Space Flight Mechanics

Prof. M. Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No. # 21
Trajectory Transfer

We start with the Trajectory Transfer, as you know once the satellite has to be injected into the orbit. So, the precise performance of the rocket it is not possible. The rocket will never perform the things that you desire. Because of the unknown wind conditions and various other factors, which contribute to the performance of the rocket? Say if you want to put the satellite in geo stationery orbit or the low earth orbit. So, in both the cases you need to may be put in the some elliptical orbit or may be in some circular orbit and the rocket where it injects. So, the amount of the velocity vector which is required in that point where the rocket is injecting. So, at that point itself may not be coinciding with the orbit which is required.

And the velocity vector also may be much of what is then, what is required in the desired orbit and therefore, the orbit correction is required. So, this orbit correction naturally the question arises how to do it, but the basic idea here is the in whichever path the satellite is moving that path must be corrected. So, correcting that path means moving the satellite from one Trajectory. That is the present path in which the satellite is moving to another Trajectory and therefore, this terminology it is called the Trajectory Transfer.

So, we start with this Trajectory Transfer and see into why? How to? What are the various regions, which are which go into the design of the trajectory transfer and what are the regions which control this trajectory transfer?

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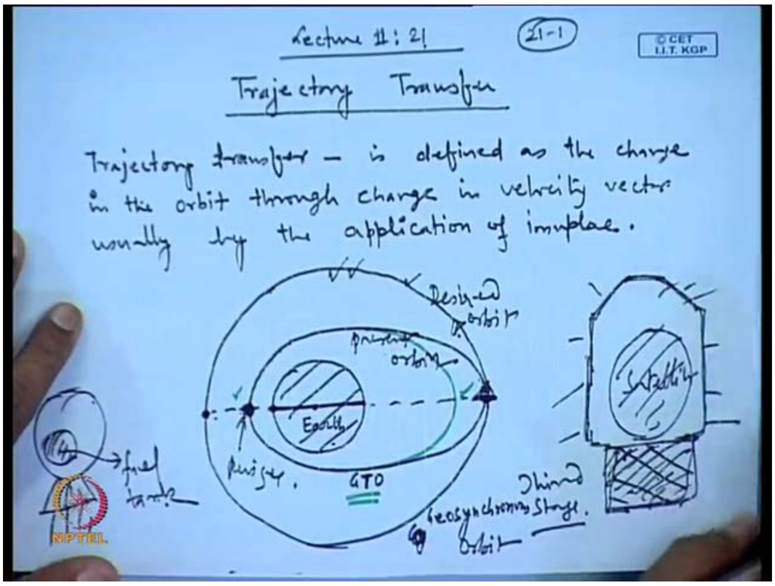

So, to start with trajectory transfer is defined as the change in the orbit through change in velocity vector usually by the application of impulse. So, often one or more impulses will be required to complete the trajectory transfer. Let us consider a very small case.

See this is the earth and by firing the rocket you have placed certain satellite in the orbit. So, the orbit of the satellite it looks like, at the present moment. Satellite is often injected at a low altitude. So, say this satellite has been injected at the perigee position. Why at low altitude? Because taking at a higher altitude. So, you will in the rocket you will have the satellite inside kept, this is your satellite. If you want to carry along with this satellite, this rocket itself will have some propellant mass. Let us say this is the last stage of the rocket which we are going to discuss toward the end of this course. So, this is little of the track, but just I am introducing it for your convenience.

So, let us say this is the third stage. In the third stage beside satellite you have this red if unnecessary weight involved with the satellite, which is the structural weight of the rocket and beside the fuel is also there. So, all these weights are together, now if you want to take it to a further height along with this rocket. So, what will happen, that you have to a spend fuel for this extra structure which is coming, because of the presence of the rocket. So, you are going to lose energy. Therefore, your propellant mass required will be much more.

And if you inject the satellite at a smaller altitude with high velocity, what will happen here? .Your rocket is gone, this part is taken off and only the satellite is remaining. In the satellite then you have the propellant system and the propulsion system is present. So, this satellite itself has certain storage of the fuel. Using this propellant the satellite can then be boosted from the present trajectory to a new trajectory. You can see that, in this case you have the lower mass involved, while in this case the larger mass involved. So, this saves a lot of energy.

Now, if you have injected the satellite suppose here in this place and your required orbit is this one. So, therefore, you are much of the desired orbit. This is the desired orbit and this is the present orbit. And therefore, you need to boost, say this is called the perigee. So, you need to boost the perigee or this is also called the raising of the perigee. Once you raise the perigee and the velocity in while raising the perigee, it matches with this circular orbit and then the satellite will really follow this course. So, this way you can precisely control the position and velocity of the satellite. And therefore, you can put it in a desired orbit what is really required.

So, which the rocket obviously, it is not possible exactly to do this thing because here you can give impulses little. So, course of time you can bring it here in this place or may be one impulse suppose, here I need to raise the perigee. So, what I will do here I will give the impulse. As I give the impulse here at the apogee position. This will make it here nothing will change. Here this distance will not change only thing, now the velocity will be higher and therefore, this satellite will go into this orbit. And what the perigee change will take place and perigee will raise to this point. So, by doing this you can raise the perigee. How much correction is required, it depends on the situation at hand.

So, the situation that I have presented here, this is also called the geo synchronous transfer orbit. Let us say, this is the geo synchronous orbit. So, this is geo synchronous or may be the geo stationery orbit. Once you are launching the satellite, you are leaving the satellite here and satellite is performing like this and its possible that the apogee also does not match with this place, rather the apogee has gone like this. You have put the satellite such that, it is in an elliptical orbit. Therefore, in that case you need to raise this apogee also and raise this perigee also. This GTO is called the geo synchronous transfer orbit. So, actually you place the satellite here in this place and thereafter you bring it to the right condition.

And doing this kind of trajectory transfer, it is a part of our study here. Next nine to ten lectures we are going to devote on this. So, various kind of trajectory transfers, how to do it? When the energy involved is minimized. Sometimes may be the small eccentricity you require while transferring, various factors are there. In fact, sometimes it may be that you catch the satellite you have to transfer it such that you have to catch the satellite in another orbit. So, that is called the chase maneuver. That is mostly in the case of chase of if you have to hit a satellite with a missile. So, during that time the maneuver you will do that will be called the chase maneuver or either the randevu problem in which one satellite is already in the orbit.

Another satellite is in different orbit and you want to do the randevu of these two satellites that is you want to approach one the second satellite. Say in this case one satellite is moving here and you have injected just a satellite here in this orbit or the already in an orbit. The satellite is moving like this and you have to catch up with this. What a strategy you plan that you catch up with this one. So, all these are part of this trajectory transfer maneuver and it is a very interesting topic.

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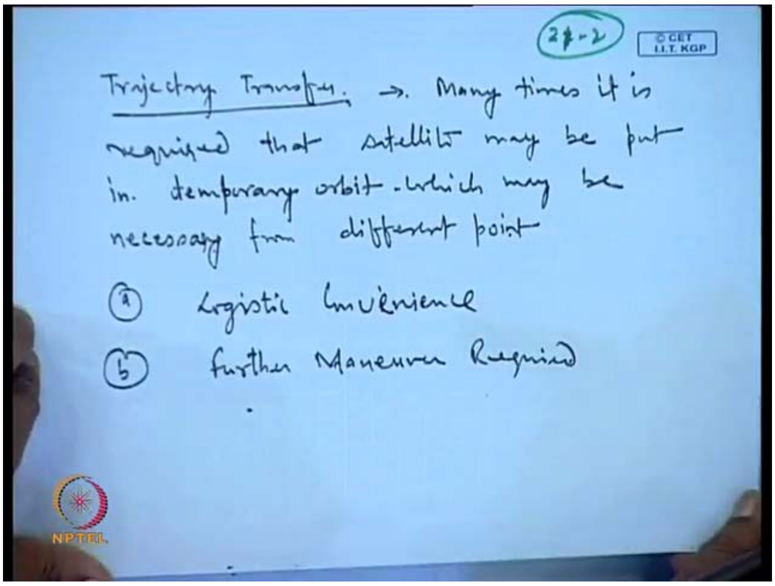

In Trajectory transfer many times it is required that satellite may be put in a temporary orbit. This may be quite necessary from the point of view that while you are launching the satellite, so at the time of the ejection from the rocket while you are injecting it in the orbit at that time it goes under severe conditions. So, after the injection then you may be able to you have then the time to check various components how the system is performing, and thereafter you really put in the desired orbit.

So, this is required from the point of view of a temporary orbit which may be necessary… The logistic point of view the logistic convenience and then may be the further maneuver required. So, already we have discussed why this satellite trajectory transfer is required.

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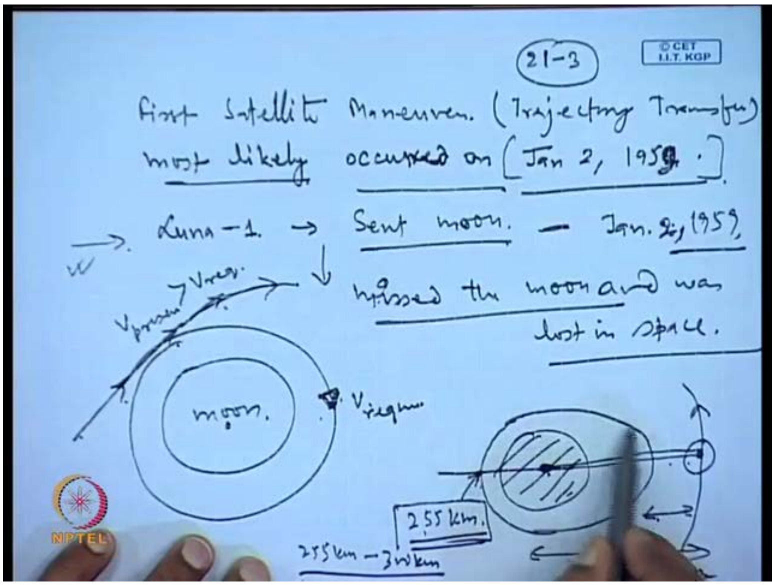

The first this trajectory transfer might have been done. The first satellite maneuver trajectory transfer most likely occurred on January 2, 1952. This is 1959 because this was done for Luna not 52.

So, soviet satellite Luna - 1 which was sent to moon and it was launched on January 2 1959 this. So, maneuvering was done for this Luna satellite to send it to moon, but it missed the moon and was lost in space. So, if you look into this, before that because the why I am telling here, the most likely because the whole history is not available in those days. The things were done very secretly. So, exactly the dates and other things it is a not very available in a very confirmed way, but it is a most likely on the January 2, 1959. Once the Luna was sent to the moon this was raised between the U S and the Russia basically. So, both the countries were trying to send their satellite to the moon first. So, then soviet route was the first to these do this operation. Thereafter America it rushed up to this business.

Once the Luna was sent to that what happened that this missing the satellite is not reaching the moon, it happens because suppose this is the moon and you are sending the satellite in a trajectory. If the velocity required to be captured by this moon is higher than that required were suppose the velocity required to be captured in this moon orbit. This is the v required and if your velocity at present, this is the v present is greater than v required. So, simply it will not go into this orbit, but it will just escape in this space. So, it will take there will be trajectory turn, but it will not go into this orbit and it will escape in the space and therefore, it is lost forever.

And of course, you know that once you send the satellite to such a large distance. So, it is not loaded with so much of fuel that you can bring it back into the desired trajectory. So, these maneuvers are manners very precisely. You know about the chandrayaan the India has been done this chandrayaan mission. For the chandrayaan mission suppose, this is the earth and this is the moon. So, moon is moving around the earth in this orbit. What was required that the satellite which was first launched in an orbit. This was a elliptical orbit here. This distance is much exaggerated. Actually, we know that the distance between the moon and the earth is 38440 kilometers. It is 3 lakhs, 84 thousands, 400 kilometers.

So, this distance is very large which we cannot show here. What was done it was put in a temporary orbit here. Now this initially perigee position was around 255 kilometers approximately. The exact values it is not known to me, but prior to this launch review which I did in that, it was shown that this is going to be launched around this position, so taking this position to be 255 kilometers to 300 kilometers. Thereafter the apogee was boosted. Slowly, this distance was increased.

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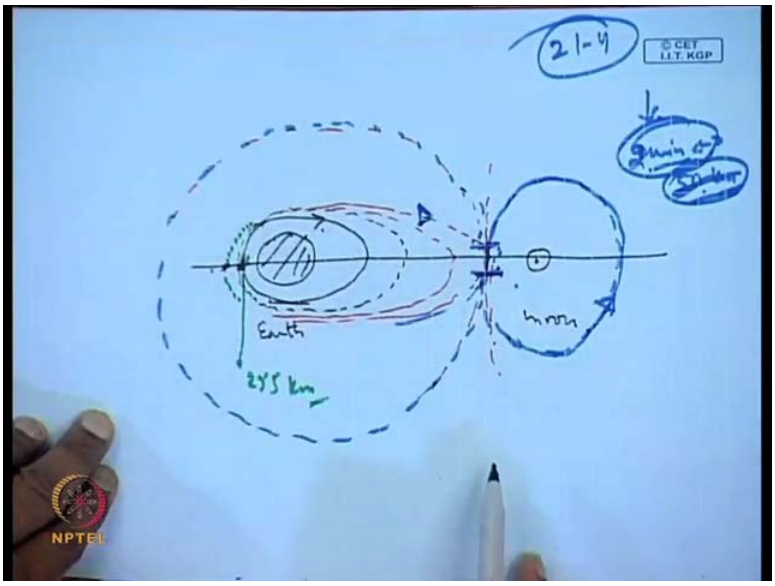

Now, we can show it here. So, this is the earth and here is the moon and the satellite you have launched. It is moving in this orbit. In this maneuver the apogee was boosted and also the perigee was raised. Whatever the perigee is here as 255 kilometers, this was also raised .why this is required? Because you know that the atmosphere is present. So, near the earth surface there is at 2 even at 200 and 55 kilometers enough atmospheres is there to kill the kinetic energy of the satellite. And therefore, this was also boosted and this helped in. if you have once you have boosted here, you lose lesser energy. Because of the aerodynamic drag.

Once this is boosted the perigee is raised and you keep on raising the apogee also. So, the apogee can be brought from here and then it can be brought here into this place. It was kept rising until unless it reached boundary. So, this is called the sphere of influence. Let me make this boundary. This is the sphere of influence of the earth and this is the sphere of influence of the moon. So, after this distance the moon gravitational force will be more dominant and here the gravitational force of earth is more dominating. So, once it reached here. There was a window of two minutes. So, this was told that, this was around 50 kilometers as far as I remember. So, let us say this was a very short period if we take in terms of time.

Only a period of two minutes was available. So, during that period the satellite was to be injected into the domain of the moon. The sphere of influence of the moon and once you are injecting the satellite into this domain. So, what is required also that the velocity should not be higher. If the velocity is higher suppose here you want to put it in this orbit. If this is a circular orbit so, there is something some velocity is required to move in this orbit and if you exceed this velocity then what will happen? So, simply the satellite will enter the domain of the moon, but it will go outside this. It will not come in the orbit around the moon and therefore, it will be lost forever. And that is kind of orbit it is called the hyperbolic orbit where the satellite will go beyond your reach, it will not be captured by the moon.

So, in this small window the satellite was to be captured, if the satellite was moving like this. So, here not the satellite is moving. So, then it must be de boosted here. That it gets captured by this moon. So, you raise the perigee and simultaneously, you have to de boost the velocity of the satellite that the moon captures it. So, doing all this operations it is a complicated operation. It has to be done in a very precise manner and doing orbit determination each and every time. So, orbit determination we have already done, but those kind of the orbit determination problem we did.

So, there you required the exact values of the position and the velocity. That is the position and velocity coordinates are required then you can find out in which orbit its moving, where it is location is with respect to the earth and with respect to the moon, but. In fact, these measurements are always faulty and therefore, we need some extra estimation or do more data processing to find out in which orbit it is moving. So, that data processing it can be done using either the calment filter, the extended calment filter or using the batch processing method.

So, those are beyond the scope of this course, but. In fact, this is required. So, the orbit determination was simultaneously, being done and once the things were observed at now it is in this position the all the proper maneuvers were done to put the satellite around the moon. So, this is one way of doing it that you are raising the perigee continuously. Another way of doing the same thing can be,

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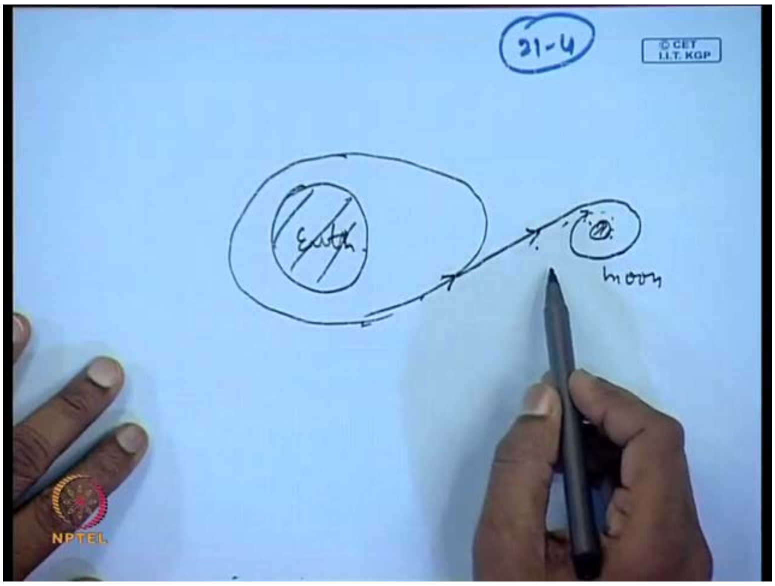

This is your earth and here is the moon. So, you have placed the satellite in an elliptical orbit and you want to capture it around the moon. So, you can give impulse in this orbit and it can be brought like this. Thereafter it can be captured.

So, another possibility is doing this. So, various kind of trajectory transfer it is possible and it depends on how much confidence you have in the maneuver. If you have done this kind of maneuver already then it is easy to do it. If you have not done this kind of maneuver. In fact, the ISRO has done for transfer from the G T O to the geo stationery orbit. So, geo transfer orbit from the geo transfer orbit to geo synchronous orbit it the satellites were already reached using this method and. So, it was very easy for them and. In fact, this technology is mastered the full proof method were utilized for doing this trajectory transfer. So, the first satellite maneuver probably it was done on second of January 1959 and that was done for the Luna - 1 satellite.

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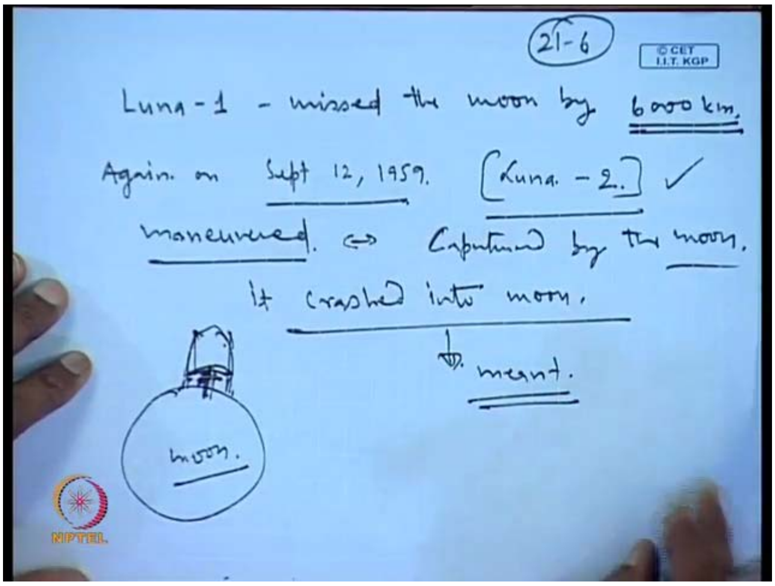

So, the Luna -1 missed the moon by 6000 kilometer. Again on September 12, 1959 Luna - 2 was maneuvered. So, after correcting the difficulties which were faced in the Luna – 1 Luna - 2 was tried out. So, this Luna - 2 was captured by the moon, but then what happened that it crashed into moon. This crashing was not incidental, but it was meant.

So, what is the reason that discussion crashing was meant. So, reason was that at that time the satellite these propulsion technology retro rockets were not in developed stage. retro rockets means once you are want to keep the satellite and you want to land the satellite on the surface of the moon then what you require that while landing if the something is landing on the surface of the moon. Slowly, it has to be brought to the surface of the moon. While landing the retro rocket must be fired to support the weight of the satellite that it lands smoothly on the surface. Otherwise it will crash and it will get destroyed. So, this retro rocket technology was not there. And therefore, this crash was meant for already that this is going to happen.

So, later on the safe landing technologies we have developed. Now the question arises that we are putting the satellite in a parking orbit or a temporary orbit. So, this kind of temporary orbit it may be useful from various point of view.

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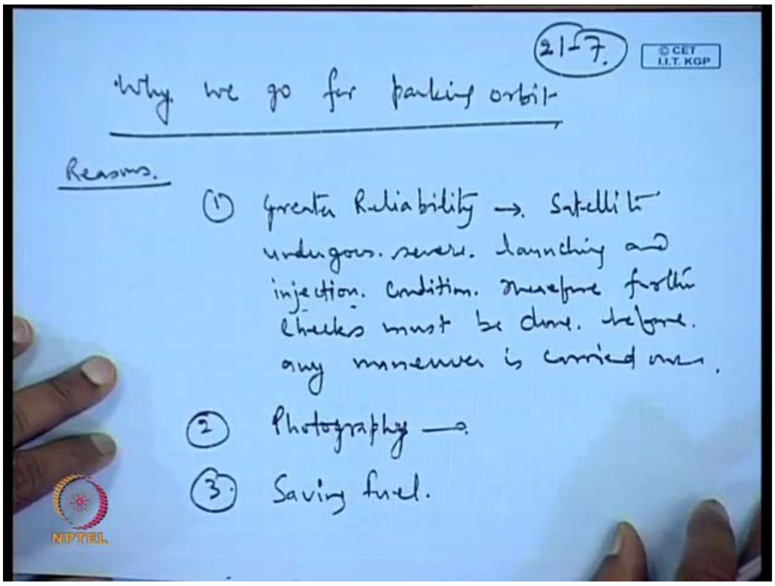

Why we go for the parking orbit, so greater Reliability because the satellite undergoes severe launching and injection conditions. This is injection on the orbit. Therefore, further checks must be done before any maneuver scattered out Parking orbit may also be used for the photography. You want to take a close photography of the things on the earth for some military secrets. Already for the military the spy satellites are there which are put on small altitude thereby the photographs can be taken which will have higher regulation and the even the using satellites the movement of the tropes are.

You have the satellites of say the regulation one meters of any object of one meter can be seen very easily using the satellites if the satellite placed on the lower altitude. So, the parking orbit in that case, it becomes very important that if the satellite is at the lower altitude. You do the necessary operation and there after you can rise to the higher orbit. Saving fuel, again there may be some unnecessary components. So, that you can reject and whatever is required that you can send it to the higher orbit, so in the case of this Apollo satellite. So, there were two models, the command models and the lunar model, so the command model while it was placed in the parking orbit around the earth. So, the command model was kept there and the lunar model was sent to the moon.

So, this way you save a lot of fuel. Because sending a larger mass to moon means you are going to spent more fuel and your then the satellite also the lunar model. That you are going to send it to the moon that also requires a large amount of fuel size increases, cost increases, complication increases, so many things that happens simultaneously.

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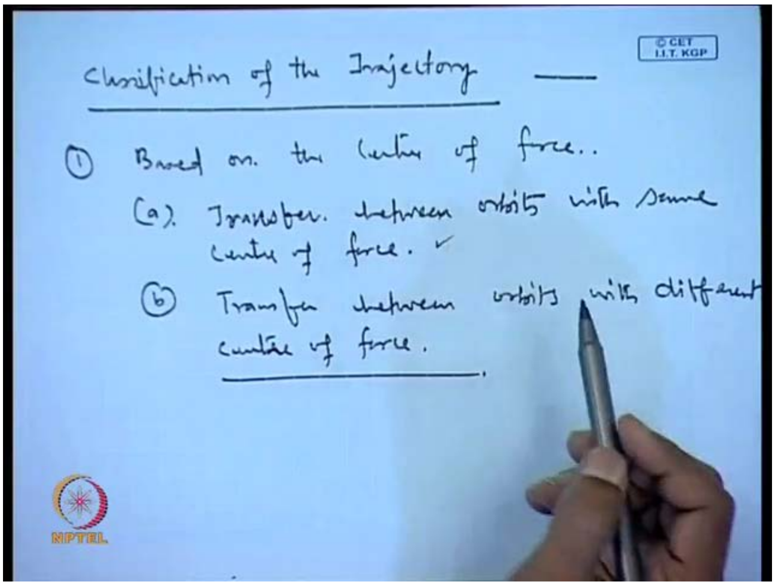

Now, the classification of the trajectory - the trajectory transfer, we do it can be done based on various factors. The first we decide on the basis of centre of force. So, here we can have transfer between orbits with same center of force transfer between orbits with different center of force.

So, in with the same center of force, this you have already seen as that example, that is from the geo transfer orbit to geo synchronous orbit. Once the satellite is raised and this transfer between the orbit with different center of force is an example, where the satellite is transferred from the earth orbit to the moon orbit or may be from earth orbit to the mars and wherever it is possible you can send the satellite anywhere in the solar system, but it obviously, it is going to take a lot of time.

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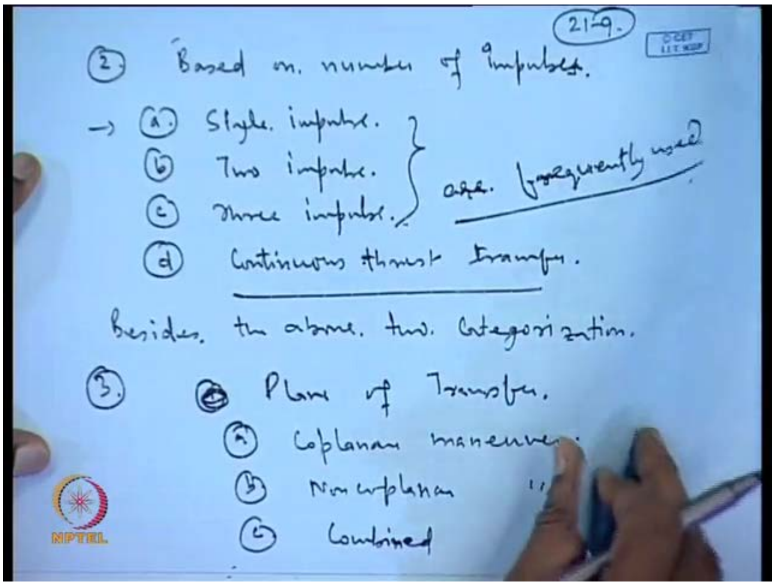

The second categorization can be done based on the number of impulses. So, we can have single impulse transfer. This is your first the minimum possible and two impulse and three impulse continuous thrust transfer. Of course, these three are frequently used.

Besides we have the above two categorization, we can also do the categorization based on the plane of transfer. So, we can say this is the based on plane of transfer. In this category will fall the coplanar maneuver. So, you are doing the maneuver in the same plane, non coplanar maneuver and the combined maneuver. So, these are the broad category in which the trajectory transfers are put. So, here while going to the trajectory transfer we will do the coplanar maneuver. We will also do the non coplanar maneuver. The combined one is obviously, the combination of these two.

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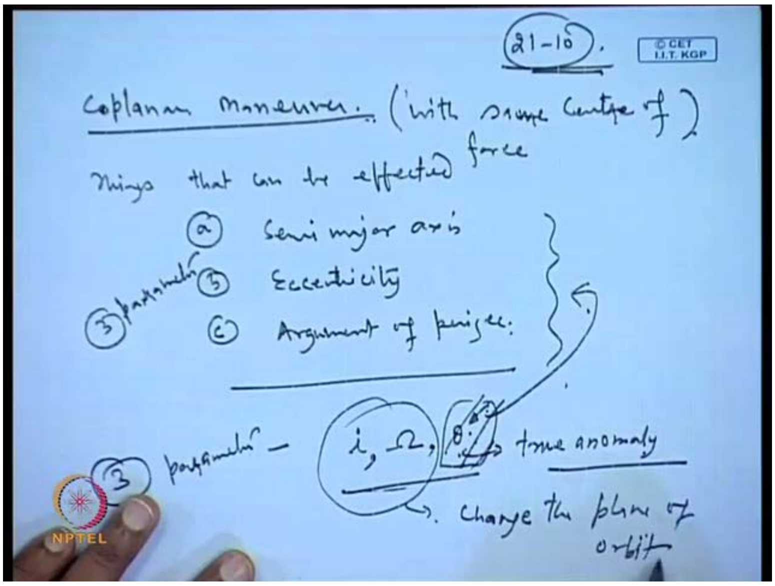

So, the coplanar maneuver what we do start with this coplanar maneuver and under this then you will have the single impulse transfer, two impulse transfers, three impulses or the continuous transfer. This is based on the center of the force, so obviously, if the transfer cases with a different center of force. This we are not going to do, but this we will cover in this course, the transfer between the orbits with the same center of force. So, in the coplanar maneuver what are the things which can be affected? So, the coplanar maneuver with same center of force.

So, what are the things that you can affect in the coplanar maneuver, so the things that can be affected, semi major axis, eccentricity and the argument of perigee. These are the things that you can do changes you can do in the coplanar maneuver. So, these are the three parameters of the orbit and rest other three parameters are the inclination of the orbit, noodle angle which is capital omega and then of course, this theta is which you call as the true anomaly. So, theta is basically it comes under this one only in the coplanar maneuver itself. So, we let us reject it, these are the two things which change plane of orbit and therefore, they do not fall in under the category of coplanar maneuver.

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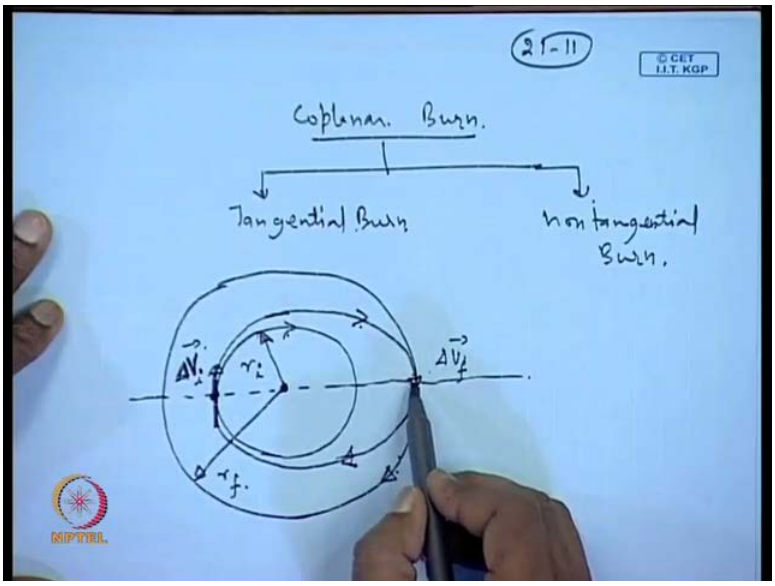

Now, the coplanar maneuver, it can be categorized as the coplanar burn. This can be categorized as the tangential burn and the non tangential burn. What does it imply say this is the center of the force and your satellite is moving in this orbit. This is r initial and you want to send it in a higher orbit. This is the radius r f. So, you can do this by various ways. Basically, this is coplanar maneuver that we require in this case. Because both the orbits we are assuming they are in the same plane. So, I can do the tangential burn means I give impulse. Here, in this place I fire the rocket in this position and therefore, I can give a tangential burn or delta v is imparted in this direction.

Once you impart the impulse or you change the velocity in this direction tangent to the trajectory. So, satellite will move away from here to here if the delta v is appropriate then it will go from this orbit to this orbit in an elliptical orbit. So, it is moving round like this. Now again you have to send the satellite. Now this is in elliptical orbit, if you want to send it in the orbit. You need to further boost it in this position. So, need to give delta v impulse. So, this delta v i and this is delta v f. So, you give an impulse of the required magnitude such that, it goes into this orbit and this point tuning can be done here to put it in the exact orbit. This is called the tangential burn because in every place we are giving the impulse tangential to the trajectory or along the velocity direction itself.

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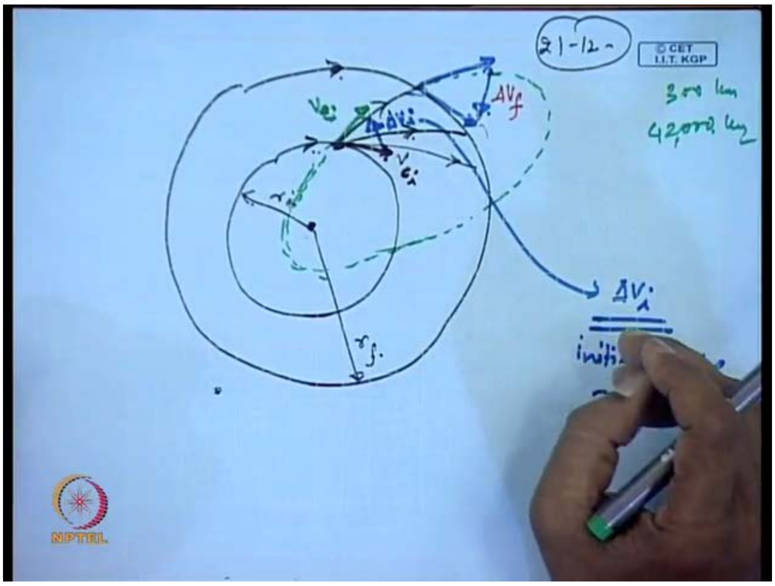

In the non tangential burn, again this is the center of the force satellite is moving in this orbit and I need to put it in a higher orbit. So, it is possible that I can give an impulse here in this place. Here, suppose this is your velocity vector v circular in the initial orbit and I can send it along any of the trajectory. I can send it like this or either I can send it in this orbit, I can send it here this way various possibilities are there.

So, if the v c i, this is the initial velocity in this circular orbit. I am sending the satellite suppose along this trajectory. So, here v elliptical this is the elliptical orbit then you are sending the satellite will move along this orbit. It will be something like this a crew presentation I am giving over here. Of course, if the earth is larger. So, we cannot show it like this because of in many cases you will see that the earth is almost your satellite is at the altitude of 300 kilometers even and at in the stationery orbit. The altitude is around the radius of the orbit is around 42000 kilometer.

We have to check for all that while the perigee does not touch the earth or not touching the earth only is not the requirement, but it should not come in the atmosphere also otherwise, it will get burned. So, this care must be taking care of. Once you are giving the impulse here in this place. What is the change in the velocity required? So, this is the final velocity vector here at this point and this is the initial velocity vector. So, the change will be done along this line. So, this is the delta v i initial required, delta v i initial impulse required to put it in this orbit and once it reaches here. So, it is going in this orbit along this direction while your velocity vector is along in this direction and therefore, you must be another impulse delta v f in this place.

So, that it comes in this orbit, but if you do this kind of maneuver. So, it is not necessary that all of them. Obviously, if here, if you give the impulse here in this place. So, the amount of energy that is required to send it in this orbit. Thereafter you give one impulse here in this place to place it in this orbit. So, the both the energies can be combined or the both the impulse can be combined. How much impulse is required? And here, also you can do the same kind of calculation, how much impulse is required here in this place to put it in this orbit.

In doing this kind of burn, so energy becomes very important factor because you do not want to spend large amount of energy. You would like to do efficient fuel management and you would like to do the minimum expense of the propellant. Such that you can keep the satellite in the orbit for a longer time, if the propellant is lost, if the propellant is not there, then later on you cannot change the orbit of the satellite over a period of time the satellite orbit drips in the low earth orbit, because of the j two component of the earth gravity, which is due to the j two gravity, it comes into the picture because of the non asphericity of the earth.

So, it rotates the orbit of the satellite. So, if say the orbit of the satellite is like this. So, because of j two it will turn like this and also there are various things involved like not only implications, but your orbit also rotates like this the nodal angle also changes. All this various factors are there because of aerodynamic drag then your semi major axis will change. So, all this things are to be taken into account.

So, if your energy spent is large means, you are going to lose all your propellant and the life of the satellite will simply go down. So, keeping the life of the satellite large that is keeping your satellite for a longer period you need to do efficient fuel management. Every maneuver has to be optimized such that, the fuel loss is minimum.

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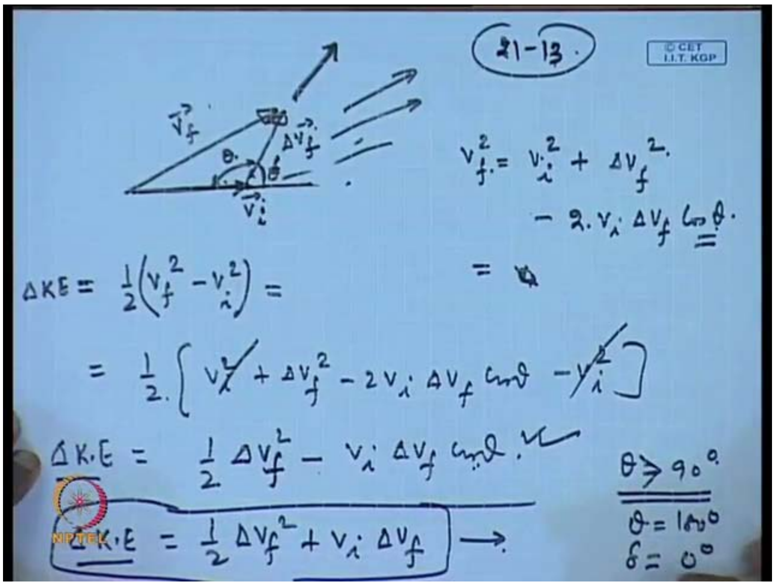

To find out for a given amount of impulse, what will be the change in kinetic energy? So, let us assume this is our v initial velocity and the amount of impulse which is available is delta v and delta v we apply at certain angle delta here. So, this is our v f the final velocity and this delta we applied. What will be the change in kinetic energy? So, change in kinetic energy will be given as delta K E 1 by 2 v f square. This is kinetic energy changed per unit minus v i square.

And this we need to find out for a given amount of impulse. So, now, this impulse given along this direction or along direction or whatever the direction, we have chosen right now. So, or either opposite to this, but in what situation for what value of this delta this in change in kinetic is going to be maximum. So, this v f can be written in terms of this angle theta. So, v f let us write in this place v f square this is equal to v i square plus delta v f square minus two v i times, delta v f cos theta and if we replace theta in terms of delta or either, we can work with theta itself here and write it this becomes 1 by 2 v i square plus delta v f square minus two v i times delta v f cos theta minus v i square.

So, this and this cancels out, what we get here, delta v f square 1 by 2 minus v i times delta v f cos theta. I can see from this place that the change in kinetic energy depends on this term because delta v f any how the delta v f magnitude v is fixed. We are taking this for a fixed amount of impulse. So, if theta equal to theta is more than 90 degree. This quantity is going to be positive. If you have theta equal to 180 degree at that time delta will be equal to 0 degree. So, then you will have this quantity as delta v f square plus v i times delta v f and this is the maximum amount of change in kinetic energy that you can register. You cannot exceed this value for a given amount of impulse. So, we will continue next time with this discussion. Thank you very much.