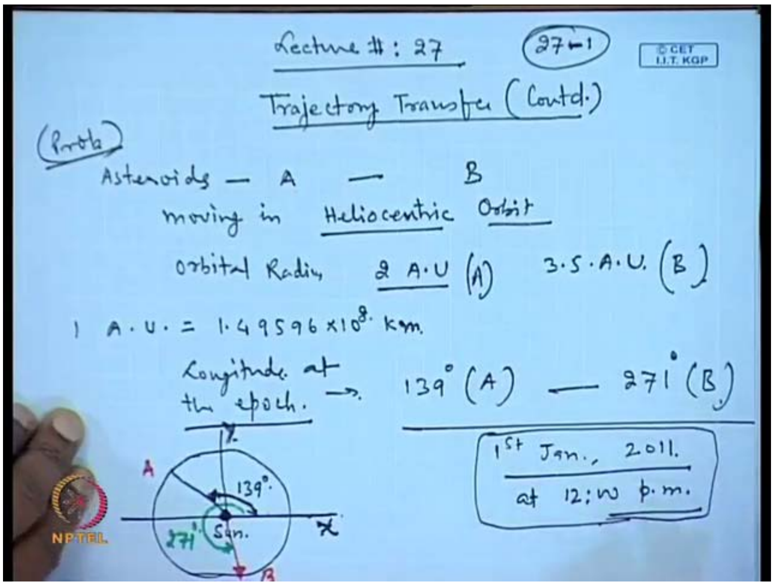

So, here today the situation is little more complicated, so let us suppose, we have two asteroids, we have two asteroids A and B, and they are moving in heliocentric orbit. Heliocentric orbit implies that its moving about the sun, now for this two asteroids the orbital radius is given to be, orbital radius for this two asteroids are, 2 astronomical units and 3.5 astronomical units, where 1 astronomical unit, this is equal to 1.49596 into 10 to the power 8 kilo meter. So, this is for asteroid A and this is for asteroid B. So, longitude at the epoch, that is at the beginning, the time from which we counted movement, so that is given to be the time or the position from which we are counting. So, suppose we have here one orbit, so I can have a difference orbit indicated by x and y. If, reference x indicated by x and y, this, is an inertial axial force. So, I can assume this point to be the sun and this is the orbit of the asteroid. So, at the epoch implies that at the beginning, once you are starting with the problem, so here the first asteroid is having around this is 139 degree longitude, so from here to here.

And, the second asteroid is having its position of 271, so this will be around this position. So, this is for asteroid A and this is for asteroid B, and the angle for this is measure from hereto here, this is 271 degree. So, the epoch for A is 139 degree, this is for A, and 271 degree, this id for B. And this epoch can be chosen to be, we can choose arbitrary in this problem, let us say that, we say that, on first of January 2011, at 12 p m this was the epoch of the asteroid. Now, what we need to do, suppose we want to send certain pro from asteroid A to asteroid B, but the radius of the two asteroids are different. Here we have shown, just we are showing the epoch not the radius, so radius are different. So, radius is given to be 2 astronomical units and 3.5 astronomical units for B.

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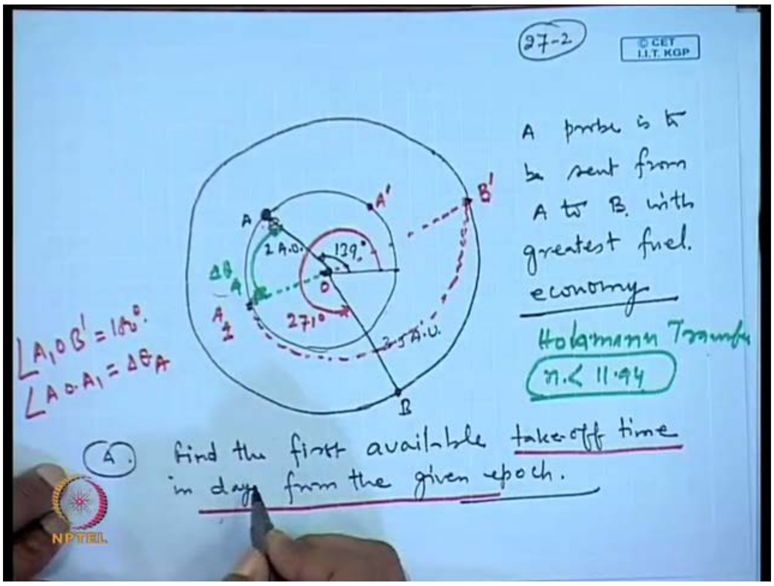

So, the real figure will appear here, so we have asteroid A, which is moving in this angle from here to here is 139 degree, and here this is asteroid A, and this is 2 astronomical units, this radius. And then we have another asteroid, which is at B and the angular position is, measured from here to here this is 271 degree, and this radius is 3.5 astronomical units. Now, a probe is to be sent from A to B, with greatest fuel economy. Fuel should be shift, so obviously we need to use the most efficient transfer in going from asteroid A to asteroid B. So, what we need to work out; find the first available, this from the given epoch. So, this is the first problem the next problem will consider this second part later on. Now, given this problem, so obviously if we look into this problem. The asteroid A is moving at a higher angular rate, while the asteroid B will be moving at a slower angular rate.

Now in going from asteroid A to asteroid B, you need to use Hohmann transfer. Hohmann transfer obviously, because for n less than 11 point, we have chosen 94. So, n less than 11.94 the Hohmann transfer is most efficient. Therefore, we need to launch in such a way, that when suppose we are going from A, and once we are launching the satellite from A. So, suppose by that time, the A has reached in this position, and from here the satellite will move, and by that time B will approach from here to here and it will catch up from this place. So let us say that, this is the amount of angle, this is the angle delta theta from here to here, delta theta A which needs to be, travels by the asteroid A before the launching can be done. And after that once it is there, and then the satellite is being sent from this place to this place, and it will go, and finally catch up at. Let us say B moves from here to here, and this is the point B prime. And during its journey from, the satellite journey from this point will write as, say some intermediate point.

Let us write this as A 1, so during its journey once the satellite is moving from here to here, this asteroid is it will move along this orbit, and it may reach suppose to this point, which we are writing as here A prime. So we have to find out, what; to find the first available take off time in days from the given epoch. So, the first available time is depending on this delta theta A, and once we divide this delta theta A by the angular rate of this asteroids, so that will give you time. Only after which, you can launch the satellite from this asteroid, so that it goes and catches up B at B prime. And B prime will be directly opposite to A 1 means this angle from here to here. Angle A, let us mark this O, so angle A O B prime, this is 180 degree. And angle A O A1 this is delta theta A.

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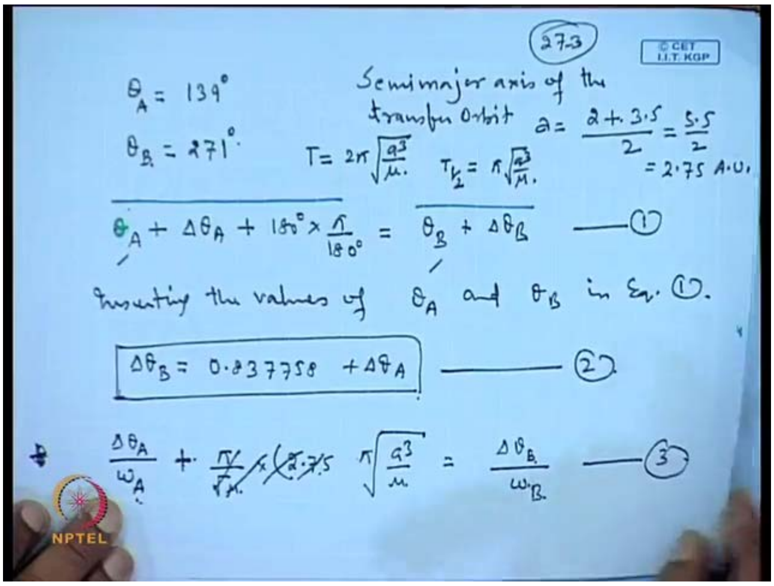

Now, the initial position of A is given by theta A, which is given to the 139 degree, and similarly, the initial position of B is given to be, how much we have written this is 271 degree. So, these are the starting values, the radius of the orbit is given, and it's moving around the sun. Therefore, the rest of the calculation can be worked out. So, we can say that theta A, which is the angle from here to here. This is theta A is than angle from this place to this place, this is theta A. Similarly, and this theta A is nothing, but 139 degree, and this is 271 degree this is nothing, but your theta B. So theta A plus delta theta A, plus 180 degree and this to be converted into radians pi, multiplies by pi and divided by 180 degree, this must be equal to theta B plus delta theta B. So, this is very obvious from, if you look into the problem what we are doing, so theta A from here to here, plus delta theta A and plus 180 degree up to this point. So, this total angle, this must be equal to, and theta B we are counting from here to here theta B. And then delta theta B is being counted from this place to, delta theta B it is being counted this place to this place, so this is your delta theta B.

Summation of this angles and plus this angles, this must be equal for catching up the asteroid at B prime by the probe, which is being launched from the asteroid A at the point A1. So, theta A is known, theta B is known. So, we will get a relationship between delta theta A and delta theta B. So, inserting the values inserting the values of theta A and theta B in equation 1, what we get delta theta B, so this is one relationship that we get for delta theta A and delta theta B. Now, we have to equalize the time also, so the time B takes from moving from this place to this place, that must be equal to the total time that A takes to move to A1 point, and there after the probe, that takes the time to move from point A1 to the point B prime. So, next we have here delta theta A divided by the angular rate of the probe A, plus, now the, if you look into this figure, So these mi major axis of this transfer orbit you can calculate.

So, let us first calculate the semi major axis of the transfer orbit. This will be 2 plus 3.5 divided by 2, here 2 is the radius of the inner orbit and 3.5 radius of the outer orbit. Therefore, this becomes 5.5 divided by 2 is equal to2.75 astronomical unit and the total time to move from this place to, point A1 to B prime, so this is half of the orbit, so this will be half the time period of the satellite, which is moving in this elliptical orbit. So, time period we know that, time period is given by equation. Now the time period equation is T equal to two pi a cube by mu under root. So, the half time period will be T1 by 2 this will be pi times a cube by mu under root, so we utilize this fact here, so this will be pi divided by mu under root times a to the power, so this a is 2.75, or either we write the equation first here, so this is pi times a cube by mu under root. So, this is the time which will be equal to, the time taken by B to move from point B to B prime, so this will be delta theta B by omega B. so, this is our equation number three.

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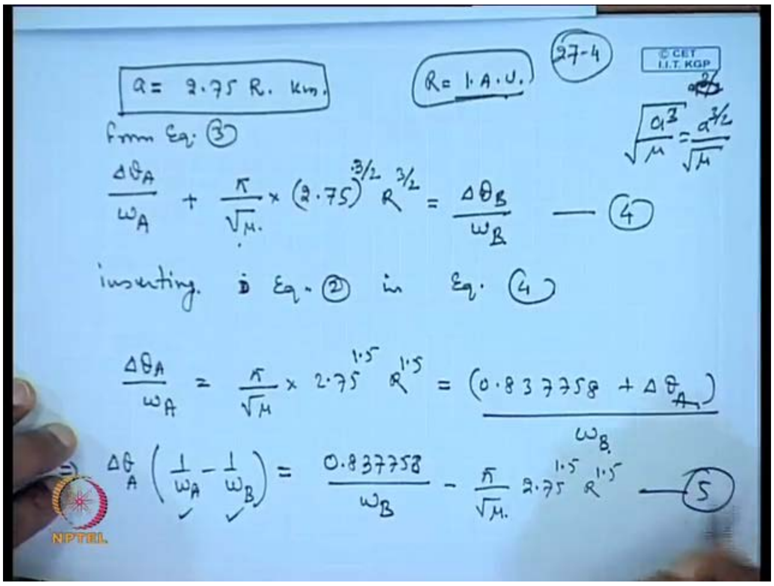

Now, a is known to us, so a is 2.75 astronomical unit, so let us say one astronomical unit we indicated by r. So, this much of kilo meters, where r is 1 astronomical unit. Now, the things are indicated in kilo meters, so delta theta A from equation 3, 2.75 to the power 3 by 2, because this is a cube to the power 3 by 2. You are talking a cube by mu under root, so this becomes a cube to the power 3 by 2 and divided by mu under root. So, mu under root we have taken here in this place and this multiplies by r to the power 3 by 2, and this is nothing, but delta theta B by omega B. So, we have the relationship in equation 2, where delta theta b is available in terms of delta theta A, so we can insert into this equation, so inserting equation 2 in equation 4. This divided by omega B. So, delta theta A we can take on this side and take this as common. So this implies delta theta A times 1 by omega A minus 1 by omega B equal to 0.837758 divided by omega B. Now, omega A and omega B can be computed, so we number this as the equation number 5.

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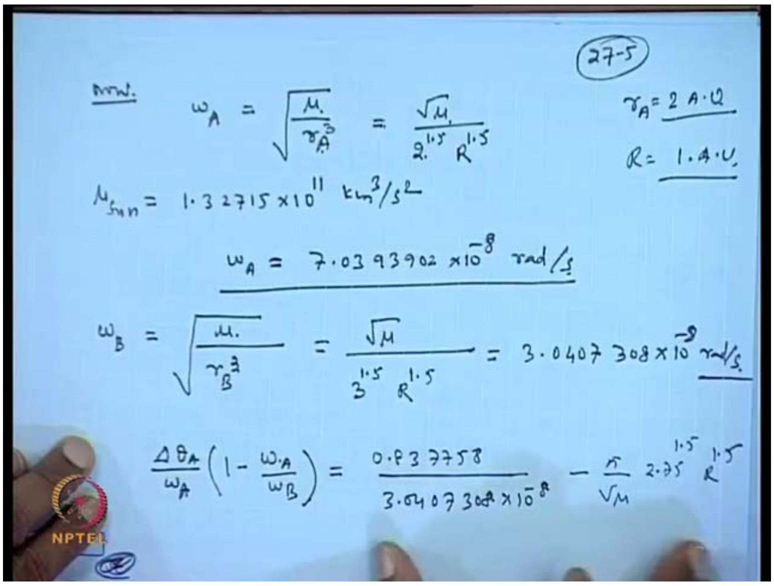

Omega A is mu by the radius of the asteroid A under root, and mu is known to you the quantity for the mu, the sum can be written as, mu sum is 1.32715 into 10 to the power 11 kilo meter cubic per second square. And r A you know it, s or A is here 2 astronomical units, so that we can convert, and insert into this equation. So, if you put all this values in this equation, so this will turn out to be. Now one more thing, after calculating this then we need to further calculate omega B. So this quantity will turn out to be mu divided by 2 to the power 1.5 mu under root divide 2 to power under 1.5 and r to the power 1.5, where r is nothing, but 1 astronomical unit.

So, insert this values and calculate, so omega a will be turn out to be 7.0393902, and you have to do this calculation very precisely, because the distance involved are very large, and little bit difference in the value of omega, it will cause a lot of error. Similarly, we can calculate omega B to be mu by r B cube under root,3 to the power 1.5, r to the power 1.5. So, this will turn out to be 3.0407308 into 10 to the power minus 8 radians per second. Now we need to insert this into equation number 5. Here r is known, rest other quantity on the right hand side now becomes known. Omega, omega B all these are known; therefore delta theta A can be calculated. So, we can write delta theta A by omega A equal to 1 minus omega A by omega B. This, minus you have pi divided by mu 2.75 to the power 1.5, r to the power 1.5. So, find out the quantities on right hand side, and then divide this by 1 minus omega A by omega B, so you get delta theta A by omega A.

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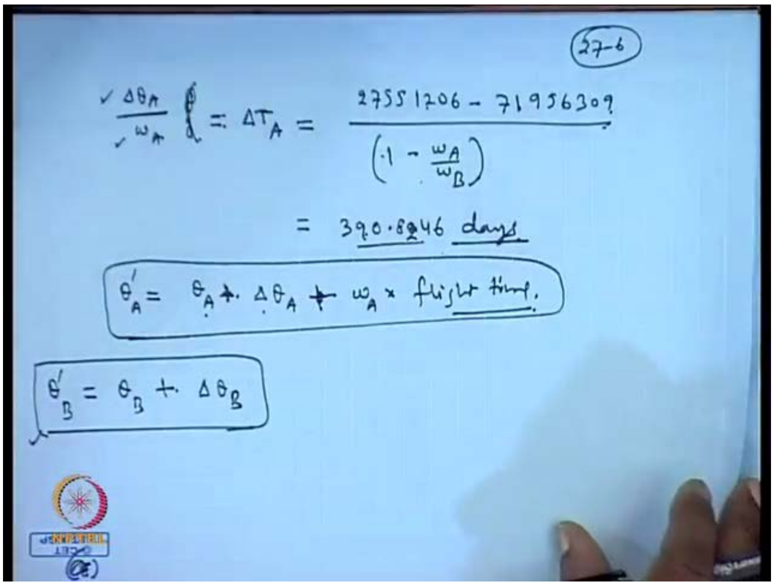

So, this quantity can be written as, so this is the time taken before the launch can be done. So delta TA, this is the angle divided by the angular rate, so this gives you the time and this is what we need. So, this becomes 2751206 minus 71, and you put the values further, so this will turn out to be 390.8246 days. So, this is the waiting period before the probe can be launched from A which has to reach to B. So, in this time now theta prime may, it can be written as theta A, plus delta theta A. so, here we see that the time taken from, for the asteroid to move from A point to another A1 point, this is turning out to be 390.8246 days, and at this point you can launch the satellite, which is going to meet at point b. this, is one part of the question, that we are trying to find it out. and if you are trying to find the position of the satellite at, once satellite catches the asteroid B. If you are trying to find the position of the asteroid at A prime, once the satellite is going to catch the asteroid at B, so you need to write here theta A plus delta theta A, plus omega A times the flight time, so this will give you the position.

So similarly, for the B, B is moving from this place, so B has already the position given by theta B, so theta B plus we can write here theta prime B equal to theta B, plus delta theta B. So, theta B is the position of the asteroid from this place to this place. And, now the asteroid is moving from this place to this place, which is the distance delta theta by B, so that you also need to work out. So, this will give you the position of the asteroid B, once the satellite is catching out. So, you can see that this is nothing, but distance where this angle theta A plus delta theta A, plus 180 degree. So, this part we are completing, and we will take up this problem again to, get into this another part. So, this another part of the problem can be, the second part of this problem can be stated as, that was the probe we has reached here, and hereafter if you at this instant, if you want to return to the asteroid A, so how long you have to wait again to reach A with maximum fuel efficiency. So, again this is a same Hohmann transfer problem, but then you have to take into account the angles properly, so we do it next time.

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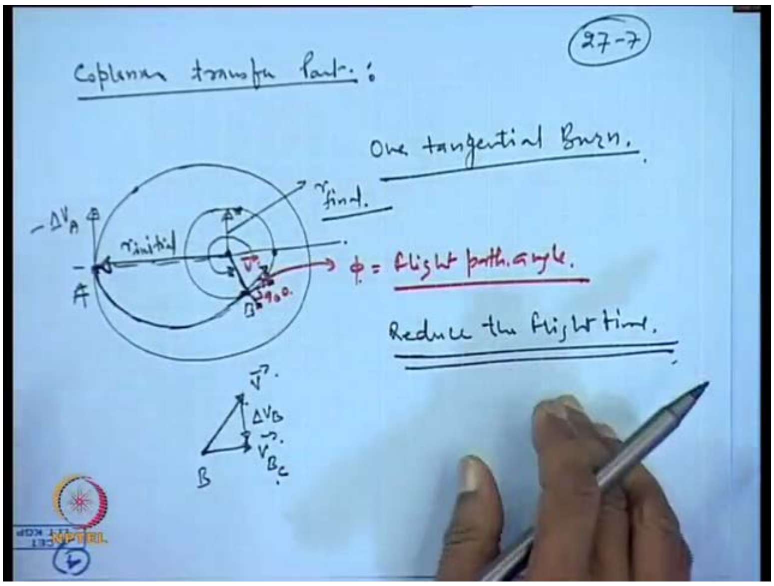

So, in the remaining time we will complete this transfer part, the co planner transfer part. So, in the co planner transfer part, we have seen that the bi elliptical and Hohmann transfer, how to do it. Now, many times it happens such that two tangential bounds are not available to you. So, suppose one is the circular orbit, another one is the elliptical orbit, and you may like to go from the outer orbit to the inner orbit or from inner orbit to the outer orbit, but the time of if you go using the Hohmann transfer, then the time of flight will be large.

And if you go by the only one tangential bond, that is if you give one tangential bond here in this place; say this is delta v A minus, this is the point A and you want to this point B here in this place, not here in this point, because if you go to this point this is the Hohmann transfer, so here we are working out only one tangential bond. So, if you want to do this one tangential bond, then fire the rocket here, de boost the satellite and it will move from this place to this place. The angle from here to here, this is ninety degree, just the local tangent, and this angle, this can be written as phi and this is called the flight path angle. Flight path angle is always measured with respect to the local tangent, and this is the velocity vector of the satellite in the elliptical orbit.

So, we have already worked out for the general case. The very first exercise that we took up, it was about the, both the bonds were; one bond started here another bond was done here in this place, so no bond was tangential, but here at least one bond tangential case we are working out, and this problem is to reduce the flight time. So, this kind of situation can arise numerous times, during that time we have to go for this kind of bond. So, this figure we can boast up here, so this is your point B, little enlarging the figure here, and this is the vector B, and this is the required velocity at the point in the circular orbit, inner circular orbit which is B C. So, you need to reduce this, so you need to give this delta v B the boost here in this place. To make the velocity tangential at point B and equal to the velocity required, to move in the inner circular orbit.

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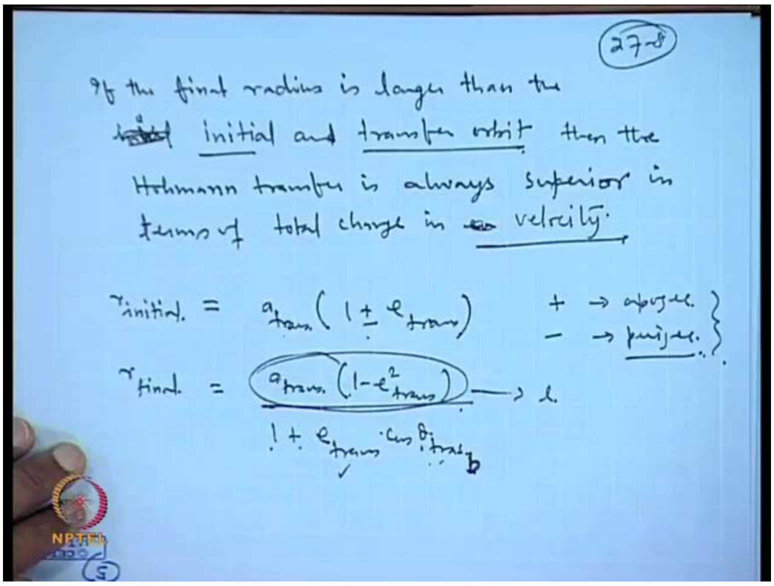

So, one thing I would like to state here, if the final, larger than the initial, and transfer orbit, that is the orbit in which you want to transfer, its radius is larger than the initial orbit and that transfer orbit itself. Then, the Hohmann transfer is always superior in terms of total change in velocity, and therefore propellant required. So, now we do this one tangent bond, so this is our radius from here to here. This is r initial and this is r final, so we have, r initial will be equal to, so r initial this we can write in terms of the; say semi major of this transfer orbit. So, what is known in this case that the two orbits are given, and you have to go and inject the satellite at point B, whose true anomaly will be given.

So true anomaly will be measured from this point to this point, so this angle will be given to you, and you need to go and inset the satellite here in this place. So that it moves in this smaller radius orbit noise, so a transfer plus minus 1 plus minus e transfer, e is the eccentricity of the transfer of the transfer orbit, and plus sign is for apogee that, that is if we are injecting the satellite from this place to this place and it has to go into this, then we take the plus sign. And if are injecting the satellite from somewhere, if the point A is lying here in this place and we are going to inject the satellite in the outer orbit, then for that the negative sign is taken.

So, negative sign is for perigee position, that is the positive sign we are talking for the outer orbit from going from outer orbit to inner orbit, and negative sign if we are talking from inner orbit to the outer orbit, and this you can see it very easily. Now, we have done this then r final can be written as, look into this equation, what we have done. This quantity which is appearing ever, this is nothing, but our l; the semi lattice rectum for this ellipse, the transfer ellipse, and this is 1 plus e cos theta, so e is the eccentricity of the transfer orbit. And theta here is the true anomaly of the point B, so B here put subscript b. So, r initial and r final we have defined in terms of right now the A transfer

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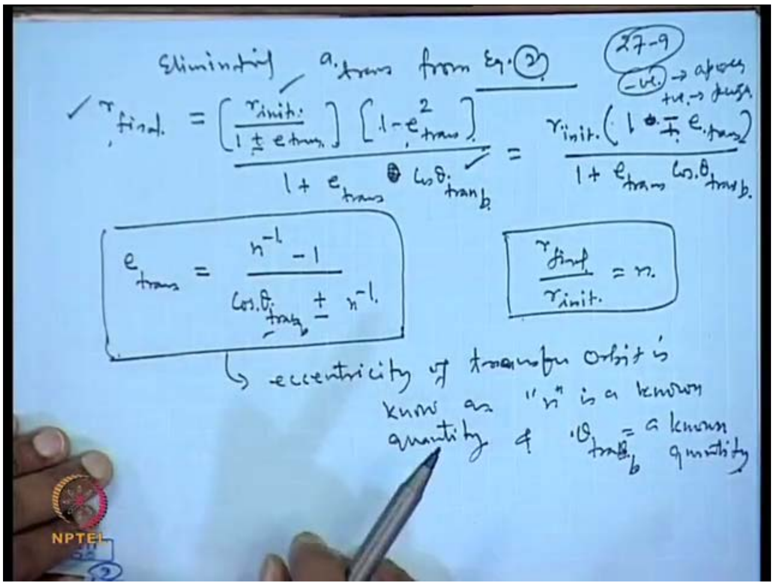

So, therefore r final, you can terms of r initial, so for this you need to eliminate a transfer. this is equation 1 and this is equation 2, so in equation 2 you eliminate a transfer, so eliminating from equation 2, so this becomes r initial divided by 1 plus minus e transfer times cos theta. Now, in this tuff radius of final orbit is known quantity, r is initial is also a known quantity, theta B is a known quantity. So, you can find out the value of e from this place, so we can separate it out, and write this in the format where e. if we separate or rearrange the terms, so this can be written as e transfer equal to n inverse minus 1 divided by. So, just we have done the rearrangement, we have taken this on this side and did the rearrangement.

You can cancel the quantities here, so this is r initial, and if you cancel it out, so this becomes 1 minus and this will become minus and plus minus plus e transfer and divided by 1 plus transfer b. So if you cancel it out, so if you divide this minus sign, here it is 1 minus c square and this is 1 plus e, so if you divide a upper sign will become negative, the positive positive cancels out and you get a negative sign, and if you are taking this here as negative sign, this gets into the positive sign. So, here in this place the negative sign is for apogee and positive sign is for perigee. And here we have utilized the fact that r final by r initial this is equal to n. this notation we are following from the beginning, so we fallow this notation. Now our eccentricity of the transfer orbit is known, as n is a known quantity, and theta transfer b this is also a known quantity, because where you would like to put the satellite in the smaller orbit that depends on you, so therefore, this is a known quantity.

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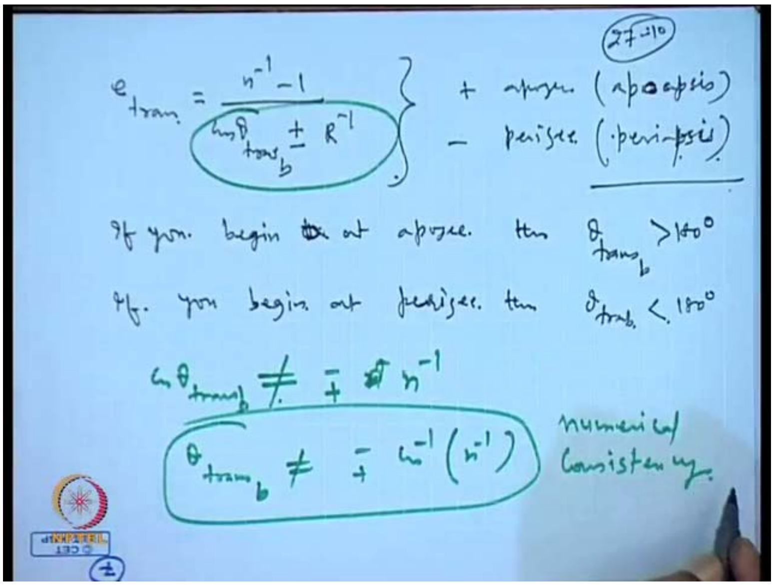

Now, once e transfer is known, so your job is over. So in the e transfer cos theta trans b plus minus r inverse, so plus sign is for apogee position and negative sign for perigee position, also this is called apoepsis. This is called general called satellite case and this is called periapsis. Now, you can see that, if you start the transfer here in point A in the outer orbit, so either you can inject here, or either symmetrically in the opposite direction here, in this place you can inject. So, here once the orbit crosses here and goes into this place, so here the orbit will cross like this, so wherever it crosses the orbit, there also you can inject. So, if you begin at apogee, as this case has been illustrated. So theta B which is theta B transfer, which is the angle from here to here, this is theta B transfer. So, this will be more than 180 degree, which is obvious that you cannot do that transfer from this place to this place, so this theta b is bound to be greater than 180 degree. So, if you begin at apogee then theta transfer b, this will be greater than 180 degree, and if you begin at perigee then, theta transfer b this will be less than 180 degree. So in that case if you are starting here in this point, and then you have to catch up here in this point so that you can see that the true anomaly in his case will be less than 180 degree. Now, in this equation you have to take care that for, you do not get into any numerical problem, that is here the quantity cos theta transfer b. This must not be equal to minus plus n inverse, or equally we can say that theta transfer b must not be equal to minus plus cos inverse. This is required for numerical consistency. Now once we have done this then we need to find the quantity, the semi major axis, but the semi major axis, as you look into this, because we are not transferring from this point to this point. We are not transferring here to here and therefore, semi major axis. We cannot find write as i initial plus i final divided by 2.

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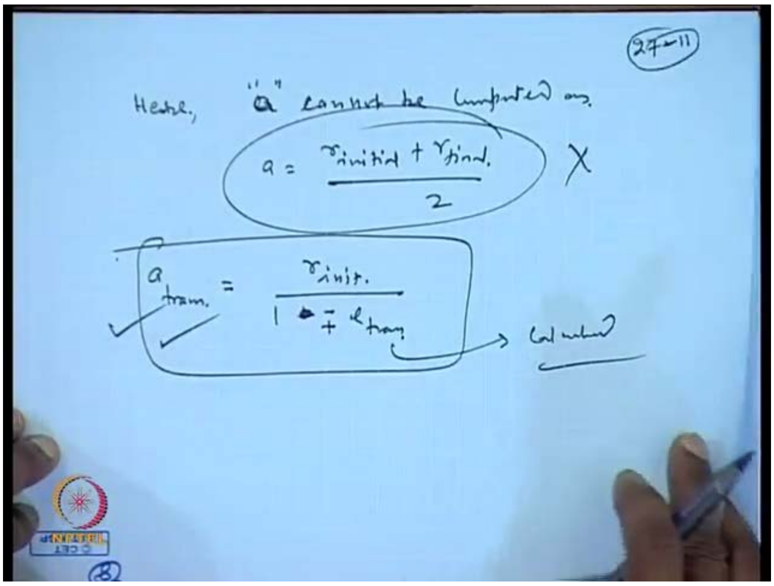

So, here a cannot be computed as a equal to r initial, plus r final divided by 2, this is not allowed, because this is not the case here. Then how to do it, so you can do it this using a transfer is equal to r initial 1 minus, e is known e we have calculated, so the a can be calculated.

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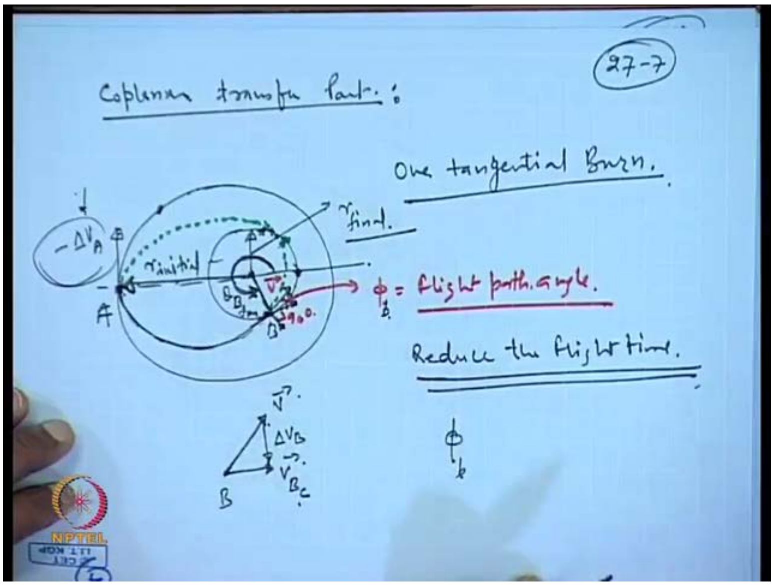

So, once we have done this job, therefore a is known to you, e is known to you. Now, you can calculate the amount of this velocity changes required here in this place at A, and amount of velocity change required at B. So at place B you need to find out the flight path angle, which is phi b, so in this we can write as phi b, so if you find the flight path angle so your job will be over. So, what we have done, we have first found out the value of b, and then we found the value of a, which clears the path for calculating the quantities which are further required.

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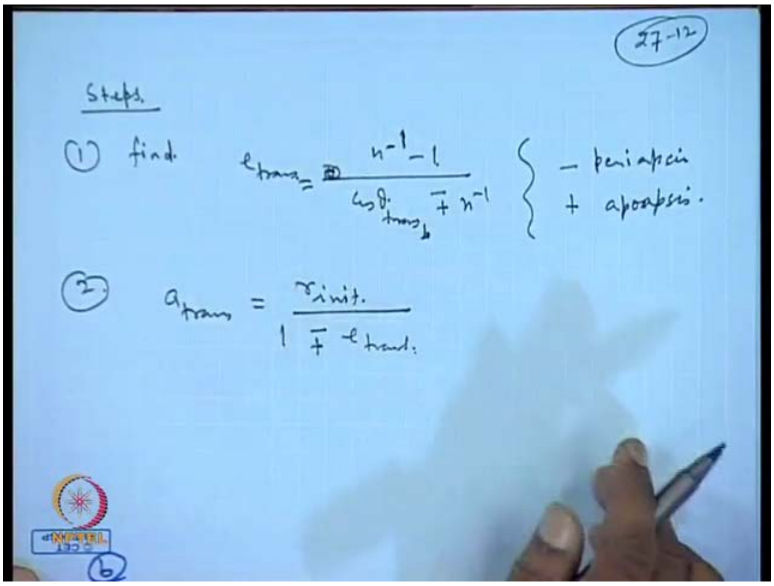

So, the steps can be written as. Next step is calculate a trans r initial 1 minus plus. Time is getting over we will continue with this exercise next time. Thank you very much.